Behavior of Extreme Dependence between Stock Markets when the Regime Shifts

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ABSTRACT

We propose a methodology based on multivariate extreme value theory, to analyze the dependence between markets during the financial crisis. We argue that extreme dependence based on block maximum is a more appropriate measure to study dependence between stock markets, when a regime shifts, than other alternatives. With this methodology, we are able to detect the increase in the extreme dependences between US and other markets during the 2008 financial crisis where traditional approaches fail to do so. In addition, the estimated dependent function allows one to quantify maximum impact of the crisis on each individual market. We then propose the use of a conditional loss distribution as a constructive tool for a stress test analysis in risk management study. Stress test levels with respect to 2008 financial data calculated from the conditional loss distribution are given.

Keywords: Extreme Value Distribution, Extreme Dependence, Risk Management, Stress Test

1. Introduction

We attempt to estimate the extreme market dependence between the US stock market index return and those of other developed and emerging markets, and show that the extreme dependences increase, when the markets enter the 2008 financial crisis. The stock markets under this study are divided into two groups: (1) developed stock markets consisting of the United Kingdom, Japan, Australia, and Hong Kong; (2) emerging stock markets consisting of Mexico, Brazil, South Korea and Thailand. Following Beltratti and Stulz (2009), we divide the observation into two periods:

- Period 1 (the period before the financial crisis), from July 2006 to June 2007;
- Period 2 (the period during the financial crisis), from July 2007 to December 2008.

The raw data we obtain are the series of closing daily indices of the stock market, in the countries under this study. Under a volatility clustering model such as GARCH, a series of indices is not stationary, but the corresponding series of returns usually is. Therefore, for each series of indices, we transform it into log returns to make the series stationary. For Japan, Korea, Australia and Thailand, the series are shifted by one day to adjust for different time zones. Figure 1 shows examples of the time series of US and Hong Kong stock index returns, from July 2006 to December 2008.

In Figure 1, for each country, the square marks show the stock returns in the period before the 2008 financial crisis, while the circular marks show the stock returns in the period during the 2008 financial crisis. It is obvious that the volatilities of the indices of the two markets increase during the crisis. This can be explained by the financial crisis causing instability in the US stock market. Since the Hong Kong economy had close ties with that of the US, the effect of the crisis inevitably spread to Hong Kong, and undermined the stability of the Hong Kong stock market, causing high volatility in the Hong Kong index. The contagion of the crisis really exists, but the mechanism of the contagion is less known. Systematic contagion theory in banking network is being developed (see e.g. Müller (2006), Amini et al. (2011) and Kley et al. (2014)). There is a hypothesis that the dependence between the markets increases during the crisis, accelerating the spread of the instability.

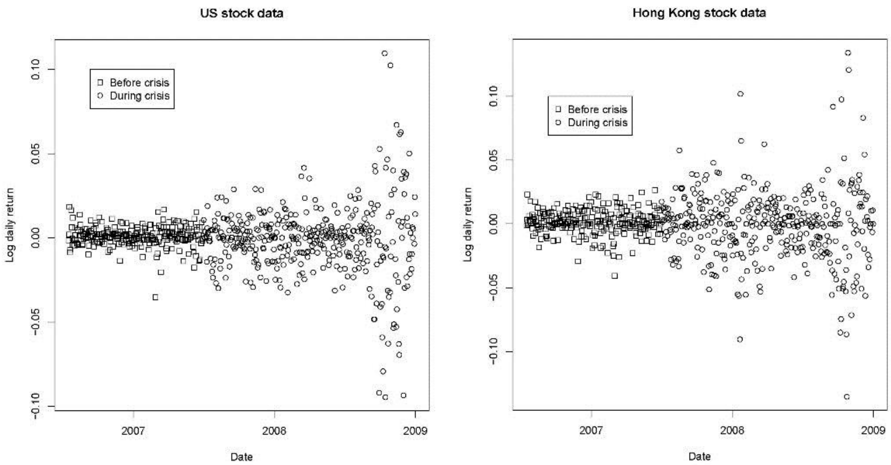

Figure 1: US and Hong Kong stock index returns from July 2006 to August 2008. The square marks show the stock returns in the period before the 2008 financial crisis. The circular marks show the stock returns in the period during the 2008 financial crisis.

However, based on the market index data, a simple dependence measurement, such as a sample correlation of index returns, is unable to provide supporting evidence for this hypothesis. For example, Figure 2 shows the scatter plots between the US and Hong Kong before and in the crisis. The correlation between the US and Hong Kong before the crisis is equal to 0.423, while the correlation in the crisis is 0.379. The sample correlation of the index returns decreases in the time of crisis. This seems to contradict the hypothesis we laid out.

We observe that the simple analysis based on index return correlations is inappropriate in at least 3 aspects.

