Sequential Calibration of Options

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Abstract

Robust calibration of option valuation models to quoted option prices is non-trivial, but as important for good performance as the valuation model itself. The standard textbook approach to option calibration is minimization of a suitably chosen measure of the prediction error, e.g. least squares minimization.

This paper interprets the total prediction error as a sum of the measurement errors and effects from the parameter dynamics. We introduce an apriori dynamics for the parameters. This will allow the parameters to change over time, while treating the measurement noise in a statistically consistent way and using all data efficiently.

We here use models for which closed form expressions or Fourier transform methods are available, e.g. exponential Levy processes or certain stochastic volatility models. We use the Heston, Bates and NIG-CIR models in this paper.

We investigate the performance and computational efficiency of standard and iterated Extended Kalman filters (EKF and IEKF) as well as Particle Filters (PF). These methods are then compared to the common practice calibration method of Weighted Least Squares (WLS) and penalised Weighted Least Squares (PWLS).

Our simulation study, using the Heston model, has shown that the introduced filter framework is capable of tracking time varying parameters and latent processes such as stochastic volatility processes. We find that the filter estimates and the PWLS estimates are much closer to the true parameters than the WLS estimates. When we use the same parameters to price Binary options we find that the prices obtained using the WLS estimates are inferior to those obtained by the other methods. When pricing exotic derivatives using models calibrated on European call option data we believe that the filter methods will perform better than the WLS.

We also apply non-linear filtering methods to daily European call option data on the S&P-500 stock index. If we price options, using yesterdays estimated parameters and todays estimated latent process and todays stock price there is a general decrease in global fit over all models and all methods. This tendency is most pronounced for the WLS method indicating overfitting. We therefore find that there is need for adaptive calibration methods to handle time varying parameters in an accurate and robust way.

Keywords: Sequential Monte Carlo filters, Calibration, Option Valuation.
1 Introduction

The theory of option valuation is almost unique among social sciences as predictions generated by the models are extremely accurate. The predictive power of parametric option valuation models was recognized immediately after the Black & Scholes model was introduced, Black and Scholes (1973), and the applicability has grown ever since, cf. Campbell et al. (1997).

More recent models have included jumps, see Merton (1976), stochastic volatility, see e.g. Hull and White (1987); Heston (1993) and state dependent diffusion terms, see e.g. Dupire (1994); Derman and Kani (1994) in order to improve the predictive power further. Today, models use a combination of jumps, stochastic volatility and local volatility, see e.g. Bates (1996); Duffie et al. (2000); Carr et al. (2003b); Chernov et al. (2004).

Parametric (option valuation) models are given as a function of variables and parameters. The main difference between these is that variables are changing while parameters are (supposed to be) fixed. Real world applications tend to violate this decomposition, letting parameters change over time as well, often using ad hoc structures for the parameter dynamics.

Option valuation models are parametric models, depending on observable quantities such as the current value of the underlying asset $S_t$, the strike level $K$, time to maturity $\tau = T - t$ etc. It also depends on parameters related to the dynamics of the underlying asset and in some cases latent states. This paper does not try to find the perfect option valuation model, if such a model does exist. Instead, we accept that the parameters will change over time due to different regimes in the economy, risk preferences changing over time or even that the imperfect model needs re-calibration.

It is essential that the theoretical value of the calibration instruments (i.e. the options used in the calibration) can be calculated with high accuracy using limited computational resources. These requirements rule out many quoted options, essentially restricting the available instruments to European call and put options.

Calibration of option valuation models to market data should be based on a sound statistical framework. The main contribution of this paper is to introduce a model for the parameter dynamics, leading to a consistent treatment of the time-varying parameters. Furthermore, we treat the parameter dynamics and measurement noise separately, using a non-linear filtering technique.

2 Option valuation

For many models used in financial mathematics it is not possible to obtain closed form expressions for the prices of derivatives. There are mainly three different types of techniques used to approximate the prices.

- Deterministic numerical solution of the pricing PDE:s or for models with jumps the corresponding PIDE:s.
- Quasi Monte Carlo or Monte Carlo methods combined with various variance reduction techniques.
- Methods based on Fourier transform techniques.

There are pros and cons with all these methods and it will not be possible to list them all in this paper. We will however mention some aspects here. The PDE/PIDE ap-
proach is a very versatile method which can be used for simple contingent claims, e.g. plain vanilla options or for exotic and path dependent derivatives. For a nice overview concerning PIDE-pricing and related numerical methods see Cont and Tankov (2004, chapter 12 and the references therein). The main drawback is that we need to approximate the prices on a grid (regular or irregular). If we are to compare model prices with observed market prices we need to calculate prices for a lot of points which will not be used in the comparison. This will lead to a considerable amount of extra work especially in the case where we have lot of observed European call and put option prices for different strike prices and maturities for which we want to match prices.

The Monte-Carlo method is the most general of the techniques and can be used to price almost any contingent claim. Another advantage is that we only need to calculate those prices we use. The main limitation is that it is very computer intensive if we need high precision in the prices. If we use an optimization method on top of the pricing method we do in general need quite accurate model prices for the optimization step to perform well. Moreover we will need a fast method to calculate the prices.

The Fourier transform based techniques are both fast and fairly accurate. The main drawback here is that we need to know the characteristic function for the log stock price in closed form. However for a fairly large class of stock price models e.g. Black-Scholes (BS) (Black and Scholes, 1973), Merton (Merton, 1976), Heston (Heston, 1993), Bates (Bates, 1996), Normal Inverse Gaussian (NIG) (see e.g. Barndorff-Nielsen, 1997, and the references wherein), Variance Gamma (VG) (Madan and Seneta, 1990), BNS (Barndorff-Nielsen and Shepard, 2001, 2002), CGMY (Carr et al., 2002), Finite Moment log-stable (FMLS) (Carr and Wu, 2002), NIG-CIR model (Carr et al., 2003b) etc. the characteristic function is available in closed form. We will consider some of these models in detail below (cf. Section 2.3). The Fourier transform based methods have been used in financial mathematics for some time now (see Carr and Madan, 1999, for a good reference to start with). A long list of references to articles using Fourier transform based methods can be found in Carr et al. (2003a).

Lee (2004) treats error bounds for a Fast Fourier Transform (FFT) implementation of the Fourier transform method and list the generalized Fourier transforms for some common pay-off functions such as e.g. the European call option. Using FFT, one can obtain prices on a regularly spaced grid of log strike levels. Options observed on the market are usually regularly spaced in the strike level, not the log strike level. If we are to evaluate the prices for a set of European call options we need to fit the corresponding log strike levels into a regular spaced grid. The accuracy of FFT depends on the grid spacing used, the interval on which we evaluate the integrand and the properties of the characteristic function of the log stock price. We need to do one FFT for each time to maturity. This will lead to extra work if we have several dates of maturity each with only a few strike levels. In order to avoid using different grids for different strike levels and maturities we will here instead consider a Gauss-Laguerre quadrature implementation for the inverse Fourier transform (cf. next subsection).

### 2.1 Implementation and numerical aspects of the inverse Fourier transform

In order to price a European call option we need to calculate the inverse Fourier transform (Lee, 2004):
\[ C_t(K, T - t) = p(t, T) \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ik\theta - \gamma} e^{i(1 + \gamma + i\theta)M_{X_T}(1 + \gamma + i\theta)} \] 
\[ \frac{1}{(\gamma + i\theta)(1 + \gamma + i\theta)} d\theta, \quad (2.1) \]

where \( t \) is the present time, \( T \) time of maturity, \( K = \exp(k) \) is the strike level, \( S_t = \exp(s_t) \) is the current stock level, \( M_{X_T} \) is the moment generating function of \( X_T = \log(S_T / S_t) \) under the risk neutral measure \( \mathbb{Q} \) given \( S_t \) and \( p(t, T) \) a discounting factor for the time interval \([t, T]\). For this to work we need that there exists a positive real number \( \gamma \) such that

\[ \mathbb{E}^{\mathbb{Q}}[S_T^{1+\gamma}] < \infty. \]

In general we can use any \( \gamma \in A_{X_T}^+ \) where

\[ A_{X_T}^+ = \{ x > 0 : \mathbb{E}^{\mathbb{Q}}[S_T^{1+x}] < \infty \}. \]

It is easy to see that if the set \( A_{X_T}^+ \) is non-empty it will be an interval of either the form \((0, \gamma_{\text{max}})\) or the form \((0, \gamma_{\text{max}}]\), where \( \gamma_{\text{max}} = \sup A_{X_T}^+ \). If it is open or closed to the right depends on the model for the log stock price. Moreover if the log stock price is modeled as a Lévy process, e.g. BS, NIG, Merton, VG, FMLS and CGMY, then the set \( A_{X_T}^+ \) will be same for all positive \( T \) (for arbitrary, but fixed and valid model parameters).

