Australian National University — Centre for Mathematics and its Applications (CMA)
Geoscience Australia — Canberra

Statistical Modeling of Extreme Values, 2008
Theoretical Assignment 4

1. If X has a generalized Pareto distribution, its distribution function is defined as

$$H(x;\gamma,\sigma) = 1 - (1+\gamma\frac{x}{\sigma})^{-\frac{1}{\gamma}}$$

where $\sigma > 0$ and $x > 0$ for $\gamma \geq 0$ and $0 < x < \sigma/|\gamma|$ for $\gamma < 0$. Let X_0 have standardized GP-distribution, i.e.,

$$H(x_0;\gamma,1) = 1 - (1+\gamma x_0)^{-\frac{1}{\gamma}}$$

with $\gamma > 0$.

(a) Calculate $E[(1+\gamma X_0)^k]$.

(b) Show that the formula obtained in a) is also true for $\gamma < 0$.

(c) Use the relation $X = \sigma X_0$ and the results in a-b) to calculate $E(X)$ and $V(X)$ for $\gamma \neq 0$.

(d) Find the method of moments estimators of γ and σ.

(e) Show that $\bar{H}(x) = (1+\gamma\frac{x}{\sigma})_+^{-\frac{1}{\gamma}}$ can be written as

$$\bar{H}(x) = \exp\{-\int_0^{x/\sigma}(1+\gamma y)_+^{-1}dy\}.$$

(f) Use the result in e) to show that

$$\frac{dH(x)/dx}{\bar{H}(x)} = (\sigma+\gamma x)^{-1}.$$

This is called hazard rate for GP-distributions.