S1: Summarizing the dependence from the whole set of data into one single number, one discards the dependence structure of the variables. Even worse, a sample correlation measures only the linear dependence, ignoring other types of dependence structure

S2: Analyzing the whole set of data simultaneously, one obscures the dependence structure at different quantiles of the variables. In this specific study, we are interested in the dependence between markets in

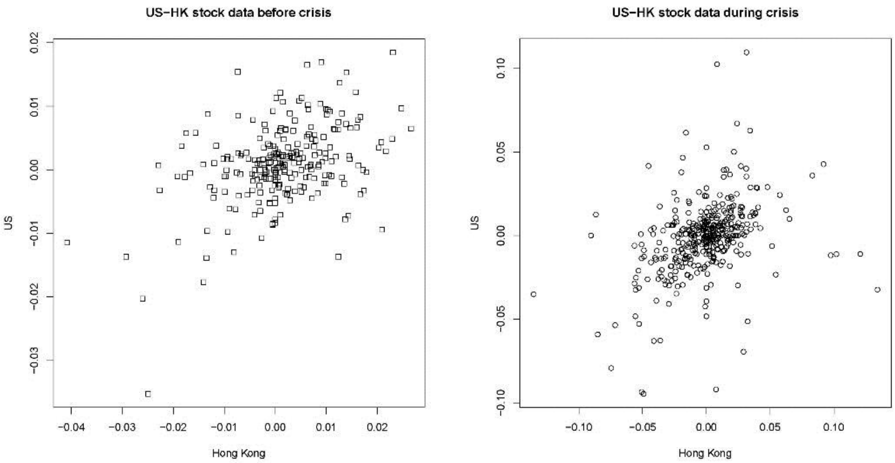

Figure 2: Scatter plots of US-Hong Kong returns before and in crisis. Left panel shows the scatter plot of the returns in the period before the 2008 financial crisis. Right panel shows the scatter plot of the returns in the period during the 2008 financial crisis.

crisis. Therefore, the market data in the low quantile are more relevant. In fact, the extreme dependence between markets should be investigated.

S3: Pairing the data by date, one assumes the synchronization of events between the crisis origin and the destination. Since the mechanism of a crisis transmission can be complex, the observation of the crisis event in the destination market may not be synchronized with the corresponding event in the origin market.

More sophisticated data analysis techniques can be employed to remove some shortcomings found in the above simple correlation analysis. Quantile regression Koenker (2005) has been used by Adrian and Brunnermeier (2008) to estimate quantiles of a system market risk factor conditional on each of its components. They define CoVaR or the conditional value at risk to measure the systemic risks contributed by the system components. Since this technique provides a complete distribution of a variable conditional on another variable, it does not suffer from the shortcoming S1. However, with its linear structure, the model assumes that, at a specific quantile of the dependent variable, the dependent structure between the dependent variable and the independent variable is the same across all quantiles of the independent variable. Specifically, for example, conditional 99 percentile of the dependent variable has the same regression coefficient for all quantiles of the independent variable. This causes the dependence at the high quantiles of the independent variable to be averaged out by the dependence at the low quantiles of the independent variable. Therefore, the technique does not provide accurately the extreme dependence between variables, and it does not fully remove the shortcoming S2.

Poon et al. (2004), Hartmann et al. (2004) and Zhou (2010) provide frameworks for applying multivariate extreme value theory to financial data. Zhou (2010) proposes the systemic impact index (SII) to measure the systemic risk in financial institutions, serving the same purpose as that of CoVaR. These frameworks of the multivariate extreme value theory are the frameworks that we found appropriate to base our analysis on. However, these studies apply the techniques to the co-exceedance of the market data, which require the data to be paired day by day. This synchronization assumption on the market data is the assumption that we found inappropriate for our purpose. We are interested in the extreme dependence between markets, when the markets enter the 2008 financial crisis, and we believe that market interdependence is so complex that the crisis transmission does not take effect synchronously between markets. Therefore, these studies still suffer from the shortcoming S3.

In our methodology, we measure the extreme dependence from block minima. Block minima (maxima) is a method to select extreme data from the original data. The process begins with grouping the original data into blocks of an equal size and selecting the minimum (maximum) in each block. Analyzing the block minima (maxima) enables us to focus our analysis only on the data that is exceptionally low (high), which is consistent with our objective of study. This eliminates the shortcoming S2. Besides, the block minima (maxima) approach requires no synchronization of events, so this measurement does not suffer the shortcoming S3. As a demonstration, we perform a correlation analysis based on the block maxima of negative returns. Figure 3 shows the scatter plots of weekly maxima between the US and Hong Kong before and during the crisis. The correlation between the US and Hong Kong before the crisis is equal to 0.517, while the correlation in the crisis is 0.664. The correlation of the block maxima of the negative returns increases in the time of crisis significantly, which is the opposite of that indicated by the correlation analysis of the returns.