This is in fact also true for the Bates model and the NIG-CIR model. Let \( g \) denote at the integrand in equation 2.1. If we view \( g \) as a function of a complex variable \( z \) we have

\[ g(z) = \frac{e^{-zk}e^{\nu(1+z)}M_{X_T}(1+z)}{z(1+z)} \]

and the integrand in equation 2.1 is thus given as \( g(\gamma + i\theta) \). We have that \( g(z) \) is analytic in the strip \( \Re z \in A_{X_T}^+ \). From a theoretical point of view the integral in equation 2.1 is the same for all \( \gamma \) belonging to the interior of \( A_{X_T}^+ \). However if we are to approximate the integral numerically it does matter which \( \gamma \) we choose. When \( s_j - k \) is large, i.e. deep in the money call options, and for large \( |k| \) the integrand will be oscillatory. This can cause serious numerical problems when we are to calculate the inverse Fourier transform. The amplitude of these oscillations are proportional to \( |g(\gamma + i\theta)| \). Looking at the denominator in \( g \) we see that these oscillations will be more pronounced when the modulus of \( \theta \) is small. If the conditional density of \( X_T \) given \( S_t = \exp(s_t) \) is very smooth (\( C^\infty \)) then \( M_{X_T}(1 + \gamma + i\theta) \) will asymptotically decay at an exponential rate as \( |\theta| \) tend to \( \infty \) for \( \gamma \) belonging to the interior of \( A_{X_T}^+ \). This further strengthens the tendency that the oscillation only will be present when \( |\theta| \) is small. Now a straightforward calculation shows that

\[ |g(\gamma + i\theta)| \leq |g(\gamma)| = \frac{M_{X_T}(1 + \gamma)\nu(1 - \gamma) - \gamma}{\gamma(\gamma + 1)}, \quad \text{for } \theta \in \mathbb{R}, \gamma \in A_{X_T}^+. \]

This upper bound is a convex function of \( \gamma \) for \( \gamma \in A_{X_T}^+ \) and therefore we have that the upper bound has a unique minimum in \( A_{X_T}^+ \). A simple rule of thumb is to choose \( \gamma \) as \( \gamma_{\text{min}} \) where

\[ \gamma_{\text{min}} = \arg \min_{\gamma \in A_{X_T}^+} g(\gamma). \]

In fact we have that \( \gamma_{\text{min}} \) will be a saddle point for the function \( g(z) = g(\gamma + i\theta) \), i.e. it is a local minimum in the \( \gamma \) direction and a local maximum in the \( \theta \) direction for \( \theta = 0 \).
In practice it is not possible to find closed form expressions for $\gamma_{\text{min}}$ for a general stock price model. There exist however very efficient numerical algorithms to find the minimum of convex functions. We use the golden-section search method to find $\gamma_{\text{min}}$, which is almost as efficient as the dichotomous-search method is for monotone functions, see e.g (Bazaraa et al., 1993, page 270–271). For numerical reasons it is actually better to work with the logarithm of $g(\gamma)$ instead, but this will also be a convex function of $\gamma$ for $\gamma \in A^+_X$, which is seen from a straightforward application of the Cauchy-Schwarz inequality to the second derivative of the log moment generating function expressed in terms of derivatives of the moment generating function.

The Binary call option

In order to price a Binary call option we need to calculate the inverse Fourier transform of (see Lee, 2004, Eq. (5.1)):

$$B_t(K, T-t) = p(t, T) \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-ik\theta - \frac{1}{2} \theta^2} e^{\gamma (\gamma + i\theta)} M_X(\gamma + i\theta)}{\gamma + i\theta} d\theta,$$

(2.2)

where $t$ is the present time, $T$ time of maturity, $K = \exp(k)$ is the strike level, $S_t = \exp(x_t)$ is the current stock level, $M_X$ is the moment generating function of $X_T = \log(S_T/S_t)$ under the risk neutral measure $Q$ given $S_t$ and $p(t, T)$ is a discounting factor for the time interval $[t, T]$. Here $\gamma$ is any positive real number such that $E^Q[S_T^\gamma] < \infty$.

2.2 The Gauss-Laguerre method

We need to make one more observation before describing the numerical integration method. Since the prices of a European call option\(^1\), $C_t(K, T-t)$ is a real number we have that

$$C_t(K, T-t) = p(t, T) \frac{1}{2\pi} \int_{0}^{\infty} \Re \left( \frac{e^{-ik\theta - \frac{1}{2} \theta^2} e^{\gamma (1 + \gamma + i\theta)} M_X(1 + \gamma + i\theta)}{(\gamma + i\theta)(1 + \gamma + i\theta)} \right) d\theta$$

$$= p(t, T) \frac{1}{\pi} \int_{0}^{\infty} \Re \left( \frac{e^{-ik\theta - \frac{1}{2} \theta^2} e^{\gamma (1 + \gamma + i\theta)} M_X(1 + \gamma + i\theta)}{(\gamma + i\theta)(1 + \gamma + i\theta)} \right) d\theta,$$

where the last equality comes from the fact that the real-part of the integrand is an even function of $\theta$. This integral can now be approximated by a Gauss-Laguerre quadrature formula. In general we have that the Gauss-Laguerre quadrature formula approximates an exponentially weighted integral from zero to infinity as

$$\int_{0}^{\infty} e^{-x} f(x) dx \approx \sum_{j=1}^{n} w_j^{(n)} f(x_j^{(n)}),$$

(see e.g. Abramowitz and Stegun, 1972, page 890). The nodes $x_j^{(n)}$, $j = 1, \cdots, n$ and the weights $w_j^{(n)}$, $j = 1, \cdots, n$ in the Gauss-Laguerre quadrature can for each fixed order $n$ be obtained from a solution to an eigenvalue/vector problem of a tridiagonal symmetric $n \times n$ matrix (Golub and Welsch, 1969).

The algorithm in Golub and Welsch (1969) has complexity of order $n^2$. It is of course possible to use the standard \texttt{eig} routine in \texttt{MATLAB}, but this will have the

\(^1\)Note that the same type of reasoning is true also for the Binary call option
complexity $n^3$ for this problem. In this paper we will use $n = 100$, which has from numerical experiments, turned out to be sufficient to get reasonably accurate prices compared to the accuracy of the observed market prices. We do not claim that this holds true in general, only for the models and options considered below. Some care should be taken for short maturities and deep in the money options. Since we use the same number weights for all option valuations these can be pre-calculated and stored, in order to increase the computational efficiency of the algorithm. The complexity of the Gauss-Laguerre pricing algorithm will be of the order $n \times n_{\text{option}}$, i.e. the number of weights times the number of different options. Error bounds for Gauss-Laguerre quadrature methods applied to analytic functions can be found in Donaldson (1973). Some important issues regarding the accuracy of Gauss quadrature formulas are also discussed in Gautschi (1983), Laurie (1999) and Laurie (2001).

The implication of these results and other numerical issues of Fourier inversion approaches to option valuation are to be addressed in detail in a forthcoming paper. Finally we state the actual option valuation formula used in this paper

$$C_t(K, T - t) = \frac{p(t, T)}{\pi} \sum_{j=1}^{n} w_j^{(n)} \mathfrak{R}(g(ix_j^{(n)} + \gamma_{\min})) \exp(x_j^{(n)}),$$

(2.3)

$g$ is of course also a function of $k$ (log-strike-level), $T - t$ (Time to maturity), $s_t$ (current log-stock-price), $r$ (short rate or zero-coupon-bond yield$^2$) and the model parameters etc, but we have suppressed this here for notational simplicity.

2.3 Stock price models

In this paper we consider the Bates model (Bates, 1996) and its sub-models (cf. below) as well as the NIG-CIR-model (cf. below).

The Bates model and its sub-models

The dynamics for the Bates model under the risk-neutral measure $Q$ is given by

$$\begin{align*}
    dS_t &= rS_t dt + \sqrt{\xi} S_t dW_t^{(S)} + S_t dJ_t \\
    dV_t &= \kappa(\xi - V_t) dt + \sigma_V \sqrt{V_t} dW_t^{(V)},
\end{align*}$$

(2.4)

where $W^{(S)}$ and $W^{(V)}$ are standard Brownian motions with correlation $\rho$. $\kappa$ is the mean reversion rate, $\xi$ is the mean reversion level and with $V_0$ as the initial value of $V$. Further, $J$ is a compound Poisson process with intensity $\lambda$, with jumps $\Delta J$ such that $\log(1 + \Delta J) \in N(\mu, \delta^2)$ and with drift $-\lambda(\exp(\delta^2/2 + \mu) - 1)$. This choice of drift for $J$ forces the discounted stock price process to be a martingale under $Q$. The Bates model contain as sub-models:

- The Heston model (set $\lambda = 0$)
- The Merton model (set e.g. $\sigma_V = 0, \xi = V_0 = \sigma^2$)
- The Black-Scholes model (set e.g. $\sigma_V = 0, \xi = V_0 = \sigma^2, \lambda = 0$).

