Behavior of Extreme Dependence between Stock Markets when Regime Shifts

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Abstract

In the past five years the world has experienced the worst recession of the last six decades with far-reaching impact on many people's lives. The world economy has become more uncertain and more volatile. Due to unprecedented nature of the recent financial crisis, the validity of old models to assess risks has been questioned. Risk managers face new challenges as to how to assess implications of these new circumstances and how to tackle similar crises in future.

Against this background we propose a methodology based on multivariate extreme value theory to analyze the increase in dependence between markets during crisis. We argue that extreme dependence based on block maximum is an appropriate measure to study dependence between stock market indexes when regime shifts. We also show that in the 2008 financial crisis, the extreme dependences between US and other markets increased significantly from those before the crisis. This insight allows one to design a reasonable stress test scenario based on 2008 financial crisis. It is also possible to use these results to quantify maximum impact of the crisis on each individual stock market.

1 Introduction

We attempt to estimate the market extreme dependence between the US stock market index return and those of the other developed and emerging markets, and show that the extreme dependences increase when the markets
enter the 2008 financial crisis. We obtained the stock market index data from Thomson Reuters Datastream. The stock markets under this study are divided into two groups: (1) developed stock markets consisting of the United Kingdom, Japan, Australia, and Hong Kong and (2) emerging stock markets consisting of Mexico, Brazil, South Korea and Thailand. Following Beltratti and Stulz [2], we divide the observation into two periods:

- Period 1 (the period before the financial crisis), from July 2006 to June 2007;
- Period 2 (the period during the financial crisis), from July 2007 to December 2008.

The raw data we obtain are the series of closing daily indices of the stock market in the countries under this study. For each series of indices, we transform it into log returns to make the series stationary. For Japan, Korea, Australia and Thailand, the series are shifted by one day to adjust for different time zones. Figure 1 show examples of the time series of US stock index returns and Hong Kong stock index returns from July 2006 to December 2008.

![US stock data](image1)

![Hong Kong stock data](image2)

Figure 1: US and Hong Kong stock index returns from July 2006 to August 2008

In Figure 1, for each country, the square marks show the stock returns in the period before the 2008 financial crisis, while the circular marks show the stock returns in the period during the 2008 financial crisis. It is obvious that the volatilities of the indices of the two markets increase during the crisis. This can be explained by that the financial crisis caused instability in the US stock market. Since Hong Kong economy had a close tie with that
of the US, the effect of the crisis inevitably spread to Hong Kong and also undermined the stability of Hong Kong stock market, causing high volatility in the Hong Kong index. Knowing the contagion of the crisis really exists, the mechanism of the contagion is less known. There is a hypothesis that the dependence between the market increases during the crisis, accelerating the spread of the instability. However, based on the market index data, a simple dependence measurement such as a sample correlation of index returns is unable to provide a supporting evidence to this hypothesis. For example, Figure 2 shows the scatter plots between US and Hong Kong before and in the crisis. The correlation between US and Hong Kong before the crisis is equal to 0.423, while the correlation in the crisis is 0.379. The sample correlation of the index returns decreases in the time of crisis. This seems to contradict the hypothesis we laid out.

Figure 2: US-Hong Kong returns before and in crisis

We observe that the simple analysis based on index return correlations is inappropriate in at least 3 aspects.

**S1:** Summarizing the dependence from the whole set of data into one single number discards the dependence structure of the variables. Even worse, a sample correlation measures only the linear dependence, ignoring other types of dependence structure.

**S2:** Analyzing the whole set of data simultaneously obscures the dependence structure at different quantiles of the variables. In this specific study, we are interested in the dependence between markets in crisis. Therefore, the market data in the low quantile are more relevant. Specifically, we are interested in the extreme dependence between markets.
S3: Pairing the data by date assumes the synchronization of events between the crisis origin and the destination. Since the mechanism of a crisis transmission can be complex, the observation of the crisis event in the destination market may not be synchronized with the corresponding event in the origin market.