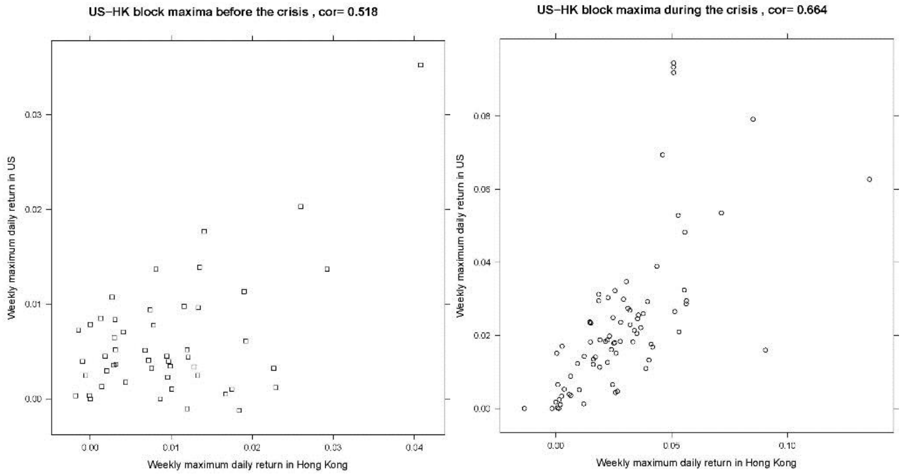

Figure 3: Scatter plot of US-Hong Kong block maxima returns before and during the crisis. Left panel shows the scatter plot of the block maxima in the period before the 2008 financial crisis. Right panel shows the scatter plot of the block maxima in the period during the 2008 financial crisis.

Even though employing block maxima eliminates the shortcomings S2 and S3, measuring only correlation does not solve S1. In this study, we instead analyze the extreme dependence of the block maxima of the negative returns between the US market and other markets, based on the multivariate extreme value theory Hall and Tajvidi (2000, 2004). The methodology enables us to elicit the whole dependence structure of the variables, eliminating S1. With this theory applied to block maxima, we remove all of the three shortcomings mentioned above. We show that the extreme dependence increased considerably when the financial markets moved into the 2008 crisis. The methodology also enables us to estimate the conditional distribution of a block maximum of a return conditional on that the market is in crisis. This conditional distribution suggests a financial stress test design that is consistent with the traditional value at risk framework. Because the dependence increases when the financial system enters a crisis, we suggest that not only the financial risk factor be stressed, but also the dependence structure be stressed when one performs a financial stress test analysis.

This study proceeds as follows. In Section 2, we present a brief introduction to extreme value theory. The goal is to outline the part of the theory that is used in our analysis. Section 3 contains the main results of our analysis on how the dependence between markets shifts during a crisis. Section 4 discusses an application of the methodology in stress test analysis. We end with final remarks and conclusions in Section 5.

2. Extreme Dependence

Extreme value theory is a method of explaining the behavior of extremes. The main theorem indicates that, for independent and identically distributed observations, the normalized maximum converges in distribution to the so called generalized extreme value (GEV) distribution. Specifically, let $X_1, X_2, \ldots, X_n$ be a sequence of iid random variables with continuous distribution function $F(x)$. Further, let $M_n = \max(X_1, X_2, \ldots, X_n), n \in N$ and suppose normalizing constants $a_n > 0$ and $b_n \in \Re$ are such that

$$\lim_{n\to\infty} P\left(\frac{M_n - b_n}{a_n} \le x\right) = \lim_{n\to\infty} F^n(a_n x + b_n) = G(x)$$

where $G(x)$ is non-degenerate. The main result indicates that $G(x)$ is in a family of distribution called the generalized extreme value (GEV) distribution as follows:

$$G(x;\gamma,\mu,\sigma) = \exp\{-(1+\gamma\frac{x-\mu}{\sigma})_+^{-\frac{1}{\gamma}}\}$$

where $\sigma > 0$, μ and $\gamma \in \Re$ (see Leadbetter et al. (1983) and Resnick (2007) for more details on these results).

The observations relevant to the present problem are negative log returns of stock market indices, and the theorem explains the behavior of these maximum negative log returns. On the other hand, if one is tempted to consider instead only those returns that go beyond some critical point, a threshold model can be applied. For independent and identically distributed observed values, when the critical point or the threshold is high enough, peak-over-threshold theorem states that the distribution of values beyond the threshold can be approximated by the generalized Pareto distribution. These theories open ways for risk measuring and monitoring involving exceptionally high losses and extremes. More sophisticated tools in the extreme value theory are devised and applied to phenomenon in financial markets, and create an advance in risk management methodology. Novak and Beirlant (2006) demonstrates how modern extreme value theory can help predict the size of a financial market meltdown. Ferrari and Paterlini (2009) created a new method based on MLqE for quantile estimation, essential for risk management purposes. However, the researches based on these theories study only the behavior of extremes of individual markets. Often time, the comovement between markets is more relevant. Less is known when it comes to the behavior of co-movement or dependence of the extremes between two markets or more. From a theoretical point of view, this requires extending the univariate extreme value theory to higher dimensions.

Multivariate extreme value distributions arise in connection with extremes of a random sample from a multivariate distribution. There are several possibilities for ordering multivariate data. For multivariate extreme values, the most widely used method is the so called marginal or M-ordering. As the name suggests, ordering or ranking here takes place within one or more of the marginal samples. Thus the maximum of a set of vectors is defined by taking
component-wise maxima.