\footnote{cf. Section 5}
Using Itô’s formula we obtain the following dynamics for $X_t = \log(S_t)$ given $S_u = e^{u}$ for $t \geq u$:

$$
\begin{align*}
  dX_t &= \left( r - \frac{V_t}{2} - \lambda (\exp(\delta^2/2 + \mu) - 1) \right) dt + \sqrt{V_t} dW_t^{(S)} + \log(1 + \Delta t), \\
  dV_t &= \kappa (\xi - V_t) dt + \sigma V \sqrt{V_t} dW_t^{(V)}, \\
  X_u &= s_u.
\end{align*}
$$

Using the results in Lee (2004) and some straightforward calculations we can write the moment generating function of $X_T = \log(S_T)$ given $S_t = e^{t}, V_t = v_t, M_{X_T}(z)$, as

$$
\begin{align*}
  d &= \sqrt{(\rho \sigma v z - \kappa)^2 + \sigma^2_v (z - \zeta^2)}, \\
  C &= \frac{\kappa \xi}{\sigma^2_v} \left( (\kappa - \rho \sigma v z - d)(T - t) \right. \\
  &\quad - 2 \log \left( \frac{(\kappa - \rho \sigma v z)(1 - e^{-d(T-t)}) + d(e^{-d(T-t)} + 1)}{2d} \right) \\
  D &= (1 - e^{-d(T-t)}) \frac{z^2 - \zeta^2}{(\kappa - \rho \sigma v z)(1 - e^{-d(T-t)}) + d(e^{-d(T-t)} + 1)}, \\
  M_{X_T}(z) &= \exp \left( z(t + r(T - t) - \lambda (T - t)(e^{\delta^2/2 + \mu} - 1)) + C + Dv \right. \\
  &\quad + \lambda (T - t)(e^{\delta^2/2 + \mu} - 1) \right). \tag{2.5}
\end{align*}
$$

We claim that this form is better suited for numerical evaluation than the form presented in Lee (2004). Moreover we have that $M_{X_T}(z)$ is analytic in the strip where

$$
(\rho \sigma v \Re z - \kappa)^2 + \sigma^2_v \Im z - (\Re z)^2 > 0,
$$

which is equivalent to

$$
\frac{\sigma v - 2\rho \kappa - \sqrt{4\kappa^2 - 4\rho \sigma v \kappa + \sigma^2_v}}{2\sigma_v(1 - \rho^2)} < \Re z < \frac{\sigma v - 2\rho \kappa + \sqrt{4\kappa^2 - 4\rho \sigma v \kappa + \sigma^2_v}}{2\sigma_v(1 - \rho^2)} \tag{2.6}
$$

We further have that the singularity at the endpoints of the strip will be a branch-type singularity. In order to be able to use the Fourier method for pricing the European call we must require that $E[|S_T^{1+\gamma}|] < \infty$ for some $\gamma > 0$, i.e. we must have

$$
\frac{\sigma v - 2\rho \kappa + \sqrt{4\kappa^2 - 4\rho \sigma v \kappa + \sigma^2_v}}{2\sigma_v(1 - \rho^2)} > 1. \tag{2.7}
$$

This is however not a severe restriction since the parameters must already satisfy the condition with un-strict inequality in order to have that the discounted stock price process is a martingale. The Heston model have exactly the same strip of analyticity while the Merton and the Black-Scholes model have moment generating functions, for the log stock price, which are entire functions.

\textsuperscript{3}Lee (2004) gets the same equation but claims that it is valid only for $\kappa > \rho \sigma v$, but that does not seem to be correct.
The NIG-CIR model

The NIG-CIR model is a Normal inverse Gaussian model that is stochastically time shifted by an integrated Cox-Ingersoll-Ross stochastic volatility process. This model was first described in Carr et al. (2003b). Here the stock price process \( S_t = S_0 \exp(X_t) \), where \( X_t \) is a NIG Lévy process and where \( f_t = \int_0^t V_s ds \), where \( V_t \) has dynamics according to Eq. (2.6). Here the process \( \{V_t\}_{t \geq 0} \) is assumed to be independent of the process \( \{X_t\}_{t \geq 0} \). The moment generating function of \( I_T \) given \( V_t = v_t \), \( M_{I_T^f}(z) \), can be written as

\[
\begin{align*}
  d &= \sqrt{\kappa^2 - 2\sigma_v^2}z \\
  C &= \frac{\kappa\xi}{\sigma_v} \left( (\kappa - d)(T - t) - 2\log \left( \frac{\kappa(1 - e^{-d(T-t)}) + d(e^{-d(T-t)} + 1)}{2d} \right) \right) \\
  D &= (1 - e^{-d(T-t)}) \frac{2z}{\kappa(1 - e^{-d(T-t)}) + d(e^{-d(T-t)} + 1)} \\
  M_{I_T^f|V_t=v_t}(z) &= \exp(C + Dv_t).
\end{align*}
\]

The moment generating function of \( X_1 \) can be written as

\[
M_{X_1}(z) = \delta \left( \sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + z)^2} \right).
\]

Finally we obtain the conditional moment generating function, for \( \log(S_T) \) given \( S_t = e^x, V_t = v_t \) as

\[
\begin{align*}
  \mu &= -M_{I_T^f}(M_{X_1}(1)) \\
  M_{X_{t+\delta j}|S_T}(z) &= M_{I_T^f}(M_{X_1}(z)) \exp(r(T-t) + \mu + s).
\end{align*}
\]

The model has six parameters \( (\xi, \kappa, \sigma_v, \alpha, \beta, \delta) \) and one latent process \( V_t \). The parameters \( (\xi, \kappa, \sigma_v) \) and \( V_t \) has the same interpretations as for the Bates model, \( \alpha \) control the tail behaviour of the log-returns and \( \beta \) controls the skewness, the ratio \( \alpha/\beta \) has similar interpretation as the correlation \( p \) in the Bates model, i.e. negative ratio gives skewness to the left and positive ratio gives skewness to the right and finally \( \delta \) is a shape parameter. When calibrating to real market data one often finds that \( \beta \) is negative giving a skewness to the left in the martingale-measure distribution of the log-returns. Straightforward calculations give that the conditional moment generating function, \( M_{X_{t+\delta j}|S_T}(z) \), is analytic in the strip

\[
\begin{align*}
  \left\{ \begin{array}{ll} 
  -\beta - \alpha < \Re z < -\beta + \alpha, & \text{for } \sqrt{\alpha^2 - \beta^2} < \frac{\kappa^2}{2\sigma_v^2\delta}, \\
  -\beta - H < \Re z < -\beta + H, & \text{for } \sqrt{\alpha^2 - \beta^2} > \frac{\kappa^2}{2\sigma_v^2\delta}, \\
  \end{array} \right.
\end{align*}
\]

where

\[
H = \sqrt{\alpha^2 - \left( \sqrt{\alpha^2 - \beta^2} - \frac{\kappa^2}{2\sigma_v^2\delta} \right)^2}.
\]

Just as for the Bates model we need to put restrictions on the parameters to be able to use the Fourier method for pricing a European call option. Here the restrictions will
be:
\[
\alpha > 1 + \beta, \quad \text{for } \sqrt{\alpha^2 - \beta^2} \leq \frac{\kappa^2}{2\alpha \delta} \\
H > 1 + \beta, \quad \text{for } \sqrt{\alpha^2 - \beta^2} > \frac{\kappa^2}{2\alpha \delta},
\]
where \( H \) is defined above. The singularity at the endpoints of the strip will be a branch-type singularity just as for the Bates model. Note that the usual exponential Lévy NIG-model is obtainable from the NIG-CIR-model by setting \( \sigma_V = 0, \nu_t = \xi = 1 \).

### 3 Calibration

Calibration is the art and science of estimating parameters and current latent states by applying standard or non-standard statistical methods to quoted option prices. This is entangled with several data problems, rendering many standard estimators suboptimal.

A serious data problem is the absence of a single quoted market price. Instead, an ask prices \( c_{\text{Ask}}(K_i, \tau_i) \) and a bid prices \( c_{\text{Bid}}(K_i, \tau_i) \) are quoted. The market price is often approximated by the mid price defined as
\[
c^*_{\tau}(K_i, \tau_i) = \frac{c_{\text{Ask}}(K_i, \tau_i) + c_{\text{Bid}}(K_i, \tau_i)}{2}, \tag{3.1}
\]
which is used when calibrating the option valuation models. Using the mid price will lead to overfitting as any parameter vector yielding option values within the bid-ask spread are acceptable with respect to the arbitrage conditions. It has been suggested to use "soft optimization" to avoid overfitting in this situation, cf. Cont and Tankov (2004).