More sophisticated data analysis techniques can be employed to remove some shortcomings found in the above simple correlation analysis. Quantile regression [9] has been used by Adrian and Brunnermeier [1] to estimate quantiles of a system market risk factor conditional on each of its components. They define CoVaR or the conditional value at risk to measure the systemic risks contributed by the system components. Since this technique provides a complete distribution of a variable conditional on another variable, it does not suffer from the shortcoming S1. However, with its linear structure, the model assumes that at a specific quantile of the dependent variable, the dependent structure between the dependent variable and the independent variable is the same across all quantiles of the independent variable. Specifically, for example, conditional 99 percentile of the dependent variable has the same regression coefficient for all quantiles of the independent variable. This causes the dependence at the high quantiles of the independent variable to be averaged out by the dependence at the low quantiles of the independent variable. Therefore, the technique does not provide exactly the extreme dependence between variables, and it does not fully remove the shortcoming S2.

Poon et.al. [12], Hartmann et.al. [8] and Zhou [15] provide frameworks for applying multivariate extreme value theory to financial data. Zhou [15] proposes the systemic impact index (SII) to measure the systemic risk in financial institutions, serving the same purpose as that of CoVaR. These frameworks of the multivariate extreme value theory are the frameworks that we found appropriate to base our analysis on. However, these studies apply the techniques to the co-exceedance of the market data, which require the data to be paired day by day. This synchronization assumption on the market data is the assumption that we found inappropriate for our purpose. We are interested in the extreme dependence between markets when the markets enter the 2008 financial crisis, and we believe that market interdependence is so complex that the crisis transmission does not take effect synchronously between markets. Therefore, these studies still suffer from the shortcoming S3.

In our methodology, we measure the extreme dependence from block minima. Block minima (maxima) is a method to select extreme data (minimum or maximum) from the original data. The process begins with grouping the original data into blocks of an equal size and selecting the minimum (maximum) in each block. Analyzing the block minima (maxima) enables us to focus our analysis only on the data that is exceptionally low (high), which
is consistent with our objective of study. Besides, the block minima (maxima) approach requires no synchronization of events. As a demonstration, we perform a correlation analysis based on the block maxima of negative returns. Figure 3 shows the scatter plots of weekly maxima between US and Hong Kong before and during the crisis. The correlation between US and Hong Kong before the crisis is equal to 0.517, while the correlation in the crisis is 0.664. The correlation of the block maxima of the negative returns increases in the time of crisis significantly, which is the opposite of what indicated by the correlation analysis of the returns.

Figure 3: US-Hong Kong block maxima returns before and during the crisis

As discussed above, correlation is not a good measure of complex dependence structures so in this study we instead analyze the extreme dependence of the block maxima of the negative returns between the US market and other markets based on the multivariate extreme value theory [6, 7]. We show that the extreme dependence increased considerably when the financial markets move into 2008 crisis regime. The methodology also enables us to estimate the distribution of a block maximum of the negative returns of a specific market conditional on that of the US market. We then define a conditional return level to be used in financial stress test analysis. By the fact that the dependence increases when the financial system enters a crisis, we recommend that not only the financial risk factor being stressed but also the dependence structure also be stressed when one performs a financial stress test analysis.

In Section 2 we present a brief introduction to extreme value theory. Our goal here is to outline part of the theory which is used later on but the presentation is far from complete. We refer the interested reader to the references therein for a detailed mathematical foundation of the theory.
Section 3 contains the main results of our analysis on how the dependence between markets shifts during a crisis. We conclude with final remarks and conclusions in Section 4.

2 Extreme Dependence

Extreme value theory is a science of explaining the behavior of extremes. The main theorem indicates that, for independent and identically distributed observations, the normalized maximum converges in distribution to the so called generalized extreme value (GEV) distribution. Specifically, let $X_1, X_2, \ldots, X_n$ be a sequence of iid random variables with continuous distribution function $F(x)$. Further, let $M_n = \max(X_1, X_2, \ldots, X_n)$, $n \in N$ and suppose normalising constants $a_n > 0$ and $b_n \in \mathbb{R}$ are such that

$$\lim_{n \to \infty} P\left(\frac{M_n - b_n}{a_n} \leq x\right) = \lim_{n \to \infty} F^{n}(a_n x + b_n) = G(x)$$

where $G(x)$ is non-degenerate. The main result which is also called the “extremal types theorem” indicates that the only possible limits for $G(x)$ are given by the following three types of distributions:

