Similarly to Fig. 6, the result shown in Fig. 7 was obtained by using the COBA and the NFOBA algorithms. From the figure, we can observe that the NFOBA algorithm may become unstable initially due to the normalization procedure. Thus, to overcome the divergence, the NFOBA algorithm was not normalized during the initial two blocks. The figure shows that the COBA algorithm has better convergence rate than the NFOBA algorithm, and their adaptation accuracies are almost the same.

To illustrate the effect of nonstationary environments on the proposed algorithms, the results shown in Fig. 8 were drawn by using the time-varying unknown system. From the results, we can see that the TOBA and COBA algorithms outperform the SOBAF and NFOBA algorithms in tracking property as well as convergence rate. In addition, the tracking property of the TOBA algorithm is superior to that of the COBA algorithm.

V. CONCLUSIONS

The TOBA, SOBA, and COBA algorithms have been developed based on the preconditioning technique. The TOBA algorithm produces fast convergence speed, and the SOBA and COBA algorithms yield computational efficiency. Through computer simulations, it has been shown that the proposed algorithms are very fast, as compared with the OBA, SOBAF, and NFOBA algorithms, and their tracking properties are superior to those of the OBA, SOBAF, and NFOBA algorithms. Furthermore, the algorithms have no instability problem existing in the SOBAF and NFOBA algorithms.

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periodograms provides the estimate in (1). We will design windows \( h_i, i = 1 \cdots K \), giving a small bias as well as a low variance estimate of \( S_x(f) \) in the neighborhood of the peak frequency. Low bias is obtained by matching the windows to the peak of \( S_x(f) \), whereas reduction of the variance is established with uncorrelated periodograms \( \hat{S}_i(f) \) at the peak. Thus, a frequency local estimate is desired. To prevent leakage from regions outside a predetermined interval of width \( B \), the Fourier transforms \( H_i(f) \) of \( h_i, i = 1 \cdots K \) have to be bandlimited to the interval \((-B/2, B/2)\). The mainlobes of the windows should be inside this band, and the sidelobes of each window should be as low as possible.

The multiple window estimation method can be considered to be a filtering procedure in a filter bank. The impulse responses of the subfilters are \( h_i, i = 1 \cdots K \). Given the input signal \( x(n) \), the power of the output signal within the frequency interval \((-B/2, B/2)\) is

\[
P_B = \sum_{i=1}^{K} \alpha_i \int_{-B/2}^{B/2} |H_i(f)|^2 S_x(f) \, df
\]

where

\[
S_x(f) = \sum_{i=1}^{K} \alpha_i h_i^T R_B h_i.
\]

The \((N \times N)\) Toeplitz covariance matrix \( R_B \) has the elements \( r_B(l) = r_x(l) + \alpha \sin(Bl) \), where \( r_x(l) \) is the covariance function of \( x(n), \sin(x) = \sin(\pi x)/\pi x \), and \( \alpha \) denotes the convolution operator. In (3), \( P_B \) is the power of \( x(n) \) within the mainlobe of the windows. We want to find the \( K \) window functions \( h_i \) that maximize \( P_B \). The optimization is performed subject to the constraint

\[
P_Z = \sum_{i=1}^{K} \alpha_i \int_{-B/2}^{B/2} |H_i(f)|^2 S_z(f) \, df
\]

where \( S_z(f) \) with the corresponding Toeplitz covariance matrix \( R_Z \) is chosen for suppression of the sidelobes of the windows.

The solution with respect to \( h_i \) is the set of eigenvectors of the generalized eigenvalue problem

\[
R_B \mathbf{q}_i = \lambda_i R_Z \mathbf{q}_i, \quad i = 1 \cdots N
\]

where \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N \). The eigenvectors corresponding to the \( K \) largest eigenvalues are used as windows \( h_i = \mathbf{q}_i, i = 1 \cdots K \), and are known as peak matched multiple windows (PM MWs). The windows are orthogonal to the covariance matrix \( R_B \), implying uncorrelated periodograms \( \hat{S}_i(f) \) at the peak frequency. The covariance matrix \( R_Z \) is chosen in order to grant certain properties to the estimate \( \hat{S}_z(f) \).