Other problems can be attributed to parametric modeling. The parametric option valuation model is based on a parametric model for the underlying asset, while the market is made up of individual, interacting, possible irrational people\(^4\). Needless to say, the postulated stochastic process will only be an approximation of the true price process. This means that two additional errors are present in all option valuation models.

- The model is incorrect, as the stochastic process used to describe the dynamics of the underlying asset is only an approximation of the true dynamics. Parameters are chosen such that the model captures qualitative features in market data but it is not possible to find the real dynamics to the underlying asset. All models will therefore include a certain degree of model error.
- The parameters, or rather the properties of the true model, are changing over time. This reflects different states in the economy and different risk preferences as well as being a re-approximation of the complex true model. Time varying parameters make adaptive calibration necessary. Note that the \( \mathbb{Q} \)-dynamics can be time-varying even when the \( \mathbb{P} \)-dynamics is time homogeneous if the market is incomplete. This corresponds to time varying risk preferences as the measure transformation \( d\mathbb{Q}/d\mathbb{P} \), is closely related to the risk preferences of agents.

These problems are usually addressed by using "simple" models that are being re-calibrated frequently. Simple models are less prone to overfitting, and it can be argued that traders, knowing the limitations of simple models, can correct most of their pitfalls. The latter claim is valid for transparent (simple) models, but not necessarily for advanced models.

\(^4\)"I can calculate the motion of heavenly bodies, but not the madness of people." - Sir Isaac Newton
3.1 Prediction error methods

The most common statistical method for calibration is the prediction error method (PEM), see Ljung (1999), although the calibration procedure often is denoted by another name. Prediction error methods minimizing a function of the prediction errors \( e_t(K_i, \tau_i; \theta) = c^*_t(K_i, \tau_i) - c^\text{Model}_t(K_i, \tau_i; \theta) \). Define a scalar-valued function

\[
L_t(\theta) = \sum_{i=1}^{N_t} \sum_{t=1}^{N_t} l(e_t(K_i, \tau_i; \theta))
\]  

(3.2)
as the measure of fit, where \( l \) is a suitable chosen norm. The first sum is taken over time and the second sum is taken over all options that day. Prediction error methods define the estimate as the parameters minimizing \( V_t(\theta) \),

\[
\hat{\theta}_t = \arg\min_{\theta \in \Theta} L_t(\theta).
\]  

(3.3)

PEM is a general class of estimation methods, commonly used in e.g. system identification and signal processing, having nice asymptotical properties and includes many estimators such as least squares estimators as a special case.

3.1.1 Least squares

The standard (textbook) approach, cf. Bates (1996); Hull (2002); Cont and Tankov (2004); Schoutens et al. (2004) on calibration of option valuation models is to choose the parameters as the weighted least squares (WLS) estimate, i.e. the parameter (vector) that minimize the weight sum of the squared difference between the observed midprice and the model predicted price

\[
L^\text{WLS}_t(\theta) = \sum_{i=1}^{N_t} \sum_{t=1}^{N_t} \lambda_{s,i} \left( c^*_t(K_i, \tau_i) - c^\text{Model}_t(K_i, \tau_i; \theta) \right)^2.
\]  

(3.4)

It is optimal from a statistical point of view to choose \( \lambda_{s,i} = 0 \forall s < t \) to increase the adaptiveness of the calibration, and to choose \( \lambda_{s,i} \) proportional to the inverse of squared bid-ask spread, relating the size of the ask-bid spread to the quality of the quoted prices

\[
\lambda_{s,i} \propto \frac{1}{\left( c^\text{Ask}_t(K_i, \tau_i) - c^\text{Bid}_t(K_i, \tau_i) \right)^2}.
\]  

(3.6)

This reduces the general WLS estimator to

\[
\hat{\theta}_t^\text{WLS}(t) = \arg\min_{\theta \in \Theta} \sum_{i=1}^{N_t} \frac{1}{\left( c^\text{Ask}_t(K_i, \tau_i) - c^\text{Bid}_t(K_i, \tau_i) \right)^2} e_t(K_i, \tau_i; \theta)^2.
\]  

(3.7)

Other popular choices of loss function is the Ordinary Least Squares (OLS)

\[
L^\text{OLS}_t(\theta) = \sum_{i=1}^{N_t} \left( c^*_t(K_i, \tau_i) - c^\text{Model}_t(K_i, \tau_i; \theta) \right)^2.
\]  

(3.8)

and Relative Least Squares (R-LS)

\[
L^\text{R-LS}_t(\theta) = \sum_{i=1}^{N_t} \frac{1}{c_t^\text{Ask}(K_i, \tau_i)^2} \left( c^*_t(K_i, \tau_i) - c^\text{Model}_t(K_i, \tau_i; \theta) \right)^2.
\]  

(3.9)
3.1.2 Other norms

The main critiques of quadratic norms is that they are sensitive to outliers and that the economic interpretation of quadratic loss is unclear. These limitations can be avoided by using a weighted $l_1$ norm instead. Popular loss functions include the Mean Absolute Error (MAE)

$$V_{t}^{\text{MAE}}(\theta) = \sum_{i=1}^{N_t} c_i^* (K_i, \tau_i) - c_i^{\text{Model}} (K_i, \tau_i; \theta),$$

(3.10)

and the Mean Absolute Relative Error (MARE)

$$V_{t}^{\text{MARE}}(\theta) = \sum_{i=1}^{N_t} \frac{c_i^* (K_i, \tau_i)}{c_i^{\tau}(K_i, \tau_i)} - c_i^{\text{Model}} (K_i, \tau_i; \theta)\right|,$$

(3.11)

The downside of using these functions are added computational difficulties. Another, economically sound but computationally cumbersome loss function is the number of predicted options within the actual bid-ask spread

$$V_{t}^{\text{IS}}(\theta) = \sum_{i=1}^{N_t} 1_{[C^{\text{bid}}(K_i, \tau_i) < C^{\text{bid}}(K_i, \tau_i)]} c_i^{\text{Model}} (K_i, \tau_i; \theta),$$

(3.12)

where the minus comes from the fact the $V_t(\theta)$ is minimized to find the estimate. This loss function cause problems for most optimization methods as it is possible to change the parameters without changing the value of the loss function. This holds especially for gradient based optimization methods.

3.2 Penalized Prediction error methods

The WLS estimator in Subsection 3.1 has one rather odd feature, if $s_i$ satisfies Eq. (3.5). An implicit assumption is in this case made that old data does not improve the estimation, as the summation only is taken over current data. This can be interpreted as the past is of limited (no) use to predict the current prices, hence are currently available information of limited use for predictions of the future!

The common practice WLS estimator is only using a finite (small) set of observations, causing noisy estimates, and sometimes difficulties in finding the correct parameters. The latter is partially discussed in e.g. Cont and Tankov (2004), where a penalty function $P(\theta, \theta_0)$ is added to the loss function

$$L_t^{P-WLS} (\theta) = L_t^{WLS} (\theta) + \zeta P(\theta, \theta_0).$$

(3.13)

Here, $\theta_0$ is a reference parameter vector and $P(\theta, \theta_0)$ is a positive function having $P(\theta, \theta_0) \geq 0$ and $P(\theta_0, \theta_0) = 0$. Given that $P(\theta, \theta_0)$ is globally convex will, for a sufficiently large $\zeta$ under some additional regularity conditions, make $L_t^{P-WLS} (\theta)$ globally convex, improving the performance of most numerical optimization schemes. However, while the penalty improves the global properties of the WLS method, questions regarding the perhaps overly strict exclusion of data remain unanswered. The penalty may influence the loss function too much, leading to biased parameters.

A particularly nice penalty to use if historical data is available (and one is able to accurately estimate the $P$-measure) is the relative entropy. Let $\theta_0$ be parameters associated with the $P$-measure and let $\theta$ be parameters associated with the $Q$-measure. The, the penalty is given by

$$P(\theta, \theta_0) = \mathbb{E}^Q \left[ \log \frac{dQ}{dP} \right] = \mathbb{E}^P \left[ \frac{dQ}{dP} \log \frac{dQ}{dP} \right],$$

(3.14)
thus choosing a $Q$-measure similar to the historical measure. The drawback of this penalty is that it is often more difficult to estimate the historical measure, than the $Q$-measure, and that we now have two measures to update each time data becomes available.

Another approach is to use the quadratic deviation from the previously estimated parameters ($\theta_0 = \theta_{t-1}$) as penalty. Hence, we get

$$P(\theta, \theta_{t-1}) = (\theta - \theta_{t-1})^2$$

(3.15)

cf. ridge regression or Tikhonov regularization (see e.g. Draper and Smith, 1998, Ch. 17). This penalty should improve hedging as the paths of the estimated parameters are smoothed over time, while being easy to implement. It is also computationally advantageous that it does not require the historical measure to be estimated. We have therefore used the quadratic loss function in this paper.