**Type I (Fréchet distribution):**

$$\Phi_{\alpha}(x) = \begin{cases} 0 & x < 0 \\ \exp(-x^{-\alpha}) & x \geq 0 \end{cases}$$

for some $\alpha > 0$,

**Type II (Weibull distribution):**

$$\Psi_{\alpha}(x) = \begin{cases} \exp(-(-x)^{\alpha}) & x < 0 \\ 1 & x \geq 0 \end{cases}$$

for some $\alpha > 0$,

**Type III (Gumbel or double exponential distribution):**

$$\Lambda(x) = \exp(-e^{-x}) \quad x \in \mathbb{R}.$$ 

The result contains contributions by Fisher, Tippet and Gnedenko. Due to a re-parametrization by von Mises it can be shown that, after location-scale changes, all these three types of distributions can be combined to a one single family which is called the generalised extreme value (GEV) distribution as follows:

$$G(x; \gamma, \mu, \sigma) = \exp\left\{-\left(1 + \gamma \frac{x - \mu}{\sigma}\right)^{-\frac{1}{\gamma}}\right\}$$
where $\sigma > 0$, $\mu$ and $\gamma \in \mathbb{R}$. In addition from the notation $x_+ = \max(x, 0)$ follows that $x > \mu - \frac{\sigma}{\gamma}$ for $\gamma > 0$ and $x < \mu - \frac{\sigma}{\gamma}$ for $\gamma < 0$. These cases correspond to type I and II distributions defined above, respectively. For $\gamma = 0$, it should be interpreted as the limit which leads to the double exponential or type III distribution; see Leadbetter et al and Resnick for more details on these results.

The observation relevant to the present problem is negative log returns of stock market indices, and the theorem explains the behavior of these maximum negative log returns. On the other hand, if one is tempted to understand instead only those returns that go beyond some critical point, a threshold model can be applied. For independent and identically distributed observed values, when the critical point or the threshold is high enough, peak-over-threshold theorem states that the distribution of values beyond the threshold can be approximated by the generalized Pareto distribution. These theories open ways for risk measuring and monitoring involving exceptionally high losses and extremes. More sophisticated tools in the extreme value theory are devised and applied to phenomenon in financial markets and create an advance in risk management methodology. Novak and Beirant [10] demonstrates how modern extreme value theory can help predict the size of financial market meltdown. Ferrari and Paterlini [5] created a new method based on MLqE for quantile estimation, essential for risk management purposes. However, the researches based on these theories study only the behavior of extremes of individual market. Often time, the comovement between markets is more relevant. Less is known when it comes to the behavior of comovement or dependence of the extremes between two markets or more. From theoretical point of view this requires extending the univariate extreme value theory to higher dimensions.

Multivariate extreme value distributions arise in connection with extremes of a random sample from a multivariate distribution. There are several possibilities for ordering multivariate data. For multivariate extreme values the most widely used method is the so called marginal or M-ordering. As the name suggests, ordering or ranking here takes place within one or more of the marginal samples. Thus the maximum of a set of vectors is defined by taking component-wise maxima.

Let $\{X_n, n \geq 1\} = \{(X_{n}^{(1)}, \ldots, X_{n}^{(d)}), n \geq 1\}$ where $X \sim F(x)$, iid and $F$ is a $d$-dimensional distribution function. As mentioned above, we define the maxima as

$$M_n = (M_n^{(1)}, \ldots, M_n^{(d)}) = (\vee_{j=1}^{n} X_j^{(1)}, \ldots, \vee_{j=1}^{n} X_j^{(d)}).$$