Let $\{\mathbf{X}_n, n \geq 1\} = \{(X_n^{(1)}, X_n^{(2)}, \ldots, X_n^{(d)}), n \geq 1\}$ where $\mathbf{X} \sim F(\mathbf{x})$ iid and F is a d-dimensional distribution function. As mentioned above, we define the maxima as

$$\mathbf{M}_n = (M_n^{(1)}, M_n^{(2)}, \ldots, M_n^{(d)}) = (\overset{n}{\underset{j=1}{\vee}} X_j^{(1)}, \overset{n}{\underset{j=1}{\vee}} X_j^{(2)}, \ldots, \overset{n}{\underset{j=1}{\vee}} X_j^{(d)})$$

In a similar manner to the univariate case, suppose we can find the normalizing constants $\sigma_n^{(i)} > 0$, $u_n^{(i)} \in \Re$ for each margin, such that as $n \to \infty$ we obtain:

$$\begin{aligned} &P[(M_n^{(i)} - u_n^{(i)}) / \sigma_n^{(i)} \leq x^{(i)}, 1 \leq i \leq d] = \\ &F^n(\sigma_n^{(1)} x^{(1)} + u_n^{(1)}, \ldots, \sigma_n^{(d)} x^{(d)} + u_n^{(d)}) \to G(\mathbf{x}) \end{aligned}$$

assuming G_i and G are non-degenerate. In this case, the limiting distribution is called a multivariate extreme value distribution (MEVD). It follows directly from the univariate results that the marginal distributions are necessarily of GEV, and so are determined by a finite number of parameters. However, the link between marginals can only be characterized by a certain measure on a simplex. A consequence of this is that there is no natural parametric family with finite number of parameters for multivariate extreme value distributions. For brevity, here we present the characterization for the bivariate case and refer the interested reader to the literature mentioned in the bibliography for the general results.

As mentioned above, the marginal distributions in MEVD are necessarily of GEV. Assuming a unit Fréchet distribution, a specific form of GEV, for the margins, i.e., $G_i(x) = \exp(-1/x)$, for $x \geq 0$, it can be shown that all the bivariate extreme value distributions, denoted by G_* below, can be written as

$$G_*(x,y) = e^{-\mu_*[\mathbf{0},(x,y)]^c}$$

where

$$\mu_*[\mathbf{0},(x,y)]^c = (\frac{1}{x}+\frac{1}{y})A(\frac{x}{x+y})$$

and

$$A(w) = \int_0^1 \max\{q(1-w),(1-q)w\}S(dq).$$

The only requirement for measure S is that

$$\int_0^1 qS(dq) = \int_0^1 (1-q)S(dq) = 1.$$

The function $A(w)$ is called a dependence function and has to satisfy the following conditions:

- $A(0) = A(1) = 1$
- $\max\{w,1-w\} \leq A(w) \leq 1$
- $A(w)$ is convex for $w \in [0,1]$.

This implies that, for statistical inference, one can either try to estimate the measure S or function $A(w)$ non-parametrically, or one can assume a specific parametric sub-family of bivariate extreme value distributions and estimate the parameters by a method such as maximum likelihood.

Some advances in this area of extreme dependence include, for example, Hall and Tajvidi (2000, 2004) and Christiansen and Ranaldo (2009). Hall and Tajvidi (2000) introduced two methods for estimating dependence function of bi-variate extremes. Hall and Tajvidi (2004) suggested a mixture of parametric and non-parametric approaches for constructing a prediction region of bi-variate extremes. These techniques have been applied successfully to environmental data. Therefore, if adjusted appropriately, one would be able to apply this bi-variate extreme value theory to financial data and to identify the behavior of extreme dependence between different financial markets.

3. Extreme Dependence between Markets

From the bivariate extreme value theory, the dependence between two extreme variables can be expressed as a dependence function. The dependence function enables one to inspect extreme dependence graphically.
Furthermore, it provides the complete information required for reconstructing the whole bivariate distribution of a pair of extreme variables.

The graph of a dependence function is convex, passing through coordinates $(0,0)$ and $(1,0)$ and contained within the triangle ∇ formed by $(0,0)$, $(1,0)$ and $(1/2, 1/2)$. If the graph of a dependence function lies along the boundary at the top of the triangle ∇, the two extreme variables are independent. If it lies along the v-shape lower boundary of the triangle ∇, the two variables are perfectly dependent. A graph of a dependence function with a high curvature towards the v-shape lower boundary represents a strong dependence structure. A flat graph represents a weak dependence structure. One way to summarize the dependence based on a dependence function is to measure the area of the convex set formed by the graph of the dependence function and the upper boundary of the triangle ∇. When comparing dependences between two dependence functions, the one with greater area can be regarded as the one with stronger dependence.