### 3.3 Self-organizing state space model

The Prediction error methods presented above does not explicitly model the time variation of the parameters, and fails to separate the effects from the measurement errors and the time varying parameters consistently. Furthermore, even though it may be possible to maximize the number of options within the spread a specific day, doing so may cause noisy parameter estimates, leading to bad or at least expensive hedges.

We suggest that the mid price is modeled by formulating the calibration problem as a filtering problem, addressing the option value ambiguity generated by the ask-bid spread. We model the mid price $c^*_t(K_i, \tau_j)$ by

$$c^*_t(K_i, \tau_j) = c^*_{Model}(K_i, \tau_j; \theta, V_t) + \epsilon_t$$

(3.16)

$$V_t \sim P(V_t|V_{t-1}; \theta)$$

(3.17)

where $\epsilon_t$ is a zero mean random vector having covariance matrix $R = \text{diag}(\lambda_t)$, where $\lambda_t$ is defined as in Subsubsection 3.1, $\theta$ is a constant parameter vector and $V_t$ is the latent factor (volatility) propagated using the exact transition kernel when possible (e.g. the Heston model or the Bates model) or else using a discretization of the continuous time dynamics.

However, it does not address the time varying parameters. This is done by reformulating the calibration as a self-organizing state space model, augmenting the latent state vector $V_t$ with the parameter vector $\theta_t$ where the parameters are supposed to evolve apriori as random walks, cf. Anderson and Moore (1979); Kitigawa (1998). Thus the calibration model is given by

$$c^*_t(K_i, \tau_j) = c^*_{Model}(K_i, \tau_j; \theta_t, V_t) + \epsilon_t$$

(3.18)

$$\theta_t = \theta_{t-1} + \eta_t$$

(3.19)

$$V_t \sim P(V_t|V_{t-1}; \theta_{t-1})$$

(3.20)

Here $\epsilon_t$ and $\eta_t$ are pairwise independent zero mean random vectors, $\eta_t$ having covariance matrix $Q$. Estimating the parameters and the latent volatility is thus transformed into filtering of latent processes, a problem for which statistical methods has been developed during the last decades.
Non-linear filtering

The filtering problem is the problem of finding the distribution of a latent, vector valued dynamic Markov process from noisy measurements. Let \( \{ y_t \} \) denote the observations \( y_1, \ldots, y_t \) from \( Y_{1:t} \) and let \( \{ x_0 \} \) be the values of the latent process \( X_{0:t} \). The filtering problem, i.e. recovering the posterior distribution \( p(x_{0:t} | y_{1:t}) \), can be solved if the measurement density \( p(y_t | x_t) \), the state transition density \( p(x_{t+1} | x_t) \) and initial state \( p(x_0) \) are known.

Starting from the simultaneous density \( p(x_{0:t}, y_{1:t}) \) and applying Bayes’ theorem yields

\[
p(x_{0:t} | y_{1:t}) = \frac{p(y_{1:t} | x_{0:t}) p(x_{0:t})}{\int p(y_{1:t} | x_{0:t}) p(x_{0:t}) \, dx_{0:t}}.
\]

This expression can be rewritten as

\[
p(x_{0:t} | y_{1:t}) = \frac{p(y_{1:t} | x_{0:t}) p(x_{0:t})}{\int p(y_{1:t} | x_{0:t}) p(x_{0:t}) \, dx_{0:t}}.
\]

The recursive filter equations may look simple but can rarely be solved analytically. Two famous examples that can be solved are the finite state filter (also known as the Hidden Markov Model filter) and the Kalman filter, which solves the filter equations for linear, Gaussian systems, see Kalman (1960); Kalman and Bucy (1961).

Extended Kalman Filter

Extended Kalman filters, see Anderson and Moore (1979); Maybeck (1982), solve the non-linear filtering problem by approximating the distributions in the filter equations by Gaussian distributions, leading to explicit updating equations. This is done by linearizing the state transition process and the measurement equation. Assume that the system of interest can be written as

\[
\begin{align*}
y_t &= h(x_t) + \varepsilon_t \quad (3.26) \\
x_t &= f(x_{t-1}, \eta_t) \quad (3.27)
\end{align*}
\]

and assume that the initial density \( p(x_0) \) is Gaussian having mean \( m_0 \) and covariance \( R_0, \varepsilon \sim N(0, R) \) and \( \eta \sim N(0, Q) \). The extended Kalman filter is the given by
Extended Kalman Filter

Initialization: Let \( p(x_0) \sim N(m_0, P_0) \).

Propagate: Propagate mean and covariance as

\[
m_{t+1|t} = f(m_t, 0) \tag{3.28}
\]
\[
P_{t+1|t} = F(m_t, 0)P_tF^T(m_t, 0) + G(m_t, 0)QG^T(m_t, 0) \tag{3.29}
\]

Updating: The mean and covariance of the filter density is given by

\[
K = P_{t+1|t}H^T (m_{t+1|t}) \left( H(m_{t+1|t})P_{t+1|t}H^T (m_{t+1|t}) + R \right)^{-1} \tag{3.30}
\]
\[
m_{t+1|t+1} = m_{t+1|t} + K(y_{t+1} - h(m_{t+1|t})) \tag{3.31}
\]
\[
P_{t+1|t+1} = P_{t+1|t} - KH^T (m_{t+1|t})P_{t+1|t} \tag{3.32}
\]

and repeat the propagation and updating steps until \( t = T \).

where

\[
F(m, \eta) = \frac{\partial f}{\partial x}(m, \eta), \quad G(m, \eta) = \frac{\partial f}{\partial \eta}(m, \eta), \quad H(m) = \frac{\partial h}{\partial x}(m). \tag{3.33}
\]

The linearization requires the system to be fairly linear to be accurate, and bad linearizations can cause the filter estimate of the latent state to diverge.

Iterated Extended Kalman Filter

A simple improvement of the extended Kalman filter is the iterated extended Kalman filter, see Maybeck (1982), replacing the update recursion by iterating the following recursions a fixed number of iterations or until convergence

\[
K_{t+1} = P_{t+1|t}H^T (\xi_{t+1}) \left( H(\xi_{t+1})P_{t+1|t}H^T (\xi_{t+1}) + R \right)^{-1} \tag{3.34}
\]
\[
\xi_{t+1} = m_{t+1|t} + K_{t+1} (y_{t+1} - h(\xi_{t+1}) - H^T (\xi_{t+1})(m_{t+1|t} - \xi_{t+1})) \tag{3.35}
\]
\[
P_{t+1|t+1} = P_{t+1|t} - K_{t+1}H^T (\xi_{t+1})P_{t+1|t} \tag{3.36}
\]

starting the recursion using \( \xi_0 = m_{0|0} \).

The iterated extended Kalman filter changes the point of linearization from \( m_{t+1|t} \) to \( m_{t+1|t+1} \), producing more accurate approximations of the filter distribution when the measurements are informative, see Lefebvre et al. (2004).

Particle filters

Particle filters, or sequential Monte Carlo filters, solve the recursive filter equations using Monte Carlo integration and resampling, thus avoiding the difficulties of integrating the equations. Monte Carlo methods are also advantageous when dealing with high dimensional systems as their rate of convergence is independent of the dimension of the integrals involved, contrary to deterministic methods.

Particle filters have been used successfully in e.g. signal processing, automatic control, image analysis and navigation, cf. Doucet et al. (2001); Künsch (2001) and
references therein, solving problems where ordinary non-linear filter algorithms, e.g. Extended Kalman filters have failed. The main limitations of particle filters are the computational requirements and the difficulty to calculate a continuous approximation (with respect to the parameters) of the likelihood. These limitations are not a major problem in this paper as the filter only is used to filter latent states, problems for which the filter for each new observation have complexity $O(N_{\text{part}})$, where $N_{\text{part}}$ is the number of particles.

The performance of the filter can sometimes be improved by using an importance sampler when updating the filter density, cf. equation 3.25. It can be shown, see e.g. Doucet et al. (2000) that the optimal choice of the importance sampler $q(x_t|x_{0:t-1},y_{1:t})$ is given by

$$q(x_t|x_{0:t-1},y_{1:t}) = p(x_t|x_{0:t-1},y_t).$$

(3.37)

This importance sampler will sample the particles in relevant parts of the state space leading to a reduction of particles for a given level of accuracy of the Monte Carlo approximation. We used the sample mean and covariance of the particles $x_{t-1}$ as input to an iterated extended Kalman filter to generate the importance sampler distribution at time $t$.

