Course Proposal: Introduction to Likelihood Methods
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Motivation: Postgraduate students frequently encounter likelihood-based inferences in their studies. However, many of them are not familiar with the basic theory justifying the likelihood methods.

Goal: The aim of the course is to introduce postgraduate students to the basic parametric likelihood methods as well as the underlying basic theory so that they may gain insights into the likelihood methods widely applied to statistical inference problems.

Organization: Many people have a habit of turning first to likelihood methods, whenever an inference problem appears. This course will present the basic likelihood theory so that the students may gain insights into the likelihood methods applied in statistics. The classical theory of likelihood inference is based on first-order asymptotic theory, such as that given by the central limit theorem, along with a higher-order asymptotic theory based on Edgeworth series expansions. The emphasis of this course is on the development of statistical methods and a description of the underlying theory rather than on the statement and proof of precise mathematical results. Many examples will be presented in classroom to demonstrate the general theory . The textbook is “Likelihood Methods in Statistics” by Thomas Severini. This course will be organized into 16 sessions each lasting two hours.

session 1: parametric models, exponential family models, moments and cumulants, sufficiency, ancillarity, multivariate normal distribution.
sections 1.1, 1.2, 1.4, 1.5, 1.6, 1.7 of the textbook.

session 2: convergence modes, central limit theorem, Edgeworth series approximations, saddlepoint approximations of densities, stochastic asymptotic expansions.
chapter 1 of Ferguson (1996); sections 2.1, 2.2, 2.3, 2.4, 2.9 of the textbook.

session 3: likelihood function, Kullback-Leibler divergence, regular models, Bartlett identities, information inequality, partial information, orthogonal parameters, likelihood-based inference methods.
sections 3.1, 3.2, 3.4, 3.5, 3.6, 3.7 of the textbook.

session 4: consistency of maximum likelihood estimates (MLEs).
section 4.2 of the textbook and Wald (1949).

session 5: asymptotic distributions of MLEs, likelihood ratio statistic.
sections 4.2, 4.3 of the textbook.

session 6: the score and Wald statistics, profile likelihood function, nonregular models.
sections 4.4, 4.6 of the textbook.

session 7: higher-order asymptotic theory of MLEs.
sections 5.2, 5.3 of the textbook.

session 8: tutorial.
some problems in homework assignments; some exercises in chapters 1,2,3,4,5 of the textbook.

session 9: higher-order asymptotic theory of likelihood ratio statistic.
section 5.4 of the textbook.

session 10: conditional inference discussing log-likelihood sample space derivatives and conditional distributions of MLEs.
sections 6.1, 6.2, 6.3 of the textbook.

session 11: conditional inference discussing normalizing transformations and modified signed likelihood ratio statistic.
sections 7.2,7.3,7.4 of the textbook.

session 12: approximations to the modified signed likelihood ratio statistics (I).
section 7.5 of the textbook.

session 13: approximations to the modified signed likelihood ratio statistics (II).
section 7.5 of the textbook.

session 14: likelihood functions for a parameter of interest: conditional likelihood functions, marginal likelihood functions, integrated likelihood functions.
sections 8.2, 8.3, 8.4 of the textbook.

session 15: modified profile likelihood function.
sections 9.2, 9.3, 9.5 of the textbook.

session 16: tutorial.
some problems in homework assignments; some exercises in chapters 5, 6, 7, 8, 9 of the textbook; preparations for the final exam.

References:

[1] Severini, T.A. (2000). Likelihood Methods in Statistics. New York: Oxford University Press.

[2] Ferguson, T.S. (1996). A Course in Large Sample Theory. London: Chapman and Hall.

[3] Sen, P.K. and Singer, J.M.(1993). Large Sample Methods in Statistics. London: Chapman and Hall.

[4] Casella, G. and Berger, R.L. (1990). Statistical Inference. California: Duxbury Press.

[5] Wald, A. (1949). Note on the consistency of the maximum likelihood estimator. Annals of Mathematical Statistics 20, 595-601.

Schedule: This course will last 4 weeks with 4 sessions each week. There will be a short home work assignment each week. A taken-home exam will also be given after all sessions are finished.