As an example of application of the theory below, we analyze the dependence between weekly maxima of the negative returns between the US and Hong Kong markets. The characterization of bivariate extreme value distributions presented in the previous section requires transforming the observations to unit Fréchet margins. We use maximum likelihood to estimate parameters of GEV for each margin. For the period before crisis we obtained $\hat{\gamma}_x = -0.231$, $\hat{\sigma}_x = 0.003$, $\hat{\mu}_x = 0.003$, $\hat{\gamma}_y = -0.126$, $\hat{\sigma}_y = 0.006$ and $\hat{\mu}_y = 0.005$, where subscripts x and y denote the US and Hong Kong, respectively. The corresponding estimates for the period after crisis are $\hat{\gamma}_x = -0.196$, $\hat{\sigma}_x = 0.012$, $\hat{\mu}_x = 0.013$, $\hat{\gamma}_y = -0.013$, $\hat{\sigma}_y = 0.018$ and $\hat{\mu}_y = 0.018$. The left panel of Figure 4 shows the weekly maxima for these markets before the crisis after transforming the margins to unit Fréchet distribution. In the right panel, the same data is depicted in log-scale to separate observations. Figure 5 shows the corresponding plots for the period during the crisis.

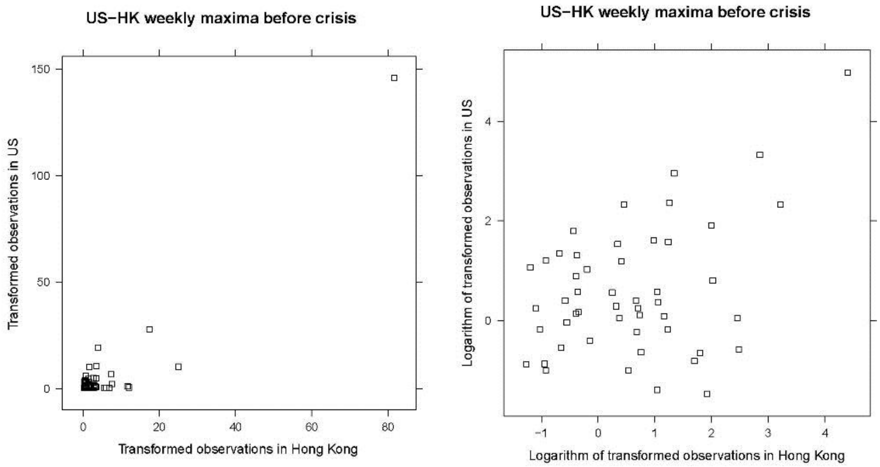

Figure 4: US-Hong Kong block maxima returns before the crisis

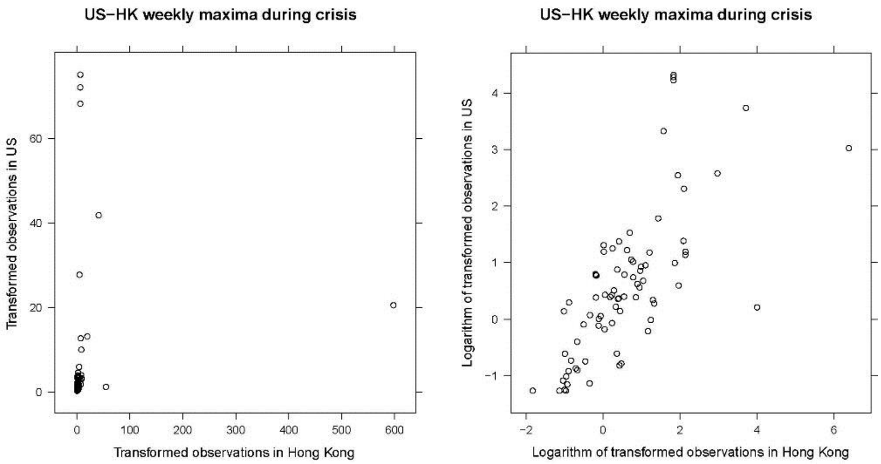

Figure 5: US-Hong Kong block maxima returns during the crisis

Although the visual impression of these figures indicates that the dependence structure has been changed in these two periods, we will quantify this by estimating dependence functions. We fitted several parametric models to the dependence function, and also computed some non-parametric estimators of the dependence functions.

The parametric models were:

1. The mixed model: $A(w) = \theta w^2 - \theta w + 1, \quad 0 \le \theta \le 1$
2. The logistic model: $A(w) = \{(1-w)^r + w^r\}^{1/r}, \quad r \ge 1$
3. The asymmetric mixed model:

$$A(w) = \phi w^3 + \theta w^2 - (\theta + \phi)w + 1, \quad \theta \geq 0, \theta + 2\phi \leq 1, \theta + 3\phi \geq 0$$

4. The asymmetric logistic model:

$$A(w) = \{(\theta(1-w))^r + (\phi w)^r\}^{1/r} + (\theta - \phi)w + 1 - \theta, \quad \theta \geq 0, \phi \leq 1, r \geq 1$$

5. The generalized symmetric mixed model:

$$A(w) = \{(1-w)^p + w^p + k((1-w)w)^{p/2}\}^{1/p}, \quad 0 \leq k \leq 2(p-1), p \geq 2$$

6. The generalized symmetric logistic model:

$$A(w) = 1 - \frac{k}{\{(1-w)^{-p} + w^{-p}\}^{1/p}}, \quad 0 \leq k \leq 1, p \geq 0$$

The asymmetric families have been developed by Tawn (1988) and the latter two models have been proposed by Tajvidi (1996). In addition to these parametric models, we also computed a few nonparametric estimators of the dependence functions. This includes Pickands estimator, smoothed splines estimator of dependence function proposed by Hall and Tajvidi (2000) and the estimator developed by Caperaa et al. (1997).