**Bootstrap filter**

**Initialization:** Sample $x^{(i)}_0 \sim p(x_0)$ for $i = 1, \ldots, N_{\text{part}}$.

**Sample:** $x^{(i)}_t \sim q(x_t|x_{t-1}^{(i)},y_{1:t})$ for $i = 1, \ldots, N_{\text{part}}$, and set $x^{(i)}_{0:t} = \left( x^{(i)}_0 \right.$ $x^{(i)}_t )$.

**Importance weights:** Evaluate $w^{(i)}_t = \frac{p(y_t|x^{(i)}_t)}{q(x^{(i)}_t|x_{t-1}^{(i)},y_{1:t})}$ for $i = 1, \ldots, N_{\text{part}}$ and normalize as $w^{(i)}_t = \frac{w^{(i)}_t}{\sum_{i' = 1}^{N_{\text{part}}} w^{(i')}_{t}}$.

**Resample:** with replacement $N_{\text{part}}$ particles $x^{(i)}_{0:t}$ from $x^{(i)}_{0:t}$ according to $w^{(i)}_t$ and repeat from step **Sample** for $t + 1$.

It can be shown, see Crisan and Doucet (2002) and references therein, that the estimate of the filter density converges under very general conditions. The Bootstrap filter does not need the model to be Gaussian or linear, although any of these properties will often improve the performance of the filter, using clever proposal densities and Rao-Blackwellization techniques, see Doucet et al. (2001) and references therein.

### 3.4 Connections between methods

The ordinary methods for calibration (found in Subsection 3.1) achieves adaptivity by only using the most recent data, failing to accumulate information over time.

It can be shown, see Tarantola (2004), that adding a quadratic penalty to the WLS corresponds to (here recursive) Bayesian estimation of parameters. More specific, the quadratic penalty is similar to postulating a Gaussian random walk dynamics for the parameters; the parameters are therefore maximum aposteriori estimates. The penalized methods will therefore accumulate data over time.
The filters in Subsection 3.3 are similar to penalized estimators as the parameters also have prior densities. The extended Kalman filter and iterated extended Kalman filter use Gaussian approximations, while the particle filter will generate optimal (when \( N_{\text{part}} = \infty \)) forecasts regardless of the distributions, although a finite \( N_{\text{part}} \) is likely to degrade the performance of the filter.

4 Simulation study

In this section we are studying the described filtering techniques on a simulated data set. The results are compared to each other as well as to the results from two weighted least squares methods.

Simulated data

Data was simulated according to the Heston model, cf. Subsection 2.3. The Heston model has five parameters, \( \Theta = (\kappa, \xi, \sigma_V, \rho, \mu) \). Prices have been generated with constant \( \kappa, \sigma_V \) and \( \rho \). \( \mu \) follows a stochastic process according to (2.6), and \( \xi \) has been varied as depicted in Figure 2. The total length of the data set is 200 days. At each day we have priced 35 European call options over a range of different strikes and maturities. We have added noise to the option prices, cf. equation (3.18), to mimic the mid price uncertainty present in real world option prices. The noise is Gaussian with a standard deviation of one fourth of the spread, where the spread was chosen to equal 1.

The set of option prices was split into an estimation set of 20 options per day and a validation set of 15 options per day. The options in the different sets were selected randomly.

Global fit measurements

We have compared the prices using the filter estimates and the simulated market prices using the Average Absolute Error as a percentage of the mean price (APE) and Average Relative Percentage Error (ARPE):

\[
APE = \frac{1}{t} \sum_{t=1}^{t} \sum_{s=1}^{N_t} \frac{N_t}{N_s} \sum_{i=1}^{N_t} \left| c_s^* (K_i, \tau_i) - c_{\text{Model}} (K_i, \tau_i; \hat{\Theta}_s) \right|
\]

\[
ARPE = \frac{1}{t} \sum_{t=1}^{t} \sum_{s=1}^{N_t} \left| c_s^* (K_i, \tau_i) - c_{\text{Model}} (K_i, \tau_i; \hat{\Theta}_s) \right| / c_s^* (K_i, \tau_i)
\]

where \( \hat{\Theta}_s \) denotes the estimated parameter values at day \( s \), \( n_s \) denotes number of options at day \( s \) and \( t \) denotes the total number of days. The performance has also been measured in terms of the proportion of option prices that are inside the spread (IS):

\[
IS = \frac{1}{\sum_{t=1}^{t} \sum_{s=1}^{N_t} \sum_{i=1}^{N_t} \left| c_s (K_i, \tau_i; \hat{\Theta}_s) - c_{\text{Model}} (K_i, \tau_i; \hat{\Theta}_s) \right| (c_s (K_i, \tau_i; \hat{\Theta}_s))}
\]

Binary options

So far all calibration and validation have been with respect to European options. However, it would be interesting to see what impact more correct parameter estimates have
for the prices of exotic types of options. Binary options can without much modification of the pricing method be priced in a fast and accurate way, c.f. Subsection 2.1. Prices have been calculated with the estimated parameters as well as with the correct parameters for a set of options. These prices have then been compared using the APE and ARPE.

Time consumption

The time consumption has been measured by the average time consumption per step (ATPS), i.e. the average time consumption of daily recalibration. Since the time consumption is dependent of factors, not specific to the actual estimation method, e.g. the performance of the computer used, we are more interested in the ordering among the methods than the actual numbers.

4.1 Model robustness and hedging

By fitting a model to option prices at a certain day it is assumed that we can achieve a very good fit at that particular day. The filtering methods collect information from market quotes from a series of days and in that way obtain parameters that are relevant not only at the present day but also for forthcoming days. We will test this assumption by using yesterday’s parameters to price today’s options and see how well these match today’s market quotes. Let the vector $\hat{Z}_t = [\hat{\theta}_t, \hat{X}_t]^T$, where $\hat{\theta}_t$ denotes the vector of estimated parameters and $\hat{X}_t$ denotes the vector of estimated latent processes at day $t$. In the case of Heston we have $\hat{\theta}_t = (\hat{k}_t, \hat{\xi}_t, (\hat{\sigma}_t), \hat{\rho}_t)$ and $\hat{X}_t = \hat{V}_t$. Prices have been calculated using yesterday’s values of the assumed constant parameters, $\hat{\theta}_t$, and today’s values of the latent processes, $\hat{X}_t$ and the stock price $S_t$. For a set of options the prices calculated in this way have been compared with the measured option prices using the measurements APE, ARPE and IS.

The above situation where we are given the stock price $S_t$ and the latent process $X_t$ can be compared with a situation where we are able to perfectly hedge all movements in the stock price and the latent process. In such a situation the difference between a constructed hedge portfolio and market quotes would stem from the difference in the parameters between days and of course the measurement noise. The test outlined in this section tests how well the estimated parameters match the market on consecutive days, which is of great importance from a hedging point of view.

In the next section when we apply this test to market data the results can be used to evaluate the models from a robustness perspective. If the results using yesterday’s parameters differ a lot from the results using today’s parameters, for the filtering methods and the PWLS method, it is likely that the assumed constant parameters are not that constant. This conclusion can not be drawn from differing results for the WLS method since this could be a result of the WLS method overfitting the model to noisy measurements.

4.2 Parameter tracking

The parameters have been estimated using three different filters: Extended Kalman Filter (EKF), Iterated Extended Kalman Filter (IEKF) and Particle Filter with an Iterated Extended Kalman Filter proposal (PFIEKF). For comparing reasons the parameters have also been estimated using Weighted Least Squares techniques with penalty term (PWLS) and without penalty term (WLS), cf. Subsection 3.1.
All routines have been implemented and run in MATLAB 7.1.0.183 (R14) Service Pack 3. The WLS and PWLS methods use Matlab’s function for non-linear least squares problems, \texttt{lsqnonlin}.

In order to make sure that estimated parameters only take values in the allowed parameter space, we are considering parameters transformed to \( \mathbb{R} \). Hence, for the Heston model we are considering the transformed parameters \( \log(k), \log(\xi), \log(\sigma_V), \) \( \text{atanh}(\rho) \) and \( \log(V_0) \). The parameters are assumed to be constant or slowly varying, why we typically set the process noise to have a small variance. For the Heston model we let the process noise covariance matrix \( Q \) have the form:

\[
Q = \begin{pmatrix}
Q_f & 0 & 0 & 0 & 0 \\
0 & Q_f & 0 & 0 & 0 \\
0 & 0 & Q_f & 0 & 0 \\
0 & 0 & 0 & Q_f & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}.
\]

In the simulation study \( Q_f \) is set to \( Q_f = 10^{-4} \) and in the empirical study is \( Q_f \) set to \( Q_f = 5 \cdot 10^{-4} \). The particle filter used 200 particles. For the PWLS we set \( \zeta \) to 10. The filters as well as the WLS were initialized with the correct parameter values.