Two examples have been investigated:

- \( R_Z = I \)
- \( R_Z = R_G \)

where \( R_G \) corresponds to a penalty frequency function. In the first example, the Karhunen-Loève basis functions are used as windows. The windows are bandpass filters and have their main power in the range \(-B/2 \leq f \leq B/2\). However, the finite length of the windows gives sidelobes that cause leakage from frequencies outside the resolution interval. In the second example, a penalty frequency function

\[
S_G(f) = \begin{cases} 
G & |f| > B/2 \\
1 & |f| \leq B/2 
\end{cases}
\]

is used to decrease the leakage from the sidelobes. The corresponding Toeplitz covariance matrix is \( R_G \). The ideal window functions fulfill the relationship

\[
\sum_{i=1}^{K} \alpha_i |H_i(f)|^2 = S_G^{-1}(f), \quad -1/2 \leq f \leq 1/2
\]

and if \( G \) is set to a large value, the sidelobes of \( |H_i(f)|^2 \) outside \(|f| > B/2\) will be suppressed by this factor. The suppression factor is indicated in parenthesis, e.g., PM MW (30 dB) for \( G = 30 \) dB.

The weighting factor \( \alpha_i \) is a parameter that can be chosen arbitrarily. We study a matched spectrum approach \( \Sigma_{K=1}^{N} \alpha_i |H_i(f)|^2 = S_x(f) \) in the local interval \(-B/2 \leq f \leq B/2\). The total filter function should have the same appearance as the peak to minimize bias as well as give a low variance of the power spectrum estimate in the neighborhood of the peak. The matched spectrum approach is fulfilled with \( \alpha_i = \lambda_i/\Sigma_{i=1}^{K} \lambda_i \), which is used in this paper.

### III. Results

The properties of the proposed method are investigated and compared with those of the Thomson spectrum estimator [2] and the single Hanning window. The windows with \( N = 128 \) are calculated from (5) with the known power density spectrum defined by

\[
S_x(f) = \begin{cases} 
e^{-2d/f|/10^{H_b(10)^*}} & |f| \leq B/2 \\
0 & |f| > B/2 
\end{cases}
\]

In (8), \( S_x(f) \) is a peaked spectrum with \( S_x(0) = 1 \) and \( S_x(B/2) = -C \) dB, where \( C = 20 \) dB throughout this correspondence. The Toeplitz covariance matrix \( R_B \) is calculated from this spectrum. The resolution parameter is \( B = 0.08 \), and the number of windows is \( K = 8 \). The windows are calculated for different values of the sidelobe suppression parameter \( G \). The Thomson multiple windows are calculated from (5) with \( r_B(l) = B \sin(Bl) \) and \( R_Z = I \). The bias, variance, and mean square error are compared for an ARMA spectrum and for white noise spectrum.

#### A. Numerical Example

In order to illustrate the proposed methods, a number of sequences \( x(n), n = 0 \cdots N-1 \) of a second-order ARMA process with poles \( p_{1,2} = 0.97 e^{\pm j\pi/2.01} \) and zeros \( z_{1,2} = 0.97 e^{\pm j\pi/0.3} \) are simulated. The number of sequences is 25, each with 128 data samples. The true spectrum has a peak at \( f = 0.1 \) and a notch at \( f = 0.3 \). The 25 estimated spectra are plotted as dotted lines in Fig. 1, where the true spectrum is depicted as the solid line. The suppression parameter \( G \) is put to \( G = 30 \) dB (PM MW (30 dB)), and the result is compared with the use of windows with \( R_Z = I \) (PM MW) and with the spectrum estimates from a single Hanning window and from Thomson multiple windows with resolution width \( B = 0.08 \) and \( K = 8 \) windows (Thomson 8 MW). The variation of the spectrum gives a clue about the variance; the closeness of the curves to the true spectrum indicates bias.

#### B. Calculation of Bias and Variance

The bias and variance can be calculated since the true ARMA spectrum is known. The calculation is made for a number of discrete frequencies. Both the bias and variance vary with the magnitude of the spectrum, and to obtain a comparable measure for different frequencies, they are divided by \( E[\hat{S}_x(f)] \) and \( E[\hat{S}_x(f)]^2 \), respectively.