Figure 6 shows the estimates of dependence function for all logistic and nonparametric models for weekly maxima returns before crisis. Corresponding estimated dependence functions for those data during crisis are depicted in Figure 7. As a measure of the strength of dependence, one can use $d = 2(1 - A(1/2))$, the scaled depth of the dependence function. Note that $d \in [0,1]$ where the lower limit implies independence, and the upper limit is reached when there is a complete dependence between observations. Table 1 summarizes the results:

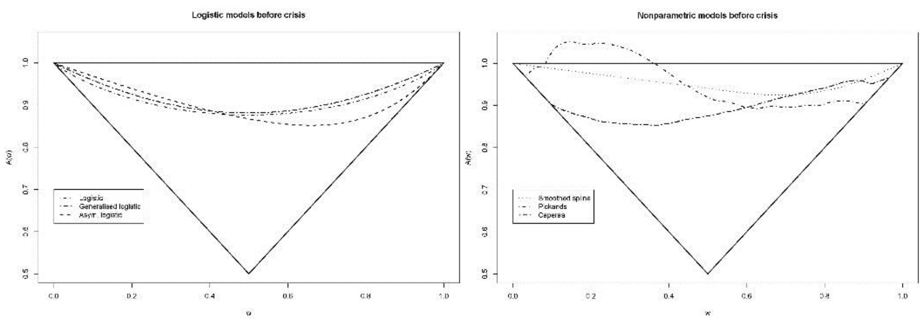

Figure 6: Dependence functions for US-Hong Kong returns before the crisis. The left panel shows the logistic models and the right panel depicts non-parametric estimators.

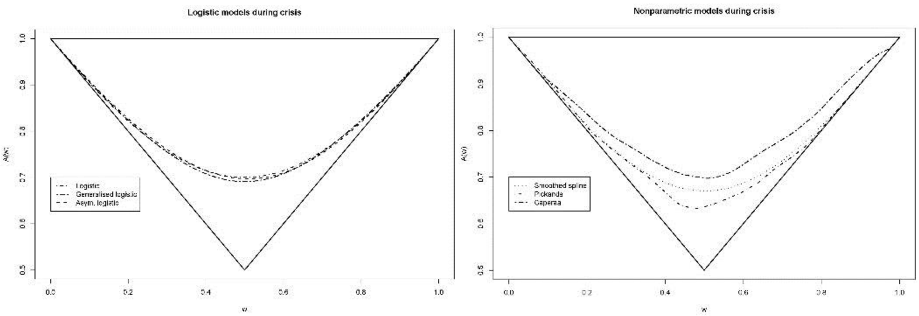

Figure 7: Dependence functions for US-Hong Kong returns during the crisis. The left panel shows the logistic models and the right panel depicts non-parametric estimators.

Table 1: Parameter estimates for the logistic and nonparametric models. $\hat{d}$ is the estimate of d , the scaled depth of the dependence function.

Model	Before crisis	During crisis
Logistic	$\hat{r} = 1.2351, \hat{d} = 0.2472$	$\hat{r} =$ 0.20592, $\hat{d} =$ 0.5998
Asymmetric logistic	$\hat{\theta} = 1.000, \hat{\phi} = 0.3180,$ $\hat{r} = 1.9614, \hat{d} = 0.2655$	$\hat{\theta} = 1.000, \hat{\phi} = 0.9436,$ $\hat{r} = 2.1703, \hat{d} = 0.6055$
Generalized logistic	$\hat{k} = 1.1077, \hat{p} = 2.00,$ $\hat{d} = 0.2371$	$\hat{k} =$ 1.0191, $\hat{p} =$ 3.4143, $\hat{d} = 0.6179$
Pickand	$\hat{d} = 0.16001$	$\hat{d} = 0.72876$
Smoothing splines	$\hat{d} = 0.11906$	$\hat{d} = 0.65923$
Caperaa	$\hat{d} = 0.24951$	$\hat{d} = 0.60403$

In Tawn (1988), the author suggests a method to test for independence between margins based on score statistics. We applied this method to the data both before and during crisis and, in both cases, p-values were practically 0. For non-parametric independence functions, we applied the test proposed by Caperaa et al. (1997) based on Tawn's measure of association. This is denoted by $\hat{d}$ in Table 1, and the results were similar. It should be mentioned that all mixed models were also fitted to the data, but the results were similar to those presented above.