In a similar manner as in the univariate case suppose we can find the normalizing constants $\sigma_n^{(i)} > 0$, $u_n^{(i)} \in \mathbb{R}$ for each margin such that as $n \to \infty$
we obtain

$$P[(M_n^{(i)} - u_n^{(i)})/\sigma_n^{(i)} \leq x^{(i)}, 1 \leq i \leq d] =$$

$$F_n^d(\sigma_n^{(1)} x^{(1)} + u_n^{(1)}, \ldots, \sigma_n^{(d)} x^{(d)} + u_n^{(d)}) \rightarrow G(x)$$

assuming marginal $G_i$ of $G$ are non-degenerate. In this case the limiting distribution is called a multivariate extreme value distribution (MEVD). It follows directly from the univariate results that the marginal distributions are necessarily of one of the three classical types presented above, and so are determined by a finite number of parameters. However, the link between marginals can only be characterized by a certain measure on a simplex. A consequence of this is that there is no natural parametric family with finite number of parameters for multivariate extreme value distributions. For brevity here we present the characterization for the bivariate case and refer the interested reader to the interested reader to see the literature mentioned in the bibliography for the general results. In particular the characterization presented here is due to Resnick and de Haan see ....

As mentioned above the marginal distributions in MEVD are necessarily one of the three possible types of extreme value distributions. Assuming unit Fréchet ($\alpha = 1$) distribution for the margins, it can be shown that all the bivariate extreme value distributions, denoted by $G_*(x,y)$ below, can be written as

$$G_*(x,y) = e^{-\mu_*[0,(x,y)]^c}$$

where

$$\mu_*[0,(x,y)]^c = \left(\frac{1}{x} + \frac{1}{y}\right)A\left(\frac{x}{x+y}\right)$$

and

$$A(w) = \int_0^1 \max\{q(1-w), (1-q)w\} S(dq).$$

The only requirement for measure $S$ is that

$$\int_0^1 qS(dq) = \int_0^1 (1-q)S(dq) = 1.$$
parametric sub-family of bivariate extreme value distributions and estimate the parameters by a method such as maximum likelihood or alike.

Some advances in this area of extreme dependence include, for example, Hall and Tajvidi [6, 7] and Christiansen and Ranaldo [4]. Hall and Tajvidi [6] introduced two methods for estimating dependence function of bi-variate extremes. Hall and Tajvidi [7] suggested a mixture of parametric and non-parametric approach for constructing a prediction region of bi-variate extremes. These techniques have been applied successfully to environmental data. Therefore, if adjusted appropriately, one is expectedly able to applied these bi-variate extreme value theory to financial data and identify the behavior of extreme dependence between different financial markets.

3 Extreme Dependence between Markets

From the bivariate extreme value theory, the dependence between two extreme variables can be expressed as a dependence function. The dependence function enables one to inspect extreme dependence graphically. The graph of a dependence function is convex, passing through with coordinates (0, 0) and (1, 0) and contained within the triangle \( \nabla \) formed by (0, 0), (1, 0) and \((\frac{1}{2}, \frac{1}{2})\). If the graph of a dependence function lies along the boundary at the top of the triangle \( \nabla \), the two extreme variables are independent. If it lies along the v-shape lower boundary of the triangle \( \nabla \), the two variables are perfectly dependence. A graph of a dependence function with a high curvature towards the v-shape lower boundary represents a strong dependence structure. A flat graph represents a weak dependence structure. One way to summarize the dependence based on a dependence function is to measure the area of the convex set formed by the graph of the dependence function and the upper boundary of the triangle \( \nabla \). When comparing dependences between two dependence functions, the one with greater area can be regarded as the one with stronger dependence.

As an example of application of the theory below we analyze the dependence between weekly maxima of the negative returns between the US and Hong Kong markets. The characterization of bivariate extreme value distributions presented in the previous section requires transforming the observations to unit Fréchet margins. We use maximum likelihood to estimate parameters of GEV for each margins. For the period before crisis we obtained \( \hat{\gamma}_x = -0.231, \hat{\sigma}_x = 0.003, \hat{\mu}_x = 0.003, \hat{\gamma}_y = -0.126, \hat{\sigma}_y = 0.006 \) and \( \hat{\mu}_y = 0.005 \), where subscripts \( x \) and \( y \) denote US and Hong Kong, respectively. The corresponding estimates for the period after crisis are \( \hat{\gamma}_x = -0.196, \hat{\sigma}_x = 0.012, \hat{\mu}_x = 0.013, \hat{\gamma}_y = -0.013, \hat{\sigma}_y = 0.018 \) and \( \hat{\mu}_y = 0.018 \).