4.3 Results

The estimated parameters are depicted in Figure 1-3. In addition to the true parameter values and the IEKF estimates the figures also contain the WLS and PWLS estimates. It can be seen that the IEKF and PWLS estimates of the parameter \( \kappa \) are closer to the true parameter values than the WLS estimates. The same holds for \( \sigma_V \) and \( \rho \). The IEKF and PWLS estimates are also much more stable over time compared to the WLS estimates, see Figure 1. Although the IEKF and PWLS estimates are in some sense restrained, these two methods handle the obstacles in the parameter path of \( \xi \) surprisingly well, see Figure 2. It is also seen that all the methods estimate the latent process \( V_t \) very well, see Figure 3.

![Figure 1: Heston simulated data. (a) Estimates and true parameter values for the parameter \( \kappa \). (---) true parameter value, (---) WLS, (---) PWLS and (---) IEKF. (b) Detail.](image-url)
Figure 2: Heston simulated data. Estimates and true parameter values for the parameter $\xi$. (---) true parameter value, (---) WLS, (---) PWLS and (•••) IEKF.

Figure 3: Heston simulated data. Estimates and true parameter values for the parameter $V_0$. (---) true parameter value, (---) WLS, (---) PWLS and (•••) IEKF.

Global fit

Table 1 displays the performance of the global fit for the different estimates. The WLS have the best fit for the estimation set, as expected. However, the IEKF and the PWLS have the best fit on the validation set. Even though the PFIEKF and the EKF estimates have less erratic behavior they can not compete with the other when it comes to global fit on the validation set. However both these methods are much faster than the WLS method, especially for the EKF which is the fastest method of them all.

Binary options

The prices of the binary options are much more accurate using the estimates from the PWLS method or IEKF method than estimates from the WLS method. In this case even the EKF gives better results than the WLS. Looking at Figure 2 and 3 it is seen that all parameter estimates are very close to the true parameters, whereas in Figure 1 the WLS estimates are further away from the true parameter value than the IEKF and WLS estimates. The same holds for the estimated parameters $\sigma_\nu$ and $\rho$. This picture together with the results indicates that the prices of binary options are more sensitive
### Table 1: The Heston model has been calibrated to a simulated estimation set using different calibration methods. The table contains the global fit error measurements for the calibrated model in sample (estimation set) and out of sample (validation set) for the different estimation methods, as well as the global fit error measurement for the calibrated model for a set of binary options. The table also contains a time consumption measurement, ATPS, for each calibration method.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Set</th>
<th>IS</th>
<th>APE</th>
<th>ARPE</th>
<th>Est. APE</th>
<th>ARPE</th>
<th>ATPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>EKF</td>
<td>Est.</td>
<td>0.8542</td>
<td>0.0027</td>
<td>0.0149</td>
<td>0.0015</td>
<td>0.0036</td>
<td>1.2 sec.</td>
</tr>
<tr>
<td></td>
<td>Val.</td>
<td>0.7987</td>
<td>0.0038</td>
<td>0.0141</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IEKF</td>
<td>Est.</td>
<td>0.9665</td>
<td>0.0016</td>
<td>0.0121</td>
<td>0.0010</td>
<td>0.0021</td>
<td>6.2 sec.</td>
</tr>
<tr>
<td></td>
<td>Val.</td>
<td>0.9390</td>
<td>0.0023</td>
<td>0.0115</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PFIEKF</td>
<td>Est.</td>
<td>0.9343</td>
<td>0.0019</td>
<td>0.0133</td>
<td>0.0015</td>
<td>0.0046</td>
<td>28.6 sec.</td>
</tr>
<tr>
<td></td>
<td>Val.</td>
<td>0.8840</td>
<td>0.0028</td>
<td>0.0124</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PWLS</td>
<td>Est.</td>
<td>0.9722</td>
<td>0.0015</td>
<td>0.0120</td>
<td>0.0010</td>
<td>0.0017</td>
<td>25.3 sec.</td>
</tr>
<tr>
<td></td>
<td>Val.</td>
<td>0.9407</td>
<td>0.0023</td>
<td>0.0114</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WLS</td>
<td>Est.</td>
<td>0.9750</td>
<td>0.0014</td>
<td>0.0115</td>
<td>0.0020</td>
<td>0.0045</td>
<td>171.1 sec.</td>
</tr>
<tr>
<td></td>
<td>Val.</td>
<td>0.9230</td>
<td>0.0024</td>
<td>0.0127</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To the values of the parameters $\kappa$, $\sigma_V$, and $\rho$ than the prices of European options. The number of European options one particular day is simply too small to extract enough information to be able to accurately price binary options. The filter estimates on the other hand uses more information, since information has been gathered over a number of days.

**Time consumption**

It has already been pointed out that the IEKF and the PWLS perform equally well. However, the IEKF outperforms the PWLS in terms of speed.

**Model robustness and hedging**

Table 2 displays the performance of the global fit for the different estimates using yesterday’s parameter values as described in Subsection 4.1.

The general lower level in the global fit measurements can be explained by the fact that we are actually moving the parameter $\xi$. However, this should affect the results for the different methods with approximately the same amount. This general decrease in the results for the filtering methods and the PWLS method should thus be seen as an indication of that the parameters are not constant.

The additional decrease for the WLS method is because the WLS method overfits the model to noisy measurements, leading to erratic parameter paths, see Figure 1.

### 5 Empirical study

We have used daily data on S&P 500 index options, from November 11th, 2001 to May 5th, 2003, thus excluding data containing a short term effect following September 11th. The original data set consisted of 38 225 quotes (date, strike, ask price, bid price,
Table 2: Model robustness and hedging results for data simulated according to the Heston model. The table contains the global fit error measurements for the calibrated model using yesterday’s estimated parameter values and today’s estimated value of the latent process and today’s stock price.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Set</th>
<th>IS</th>
<th>APE</th>
<th>ARPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>EKF</td>
<td>Est.</td>
<td>0.7397</td>
<td>0.0049</td>
<td>0.0181</td>
</tr>
<tr>
<td>IEKF</td>
<td>Est.</td>
<td>0.8171</td>
<td>0.0039</td>
<td>0.0158</td>
</tr>
<tr>
<td>PFIEKF</td>
<td>Est.</td>
<td>0.6857</td>
<td>0.0048</td>
<td>0.0179</td>
</tr>
<tr>
<td>PWLS</td>
<td>Est.</td>
<td>0.8249</td>
<td>0.0040</td>
<td>0.0159</td>
</tr>
<tr>
<td>WLS</td>
<td>Est.</td>
<td>0.6709</td>
<td>0.0052</td>
<td>0.0187</td>
</tr>
</tbody>
</table>

index level, time to maturity and risk free interest rate). The raw data was processed according to the following rules:

1. All options having time to maturity less than 6 trading days was excluded. These options are illiquid and highly sensitive to changes in any variable.

2. Options having bid-ask spread larger than 5 have been excluded, in order to eliminate illiquid options. Also options having zero or negative bid-ask spread have been eliminated as these are an effect of non-synchronous trading.

3. The least liquid option when call and put options having identical strike and time to maturity is available, has been eliminated. It is well known that in-the-money are traded infrequently compared to at-the-money or out-of-the-money options. We have therefore excluded in-the-money options when at-the-money or out-of-the-money options are available.

A total of 24,930 unique quotes remain after the processing. It is convenient to transform all quoted put options into call options, using the put-call parity.

In practice, the continuously compounded short rate (c.f. Equation (2.4)), is not constant and known. If we want to price a claim at time $t$ with maturity at time $T$, it is possible to replace $r$ with the zero-coupon-bond (ZCB) yield over the corresponding time period up to maturity $(t, T)$ and replace $\mathbb{Q}$ with the corresponding forward measure, $\mathbb{F}^T$ instead. A drawback with this approach is that we get different dynamics for the underlying stock for different maturities, i.e. for each combination of $t$ and $T$. This is of course due the fact that the ZCB yield varies with $t$ and $T$. From a valuation perspective this causes no serious problem, simply replacing $r$ with the corresponding zero-coupon-yield and set $p(t, T)$ equal to the ZCB price. The pricing formula equation 2.1 will look exactly the same.