As discussed in Section 2, any dependence function corresponds to a distribution function for a bivariate extreme value distribution. This in turn defines a bivariate density function. Of course, any of the estimated dependence functions above can be used to estimate corresponding distribution and density functions. Figure 8 shows the density function and its contour plot for returns during crisis, based on the smoothing splines method. The advantage is that one can construct compact prediction regions for returns by using density function.

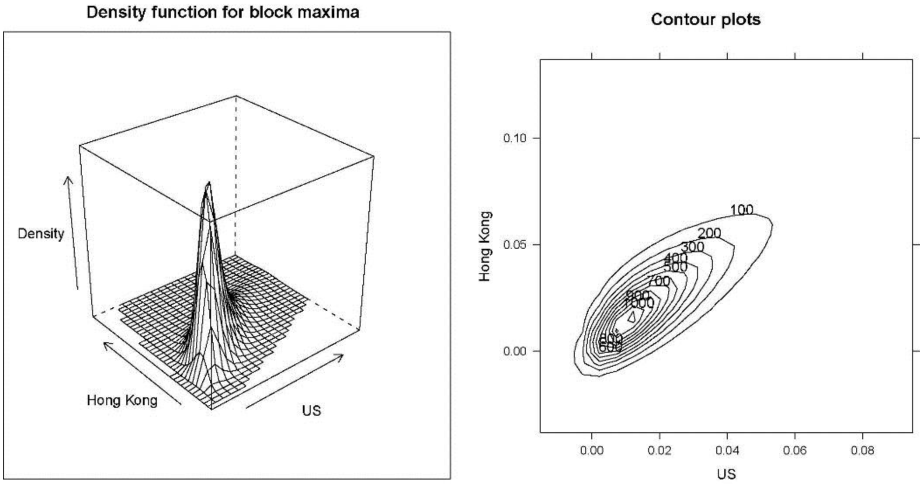

Figure 8: US-Hong Kong block maxima density function and contour plots during the crisis. Left panel shows the block maxima density function. Right panel depicts the contour plots of the block maxima.

It should be emphasized that we also analyzed the dependence between the US stock market index, plus some other developed markets and emerging markets. This included developed markets such as the UK, Australia and Japan. Even the dependence between the US and some emerging markets such as Thailand, Korea and Mexico was analyzed. For brevity, these results are not reported here but, in all cases, the above pattern was evident in all analyses. That is the dependences between the block maxima of the negative returns between markets in the crisis are higher than those before the crisis.

Table 2 summarizes the dependence measurements based on the correlation and the extreme dependence. Observe that the correlation analysis on paired log returns can capture the increase in the dependence only in some cases. However, the extreme dependence can capture the increase in all cases.

Table 2: A summary of the two dependence measurements when applied to cross market data: correlation based on pair log returns; and $\hat{d}$, the scaled depth of the dependence function, based on block maxima of negative returns.

Pair	Correlation (Range = [-1,1])		$\hat{d}$ (Range = [0,1])	
	Before crisis	During crisis	Before crisis	During crisis
US-UK	0.436	0.501	0.286	0.603
US-Australia	0.509	0.642	0.164	0.551
US-Japan	0.417	0.602	0.144	0.659
US-Hong Kong	0.423	0.379	0.118	0.728
US-Brazil	0.758	0.728	0.380	0.668
US-Mexico	0.736	0.773	0.490	0.700
US-Korea	0.416	0.364	0.165	0.594
US-Thailand	0.150	0.274	0.125	0.549

4. An Application of Extreme Dependence

For risk management purposes, it is sometime more relevant to calculate distribution of return in one market conditioned on the return in another market. The dependence function estimated in the previous section enables one to calculate a conditional distribution. As an example, different panels of Figure 9 shows the related densities and distributions for the US market, and also conditional densities and distributions for Hong Kong, given that the return in the US is 0.06.

The conditional distribution paves the way for a stress test design that is consistent with traditional value at risk measurement. Recall that 100(1-α)% value at risk is defined as the 100(1-α)% quantile of the loss distribution. To make a stress test consistent with the value at risk paradigm, we propose to measure the loss under stress to be the 100(1-α)% quantile of the loss distribution, conditioned on a major market under stress.