Left panel of Figure 4 shows the weekly maxima for these markets before the crisis after transforming the margins to unit Fréchet distribution. In the
right panel the same data is depicted in log-scale to separate observations. Figure 5 shows the corresponding plots for the period during the crisis.

![Figure 4: US-Hong Kong block maxima returns before the crisis](image)

Figure 4: US-Hong Kong block maxima returns before the crisis

![Figure 5: US-Hong Kong block maxima returns during the crisis](image)

Figure 5: US-Hong Kong block maxima returns during the crisis

Although the visual impression of these figures indicate that the dependence structure has been changed in these two periods we will quantify this by estimating dependence functions. We fitted several parametric models to the dependence function and even computed some non-parametric estimators of the dependence functions.

The models were
1. The mixed model: \( A(w) = \theta w^2 - \theta w + 1, \quad 0 \leq \theta \leq 1 \)

2. The logistic model: \( A(w) = \{(1 - w)^r + w^r\}^{1/r}, \quad r \geq 1 \)

3. The asymmetric mixed model:
   \[
   A(w) = \phi w^3 + \theta w^2 - (\theta + \phi)w + 1, \quad \theta \geq 0, \theta + 2\phi \leq 1, \theta + 3\phi \geq 0
   \]

4. The asymmetric logistic model:
   \[
   A(w) = \{\{(\theta(1 - w))^r + (\phi w)^r\}^{1/r} + (\theta - \phi)w + 1 - \theta, \theta \geq 0, \phi \leq 1, r \geq 1
   \]

5. The generalized symmetric mixed model:
   \[
   A(w) = \left(\frac{(1 - w)^p + w^p + k ((1 - w) \ w)^{\frac{k}{2}}}{(1 - w)^p + w^p}\right)^{\frac{1}{p}}, \quad 0 < k \leq 2(p - 1), p \geq 2
   \]

6. The generalized symmetric logistic model:
   \[
   A(w) = 1 - \frac{k}{(1 - w)^p + w^p}, \quad 0 < k \leq 1, p \geq 0.
   \]

The asymmetric families have been developed by Tawn [14] and the latter two models have been proposed by Tajvidi [13]. In addition to these parametric models, we also computed a few nonparametric estimators of the dependence functions. This includes Pickands estimator [11], smoothed splines estimator of dependence function proposed by Hall and Tajvidi [6] and the estimator developed by Caperaa et.al. [3].

Figure 6 shows the estimates of dependence function for all logistic and nonparametric models. Corresponding estimated dependence functions for weekly maxima returns during crisis are depicted in Figure 7. As a measure of the strength of dependence one can use \( d = 2(1 - A(1/2)) \). Note that \( d \in [0,1] \) where lower limit implies independence and upper limit is reached when there is a complete dependence between observations. The following table summarizes the results:

<table>
<thead>
<tr>
<th>model</th>
<th>before crisis</th>
<th>during crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>logistic</td>
<td>( \hat{r} = 1.2351, \hat{d} = 0.2472 )</td>
<td>( \hat{r} = 2.0592, \hat{d} = 0.5998 )</td>
</tr>
<tr>
<td>generalized logistic</td>
<td>( \hat{k} = 1.1077, \hat{\rho} = 2.00 ) ( \hat{d} = 0.2371 )</td>
<td>( \hat{k} = 1.0191, \hat{\rho} = 3.4143 ) ( \hat{d} = 0.6179 )</td>
</tr>
<tr>
<td>asymmetric logistic</td>
<td>( \theta = 1.000, \phi = 0.3180 ) ( \hat{r} = 1.9614, \hat{d} = 0.2655 )</td>
<td>( \theta = 1.000, \phi = 0.9436 ) ( \hat{r} = 2.1703, \hat{d} = 0.6055 )</td>
</tr>
<tr>
<td>pickand</td>
<td>( \hat{d} = 0.16001 )</td>
<td>( \hat{d} = 0.72876 )</td>
</tr>
<tr>
<td>smoothing splines</td>
<td>( \hat{d} = 0.11906 )</td>
<td>( \hat{d} = 0.65923 )</td>
</tr>
<tr>
<td>caperaa</td>
<td>( \hat{d} = 0.24951 )</td>
<td>( \hat{d} = 0.60403 )</td>
</tr>
</tbody>
</table>

In [14] Tawn suggests a method to test for independence between margins based on score statistics. We applied this method to the data both before
and during crisis and in both cases p-values were practically 0. For non-
parametric independence functions we applied the test proposed by Caperaa
et.al [3] based on Tawn’s measure of association, denoted by $\hat{d}$ in Table 1,
and the results were similar. It should be mentioned that all mixed models
were also fitted to the data but the results were similar to those presented
above.