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is the Fourier transform matrix. The variance of the spectrum estimate
as large valued estimates. Bias is defined as

\[ \text{Bias } \hat{S}_x(f) = \frac{E[\hat{S}_x(f)] - S_x(f)}{E[\hat{S}_x(f)]} \tag{9} \]

where the expected value of the spectrum estimate is calculated to be

\[ E[\hat{S}_x(f)] = E\left[ \sum_{i=1}^{K} \alpha_i \mathbf{h}_i^T \Phi^H(f) \mathbf{x} \mathbf{x}^T \Phi(f) \mathbf{h}_i \right] \]
\[ = \sum_{i=1}^{K} \alpha_i \mathbf{h}_i^T \Phi^H(f) R_x \Phi(f) \mathbf{h}_i. \tag{10} \]

In (10),

\[ \Phi = \text{diag}[e^{-j2\pi f} \ldots e^{-j2\pi(N-1)f}] \]

is the Fourier transform matrix. The variance of the spectrum estimate is given by all combinations of the different periodogram covariances

\[ \text{Variance } \hat{S}_x(f) = \sum_{i=1}^{K} \sum_{j=1}^{K} \alpha_i \alpha_j \text{cov}(\hat{S}_i(f)\hat{S}_j(f)) \]
\[ = \frac{E[\hat{S}_x(f)]^2}{E[\hat{S}_x(f)]^2}. \tag{11} \]

Denoting \( \mathbf{h}_i^T \Phi^H(f) \mathbf{x} = A_i \) and assuming \( \mathbf{x} \) to be Gaussian gives the covariance as

\[ \text{cov}(\hat{S}_i(f)\hat{S}_j(f)) = \text{cov}(A_i A_j^H) \]
\[ = E[A_i^H A_j] - E[A_i^H A_i] E[A_i^H A_i] \]
\[ = |\mathbf{h}_i^T \Phi(f) R_x \Phi(f) \mathbf{h}_i|^2 + |\mathbf{h}_j^T \Phi(f) R_x \Phi(f) \mathbf{h}_j|^2 \]
\[ = |\mathbf{h}_i^T \Phi(f) R_x \Phi(f) \mathbf{h}_i|^2 + |\mathbf{h}_j^T \Phi(f) R_x \Phi(f) \mathbf{h}_j|^2 \]
\[ = |\mathbf{h}_i^T \Phi(f) R_x \Phi(f) \mathbf{h}_i|^2 + |\mathbf{h}_j^T \Phi(f) R_x \Phi(f) \mathbf{h}_j|^2 \]
\[ \text{cov}(\hat{S}_i(f)\hat{S}_j(f)) \]
\[ = |\mathbf{h}_i^T \Phi(f) R_x \Phi(f) \mathbf{h}_i|^2 + |\mathbf{h}_j^T \Phi(f) R_x \Phi(f) \mathbf{h}_j|^2 \]
\[ = |\mathbf{h}_i^T \Phi(f) R_x \Phi(f) \mathbf{h}_i|^2 + |\mathbf{h}_j^T \Phi(f) R_x \Phi(f) \mathbf{h}_j|^2 \]
\[ = |\mathbf{h}_i^T \Phi(f) R_x \Phi(f) \mathbf{h}_i|^2 + |\mathbf{h}_j^T \Phi(f) R_x \Phi(f) \mathbf{h}_j|^2 \]
\[ = |\mathbf{h}_i^T \Phi(f) R_x \Phi(f) \mathbf{h}_i|^2 + |\mathbf{h}_j^T \Phi(f) R_x \Phi(f) \mathbf{h}_j|^2 \]
\[ = \sum_{i=1}^{K} \sum_{j=1}^{K} \alpha_i \alpha_j \text{cov}(\hat{S}_i(f)\hat{S}_j(f)) \]
\[ = \frac{E[\hat{S}_x(f)]^2}{E[\hat{S}_x(f)]^2}. \tag{12} \]

according to Walden et al. [6].

The calculated normalized bias of the ARMA process in Fig. 1 is depicted in Fig. 2(a) for the different methods. The single Hanning window (dotted line) shows small bias. The Thomson8 MW (dashed-dotted line) has a fairly large bias at the peak \( f = 0.1 \) and the notch \( f = 0.3 \). The behavior of the peak matched multiple windows PM MW (dashed line) shows a reduced bias at \( f = 0.1 \), but the sidelobe leakage causes a large bias at the notch. With the use of a penalty function with \( G = 30 \) dB (solid line), the leakage is reduced, giving a smaller bias.

