1. TITLE OF PRESENTATION:

Constrained Statistical Inference in Parametric Models

2. NAMES AND ADDRESSES OF PRESENTERS:

Professor Mervyn J. Silvapulle

Department of Econometrics and Business Statistics,
Monash University, Melbourne, Australia 3145.

Tel: +61-3-9903-2237; +61-3-9903-1024. *Fax: +61-3-9903-2007*
Email: mervyn.silvapulle@buseco.monash.edu.au

3. ABSTRACT

Inequality constraints on parameters are encountered frequently in areas such as biostatistics, clinical trials and econometrics. For example, treatments may be ordered. Statistical methods for incorporating such constraints have advanced significantly over the past two decades. This course provides an introduction to this area. It starts with inequality constrained statistical inference [CSI] on multivariate normal mean, and then extends to cover linear regression models. The essentials for more general parametric models, such as generalized linear models, would also be discussed briefly.

Prerequisites:

The minimum requirement is statistical inference at the upper undergraduate level, but it would be more suitable for those with some graduate level training in mathematical statistics. A basic knowledge of the following would be assumed: inference on the mean of multivariate normal mean, linear algebra, the matrix approach to generalised least squares in linear regression, and the basics of projections to interpret GLS estimates/test statistics. Further, the second part of the course on general parametric models would assume familiarity with standard Taylor series type arguments used in the large sample theory of maximum likelihood. No prior knowledge of CSI is assumed.

4. BRIEF OUTLINE

The emphasis of the course is the *theory* of statistical inference when parameters are subject to order restrictions. The lectures will based on selected parts of the recent book, Silvapulle and Sen (2005), on Constrained Statistical Inference [CSI] published by J Wiley. The main emphasis will be inference on the mean of a multivariate normal distribution, and extension to cover general parametric models. The course notes in point form, with reference to the page numbers of the book, will be available.

Target audience:

PhD students and researchers in mathematical statistics. This course is suitable for those who are not familiar with CSI and wish to learn some of the important theoretical results and read journal papers on the topic.

Detailed outline:

The topics that may be covered are listed below, but the topics that will be covered may change slightly. It would be expected that the attendees will have access to the book, Silvapulle and Sen (2005). [The section numbers given within square brackets refer to those in the book, Silvapulle and Sen (2005)].

Part 1

1. Motivate CSI by briefly mentioning a range of examples where constraints on parameters arise, and the importance of incorporating such statistical information. [Section 1.2; pp 2- 23].

2. Detailed discussion of CSI in the simplest possible case, namely inference on the mean of a bivariate normal distribution with identity covariance matrix.
[Examples 3.3.1 and 3.3.2.; pp 63 – 66].

3. Extend the results in the previous part to the case when the mean is restricted to a cone [Example 3.3.3; pp 66-68].

4. Then extend these further to the case when the covariance matrix is not the identity matrix. To this end, make use of geometric interpretations of MLE and LRT in two-dimensions, use diagrams to provide an intuitive way of visualizing the MLE and LRT, and illustrate their connections to projections, similar to those in generalised least squares
[pp 69—72; ignore Example 3.3.4, use Figures 3.5, 3.6 and 3.7].

5. Introduce chi-bar square distribution in the general setting, and discuss methods of computing chi-bar square weights and the tail probabilities of chi-bar square distributions. [Section 3.4; pp 75 – 78; state Proposition 3.4.1 and Theorem 3.4.2; Section 3.5 (ignore Simulation Methods 2 and 4), pp 78 – 81.]

Part 2

1. Likelihood Ratio Test [LRT] for Type A problems (i.e. test of linear equality against linear inequality) on the mean of a multivariate normal with known covariance matrix.
[Corollary 3.7.2; Figure 3.10; pp 85--86].

   Use this to introduce a standard technique for constructing tests for Type A testing problems on a parameter when it has an asymptotically normal estimator.
[Proposition 3.7.3, (equation 3.33), Example 3.7.1; pp 86—89]

2. The linear model. Use the foregoing results to develop corresponding CSI results for the regression parameter of the linear model. Illustrate by a worked example.

   [Section 3.9; pp 95 –97; 98—99; Corollary 3.9.3 (a) and the connection to Corollary 3.7.2].

3. Recall several asymptotic results for unconstrained maximum likelihood inference.
   [Section 4.2; pp 145—148; Propositions 4.2.1 and 4.2.2].

4. LRT of linear equality against linear inequality hypotheses in a general regular parametric model when the observations are iid. Extend these to regression type settings.
   [pp 148—149; pp156—157].

Part 3

1. Illustrate the foregoing important general results using worked data examples.
   [Section 4.3.5; Examples 4.3.1 and 4.3.2].

2. Tests other than LRT: Distance and quasi-likelihood ratio tests [QLRT].
   [D in equations 4.22 and 4.23 on page 154; QLRT in equation 4.32 on page 159; and the statistic D in equation 4.33 on page 159].

3. A generalization of Rao's score test against inequality constraints.
   [The tests in equations 4.75 and 4.81; Sections 4.6.1 and 4.6.2; pp 175—178.]

4. Illustrate the foregoing one-sided score test using a worked example.
   [Worked Example in section 4.6.6, pp 181—183].