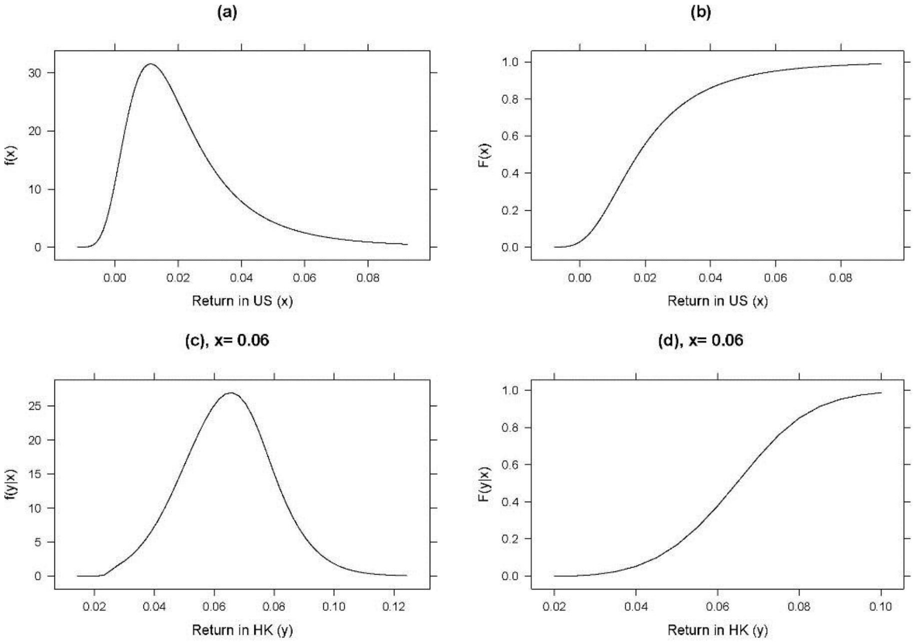

Figure 9: Panels (a) and (b) show the density and distribution functions for maximum weekly returns in the US during the crisis. Panels (c) and (d) depict the conditional density and distribution functions of returns in Hong Kong given that the return in the US is 0.06.

To be explicit, let X_1 and X_2 be the block maxima of the negative returns of the US market and the market under study, respectively. The loss under stress is defined as the value l such that

$$P(X_2 > l \mid X_1 > t) = \alpha \tag{1}$$

where α is implied from the confidence level in the adopted value at risk standard, and t is a stress parameter. The parameter t expresses the threshold, where its exceedance indicates stress condition. Assume that X_1 and X_2 have marginal distributions F_1 and F_2, respectively. Equation (1) can be equivalently rewritten as

$$P(F_2(X_2) > q_{p,\alpha} \mid F_1(X_1) > t) = \alpha \tag{2}$$

where $q_{p,\alpha} = F_1(l)$ and $p = F_2(t)$.

To make the connection with dependence measurement in the extreme value literature, recall that the coefficient of (upper) tail dependence λ of X_1 and X_2 is defined as Embrechts et al. (2002)

$$\lambda = \lim_{q \to 1-} P(X_2 > F_2^{-1}(q) \mid X_1 > F_1^{-1}(q)).$$

The measurement $q_{p,\alpha}$ in (2) answers a question opposite to that of λ. While λ tells us the dependence at the high quantile of X_1 and X_2, $q_{p,\alpha}$ tells us at what (supposedly high) quantile the tail dependence emerges.

Table 3 contains $q_{0.95,\alpha}$ at different levels of α, where X_1 is the block maximum of US's negative index returns, and X_2 varies across different markets. The table can be employed to determine a stress test level, based on the 2008 financial crisis scenario.

As Table 3 suggests, to perform a stress test in the Hong Kong market based on 2008 financial crisis when 99% value at risk is adopted, one should stress the loss at its 99.93% quantile. This level of loss is considerably higher than the value at risk level, which is at 99%, and significantly higher than the conditional loss level in a normal time, which is at 99.30%. The increase in loss is due to (i) the condition of the US market under stress and (ii) the increase in the dependence structure between the Hong Kong market and the US market. The proposed stress test design implies that a stress test should be performed not only in the distribution, but also in the dependence structure.

Table 3: $q_{0.95,\alpha}$ the proposed stress test levels based on conditional loss distribution, at different level of confidence levels. The levels are computed with respect to the dependence functions before the crisis, and those during the crisis, to show the contrast caused by the differences in dependence structure.

Market	Before crisis 100(1-α)%			During crisis (100(1-α)%		
	90%	95%	99%	90%	95%	99%
UK	0.9586	0.9813	0.9971	0.9879	0.9949	0.9992
Australia	0.9386	0.9695	0.9940	0.9882	0.9952	0.9992
Japan	0.9340	0.9682	0.9938	0.9913	0.9960	0.9993
Hong	0.9297	0.9649	0.9930	0.9919	0.9961	0.9993

Kong						
Brazil	0.9726	0.9880	0.9977	0.9889	0.9947	0.9990
Mexico	0.9810	0.9914	0.9986	0.9912	0.9959	0.9993
Korea	0.9383	0.9703	0.9946	0.9873	0.9942	0.9990
Thailand	0.9312	0.9656	0.9932	0.9810	0.9905	0.9981

5. Conclusion

In this paper, we study how the dependence between different markets changes during a crisis. We use data from the recent financial crisis and fit bivariate extreme value distributions to maximum weekly returns, in both developed and emerging markets. Our analysis shows that it is indeed the case, that dependence between certain markets during crisis is much stronger than during "normal" times. We argue that using a measure such as correlation could be misleading for studying the relationship in these circumstances.

As a practical guidance for risk managers, we discuss in detail how extreme value theory can be used to estimate the joint distribution of weekly maximum returns. We also suggest that conditional densities and conditional distribution functions can be seen as powerful tools for designing a stress test analysis.

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