The variance is depicted in Fig. 2(b). The multiple window methods show their superiority in variance reduction. On average, they all have about one fourth of the Hanning window variance (dotted line), which is one for all frequencies. The PM MW (dashed line) has a large variance in the region of the notch \( f = 0.3 \) caused by sidelobe leakage of the windows. The PM MW (30 dB) (solid line) shows, on average, comparable results with the Thomson8 MW (dashed-dotted line). At the peak of the spectrum \( f = 0.1 \), the Thomson8 MW has the smallest variance as the bias of the peak is not considered. However, a closer study of the peak as well as the surroundings shows that the average variance of the peaked matched methods is smaller. The disadvantage of suppressing the sidelobes in the PM MW is that it gives a slightly worse variance in the area of the peak \( f = 0.1 \). The variance at the notch \( f = 0.3 \) are, however, small for the PM MW (30 dB) compared with the others and show the gain in the sidelobe suppression. For the flat spectrum segment \( f > 0.3 \), the Thomson8 MW is close to the theoretical value for white noise, \( 1/8 = 0.125 \).

C. Comparison of Mean Square Error

Both low variance and small bias can be evaluated with the use of the mean squared error, as this measure includes both variance and squared bias. The normalized mean squared error MSE \( \hat{S}_x(f) \) is defined as

\[ \text{MSE } \hat{S}_x(f) = E[\hat{S}_x(f) - S_x(f)]^2 / E[\hat{S}_x(f)]^2 \]
\[ = \text{Variance } \hat{S}_x(f) + \text{Bias}^2 \hat{S}_x(f) \]
\[ \text{and an average value is calculated as} \]
\[ \text{MSE}_{\text{ave}} = \frac{1}{N} \left( \sum_{k=0}^{N} \text{MSE } \hat{S}_x(k/N) \right) \]
\[ \text{where } N = 128 \text{ is the number of frequencies in the DFT.} \]

The windows tested are the PM MW (30 dB) and the PM MW without sidelobe suppression. Two more cases with different \( G \) values are included: \( G = 10 \) dB and \( G = 50 \) dB. The suppression of 50 dB reduces the number of windows to \( K = 6 \) since the weighting factor
IV. CONCLUSIONS

A peak matched multiple window method for peaked spectra is proposed. The resulting spectrum estimate has low variance and bias in the neighborhood of the peak. The proposed method shows, however, a large bias at a notch. This is due to large sidelobes of the windows that cause leakage from frequencies outside the resolution width. This leakage is suppressed with the use of a penalty function. A local spectrum estimate with leakage control is achieved as the sidelobe suppression is chosen with a parameter $G$. This makes the method suitable for estimation of peaked spectra as well as for spectra with notches, e.g., ARMA processes. The result is also satisfactory for white noise spectrum.

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Recursive versus Nonrecursive Correlation for Real-Time Peak Detection and Tracking

Carl During

Abstract—This correspondence compares the recursive versus the nonrecursive algorithms of correlation for a fast varying peak position by simulations. The recursive algorithms give better response and accuracy of time delay estimation, even though the sampling period decreases. The average magnitude difference function (AMDF) is used as an application example.

I. INTRODUCTION

This correspondence concerns real-time correlation peak detection using the well-known average magnitude difference function (AMDF) (see for example, [1]–[4]) as an application example. We compare the recursive versus the nonrecursive algorithms of AMDF correlation for a fast varying peak position by simulations. Discrete correlation in the frequency or time domain are old techniques (see, for example, [5]–[7]) used for comparing information (see, for example, [8]–[12]) but still useful in some applications (e.g., [13]–[16]). Even though the following discussion will concentrate on measurements of mechanical variables, a translation may be done to other case studies. The AMDF is, for example, used by velocity sensors like the one illustrated in Fig. 1 [17] or by the scene-matching technique implemented in the cruise-missile guidance system (e.g., [18] and [19]). The required information is here given by detecting, in real time, the extreme value (peak) of a 1-D or 2-D discrete AMDF, respectively. The location of the extreme value within the AMDF expresses how much a sequence is shifted in time or space with respect to another sequence. This gives

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The author is with the Department of Machine Design, Division of Mechatronics, Royal Institute of Technology, Stockholm, Sweden.

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