Figure 6: Dependence functions for US-Hong Kong returns before the crisis.
Left panel shows the logistic models and right panel depicts non-parametric
estimators.

Figure 7: Dependence functions for US-Hong Kong returns during the crisis.
Left panel shows the logistic models and right panel depicts non-parametric
estimators.

As discussed in Section 2 any dependence function corresponds to a dis-
bution function for a bivariate extreme value distribution which in turn
defines a bivariate density function. Of course, any of the estimated depen-
dence functions above can be used to estimate corresponding distribution
and density functions. Figure 8 shows the distribution function for returns
during crisis based on smoothing splines method. The right panel depicts
the contour plots which can be considered as semi compact prediction re-
gions for maximum returns. We used numerical differentiation to estimate
the density function for this case which is shown in Figure 9. The advantage
is that one can construct compact prediction regions for returns by using density function. The contour plots of density are shown in the right panel of Figure 9.

Figure 8: US-Hong Kong block maxima distribution function and contour plots during the crisis

Figure 9: US-Hong Kong block maxima density function and contour plots during the crisis

It should be emphasized that we even analyzed the dependence between the US stock market index and both some other developed markets and emerging markets. This included a few developed markets such as the UK,
Australia and Japan. Even the dependence between US and some emerging markets such as Thailand, Korea and Mexico was also analyzed. For brevity these results are not reported here but in all cases, the above pattern was evident in all analyses namely that the dependences between the block maxima of the negative returns between markets in the crisis are higher than those before the crisis.

Table 2 summarizes the dependence measurements based on the correlation and the extreme dependence. Observe that the correlation analysis on paired log returns can capture the increase in the dependence only in some cases, while the extreme dependence can capture the increase in all cases.

Table 2: A summary of the two dependence measurements when applied to cross market data: correlation based on pair log returns; and dependence function area based on block maxima of negative returns.

<table>
<thead>
<tr>
<th>Pair</th>
<th>Correlation (range = [-1, 1])</th>
<th>Dependence function area (range = [0, 0.25])</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>before crisis</td>
<td>in crisis</td>
</tr>
<tr>
<td>US-UK</td>
<td>0.436</td>
<td>0.501</td>
</tr>
<tr>
<td>US-Australia</td>
<td>0.509</td>
<td>0.642</td>
</tr>
<tr>
<td>US-Japan</td>
<td>0.417</td>
<td>0.602</td>
</tr>
<tr>
<td>US-Hong Kong</td>
<td>0.423</td>
<td>0.379</td>
</tr>
<tr>
<td>US-Brazil</td>
<td>0.758</td>
<td>0.728</td>
</tr>
<tr>
<td>US-Mexico</td>
<td>0.736</td>
<td>0.773</td>
</tr>
<tr>
<td>US-Korea</td>
<td>0.416</td>
<td>0.364</td>
</tr>
<tr>
<td>US-Thailand</td>
<td>0.150</td>
<td>0.274</td>
</tr>
</tbody>
</table>

The methodology does not only provide a way to summarize the extreme dependence as shown in Table 2, but also allows one to evaluate the entire range of the joint distribution between the two variables. Conditional distribution, a useful tool for risk management purpose, can also be obtained from the dependence function. As an example Figure 10 shows the conditional density of return in Hong Kong given that the US return is less than 0.09 during crisis.

For risk and portfolio management purposes probably it is of more interest to calculate distribution of return in one market given the return in another market. Our methodology makes this straight forward. As an example different panels of Figure 11 show the related densities and distributions for US market and conditional densities and distributions for Hong Kong given that return in US is 0.06.

Recall the relation between a dependence function and a copula function. Let $X_1$ and $X_2$ be a bivariate random variable with marginal $F_1$ and $F_2$, 

@@@ below this should be checked
Figure 10: Conditional density of return in Hong Kong given that the US return is less than 0.09 during crisis.

respectively.

\[
\tilde{C}(p, q) = P(F_1(X_1) > p, F_2(X_2) > q) = \exp \left( \left( \ln(1 - p) + \ln(1 - q) \right) A \left( \frac{\ln(1 - q)}{\ln(1 - p) + \ln(1 - q)} \right) \right)
\]

where \(0 \leq p, q \leq 1\) are probabilities. The conditional survival probability is then

\[
P(F_2(X_2) > q_{p,\alpha} | P(F_1(X_1) > p) = \tilde{C}(p, q_{p,\alpha})/(1 - p) = \alpha.
\]

When \(X_1\) and \(X_2\) represent negative market index returns, with fixed \(q\) and \(\alpha\), the quantity \(F_2^{-1}(q_{p,\alpha})\) measure the risk of market index \(X_1\) relative to the risk of market index \(X_2\). This is in fact the CoVaR defined in [1]. Applied to our context, this measurement can be used to measure crisis spill over effect.

With \(X_1\) and \(X_2\) as block maxima of US’s and Hong Kong’s negative index returns, respectively, Figure 12 shows their conditional survival probability \(q_{p,\alpha}\) at various \(1 - \alpha\) when \(p\) is equal to 0.95\(^1\) over 2 different regimes:

\(^1\)Matching return periods, 99% daily VaR is equivalent to 95% VaR based on 5-day block maximum of negative daily returns.
Figure 11: Panel (a) and (b) show the density and distribution function for maximum weekly return in US during crisis. Panel (c) and (d) depict the conditional density and distribution function of return in Hong Kong given that the return in US is 0.06.

Figure 12: Conditional survival probability when $p = 0.95$ at different $\alpha$. 
before-crisis and in-crisis. The two lines in Figure 12 translate the US-Hong Kong dependence functions in Figure 7 (d) into $q_{p,\alpha}$. The dashed line illustrates $q_{p,\alpha}$ induced by the before-crisis dependence function. The solid line illustrates $q_{p,\alpha}$ induced by the in-crisis dependence function. Observe that, at $1 - \alpha = 0.95$, the before-crisis $q_{0.95,0.05}$ is at 0.9649, while the in-crisis $q_{0.95,0.05}$ is at 0.9961. Suppose $p = 0.95$ represents that US market is in distressed. This means, given US market is in distressed, the 95% value at risk of Hong Kong increases from 96.5 percentile before crisis to 99.61 percentile in the crisis. The value at risk increases much higher in the time of crisis due to the increase in market dependence. Note that this increase in percentile accounts only for the effect of the higher market dependence. The effect of higher volatility in the time of crisis has not incorporated into the analysis.

Table 3 contains $q_{0.95,\alpha}$ at different level of $\alpha$, where $X_1$ is the block maximum of US’s negative index returns and $X_2$ varies across different markets. The table can be employed to determine a stress test level based on the 2008 financial crisis scenario.

<table>
<thead>
<tr>
<th>Market</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>UK</td>
<td>0.9586</td>
<td>0.9813</td>
<td>0.9971</td>
<td>0.9879</td>
<td>0.9949</td>
<td>0.9992</td>
</tr>
<tr>
<td>Australia</td>
<td>0.9386</td>
<td>0.9695</td>
<td>0.9940</td>
<td>0.9882</td>
<td>0.9952</td>
<td>0.9992</td>
</tr>
<tr>
<td>Japan</td>
<td>0.9340</td>
<td>0.9682</td>
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4 Conclusions

In this paper we study how the dependence between different markets change during a crisis. We use data from the recent financial crisis and fit bivariate extreme value distributions to maximum weekly returns in both developing and emerging markets. Our analysis shows that it is indeed the case that dependence between certain markets during crisis is much stronger than “normal” times. We argue that using a measure such as correlation could be indeed misleading for studying the relationship in these circumstances.

As a practical guidance for risk managers we discuss in detail how extreme value theory can be used to estimate the joint distribution of weekly
maximum returns. We also suggest that conditional densities and distribution functions can be seen as powerful tool for risk management under these circumstances.

References