However, we need to discuss how the transformation is done as real world indices pay dividends. It is common to approximate the dividend by a dividend yield $q$. The dividend yield depends on time and on time to maturity. We therefore need to estimate the dividend yield accurately and such that the options prices are continuous in the strike level.
It is shown in e.g. Hull (2002) that dividends can be modeled by replacing the current value of the underlying equity or index \( S_t \) by \( S_t e^{-q(t,t+\tau)} \), when modeling a European call option maturing at \( t+\tau \). However, \( q(t,t+\tau) \) is not quoted on the market, why we derive it from option quotes. It is well known from the put-call parity that

\[
C_t(K_i, \tau) - p_t(K_i, \tau) = S_t e^{-q(t,t+\tau)} - K_i p(t,t+\tau). \tag{5.1}
\]

It is possible to estimate the corrected index level \( S_t e^{-q(t,t+\tau)} \) by a sample mean. Rearranging equation 5.1 and adding a noise term \( \xi \) gives

\[
S_t e^{-q(t,t+\tau)} = c_{K_i, \tau}(t) - p_{K_i, \tau}(t) + K_i p(t,t+\tau) + \xi. \tag{5.2}
\]

The corrected index level \( \tilde{S}_t = S_t e^{-q(t,t+\tau)} \) can now be estimated by the sample mean

\[
\tilde{S}_t = \frac{1}{N} \sum_{i=1}^{N} c_{K_i, \tau}^*(t) - p_{K_i, \tau}^*(t) + K_i p(t,t+\tau). \tag{5.3}
\]

The mean is taken over overlapping mid prices. The estimate is stable over different \( K_i \) and reasonably stable over time.

### 5.1 Parameter tracking

We have estimated three models on real data from options on the S&P 500 index, namely Heston, Bates and NIG-CIR. As a first step the data was processed according to the previous section. Thereafter the data was split into an estimation set and a validation set. At each day 20% of the options (randomly selected) were put into a validation set and the rest were put into an estimation set.

In order to investigate the robustness of the estimation methods a number of different estimation-validation set pairs were created. The figures presented in the tables of this section are the means taken over these different sets. For the inside spread measure we also present the measured standard deviation.

It should also be mentioned that the WLS method failed to converge for some sets. In those cases the result was not recorded. However, even though these results do not affect the numbers in the tables, the fact that the WLS method do not always converge to reasonable values should always be mentioned in any discussion regarding robustness issues.

### 5.2 Results

Global fit of the models have been measured in terms of IS, APE and ARPE in the same way as in the simulation study. These results are presented in tables 3, 4 and 5. No evaluation with respect to more exotic contracts has been made.

We have also estimated option prices for the different models using yesterday’s estimated parameter values and present estimated values of the latent process and the stock price. The estimated prices are compared with observed market prices as in the simulation study. The results are presented in tables 6, 7 and 8.

**Global fit and estimation method robustness**

The best fit, in sample and out of sample, is obtained with the WLS method, followed by the WLSP, PFIEKF and IEKF. For the Heston and Bates models we are able to get
good WLS estimates over all of the data sets, whereas the WLS method seems to have problems converging for most of the data sets for the NIG-CIR model.

For the Heston and Bates models it is also seen that the WLS method has higher standard deviation than the other methods, thus indicating a lower level of robustness with respect to the choice of estimation set. Among the other methods it is hard to find any ordering with respect to the robustness deduced from the measured standard deviations.

Model robustness and hedging

Applying the test outlined in Subsection 4.1 to real option data we are now in a situation where we can test the robustness of a market model, as well as investigating which estimation method is most suitable from a hedging perspective. From the tables 6, 7 and 8, it is possible to see some important differences between the methods and models.

First of all we observe a general decrease in global fit among all the models and methods compared with the figures in table 3, 4 and 5. The lower level of global fit for the filtering methods indicates that this decrease is partly due to moving parameters.

We also observe that the difference between the results are even greater for the WLS method. This indicates that the WLS overfits the model to noisy data, while the filtering techniques are better of estimating the constant parameters from the noisy data. Whereas the WLS estimates were very good at explaining the present option prices, they have problems explaining option prices the next day. Further more we see that the results get worse with increased model complexity. This even strengthens the assumption about overfitting. Also it can be seen that the simplest model (Heston) that was the least capable at fitting today’s data was one of the best at predicting tomorrow’s prices.

As a last point we make the observation that there is actually a noticeable difference between the results for the IEKF and the PWLS method. Whereas they perform equally well for the Heston model there is a big difference between the results for the Bates and NIG-CIR models. We make the interpretation that there is a need for adaptive models in order to find sound parameter estimates.

6 Conclusions

The filter technique proposed in the paper has been evaluated on simulated and real option data, applied to three different models. The filter estimates have been compared to the standard WLS estimates as well as to estimates from a penalized WLS method. For simulated data using the Heston model we get the results:

- The filter and PWLS estimates are much closer to the true parameter values of the parameters $\kappa$, $\sigma_v$ and $\rho$ than the WLS estimates. This indicates that the WLS overfits the model to noisy measurements. For the parameter $\xi$ and the latent process $V_t$ all of the calibration methods gives accurate estimates.

- The WLS method have the best fit in sample as expected. However, out of sample the PWLS method and the IEKF outperforms the WLS method. The advantage of the IEKF and the PWLS over the WLS method is even more pronounced when a set of binary options is used as validation set. Thus, when pricing more exotic contracts there is be a need for this kind of estimation methods.
Filter | Set | IS | APE | ARPE
---|---|---|---|---
EKF | Est. | 0.7270 (0.0018) | 0.0072 | 0.0190
     | Val. | 0.7164 | 0.0076 | 0.0197
IEKF | Est. | 0.7461 (0.0019) | 0.0068 | 0.0176
     | Val. | 0.7360 | 0.0073 | 0.0182
PFIEKF | Est. | 0.7545 (0.0023) | 0.0067 | 0.0173
      | Val. | 0.7433 | 0.0072 | 0.0180
WLSP | Est. | 0.7525 (0.0022) | 0.0067 | 0.0173
      | Val. | 0.7420 | 0.0072 | 0.0180
WLS | Est. | 0.7780 (0.0035) | 0.0063 | 0.0166
     | Val. | 0.7602 | 0.0069 | 0.0177

Table 3: The Heston model has been calibrated to a number of S&P 500 data sets using the different estimation methods. The table contains the global fit error measurements for the calibrated model in sample (estimation set) and out of sample (validation set) for the different estimation methods. The numbers presented are the mean values taken over the results from the estimation on the different data sets. Standard deviations have been calculated for the inside spread measure and are displayed within brackets.

Filter | Set | IS | APE | ARPE
---|---|---|---|---
EKF | Est. | 0.7894 (0.0014) | 0.0063 | 0.0158
     | Val. | 0.7799 | 0.0067 | 0.0168
IEKF | Est. | 0.8388 (0.0016) | 0.0054 | 0.0137
      | Val. | 0.8269 | 0.0058 | 0.0147
PFIEKF | Est. | 0.8375 (0.0037) | 0.0054 | 0.0134
      | Val. | 0.8244 | 0.0058 | 0.0145
WLSP | Est. | 0.8384 (0.0017) | 0.0053 | 0.0129
     | Val. | 0.8250 | 0.0057 | 0.0144
WLS | Est. | 0.8842 (0.0059) | 0.0051 | 0.0129
      | Val. | 0.8675 | 0.0055 | 0.0147

Table 4: The Bates model has been calibrated to a number of S&P 500 data sets using the different estimation methods. The table contains the global fit error measurements for the calibrated model in sample (estimation set) and out of sample (validation set) for the different estimation methods. The numbers presented are the mean values taken over the results from the estimation on the different data sets. Standard deviations have been calculated for the inside spread measure and are displayed within brackets.
Table 5: The NIG-CIR model has been calibrated to a number of S&P 500 data sets using the different estimation methods. The table contains the global fit error measurements for the calibrated model in sample (estimation set) and out of sample (validation set) for the different estimation methods. The numbers presented are the mean values taken over the results from the estimation on the different data sets. Standard deviations have been calculated for the inside spread measure and are displayed within brackets. The standard deviation is missing for the WLS because the method did not converge for enough data sets.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Set</th>
<th>IS</th>
<th>APE</th>
<th>ARPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>EKF</td>
<td>Est.</td>
<td>0.8264</td>
<td>(0.0013)</td>
<td>0.0056</td>
</tr>
<tr>
<td></td>
<td>Val.</td>
<td>0.8167</td>
<td></td>
<td>0.0060</td>
</tr>
<tr>
<td>IEKF</td>
<td>Est.</td>
<td>0.8526</td>
<td>(0.0021)</td>
<td>0.0051</td>
</tr>
<tr>
<td></td>
<td>Val.</td>
<td>0.8434</td>
<td></td>
<td>0.0055</td>
</tr>
<tr>
<td>PFIEKF</td>
<td>Est.</td>
<td>0.8636</td>
<td>(0.0016)</td>
<td>0.0049</td>
</tr>
<tr>
<td></td>
<td>Val.</td>
<td>0.8526</td>
<td></td>
<td>0.0053</td>
</tr>
<tr>
<td>WLSP</td>
<td>Est.</td>
<td>0.8673</td>
<td>(0.0011)</td>
<td>0.0049</td>
</tr>
<tr>
<td></td>
<td>Val.</td>
<td>0.8549</td>
<td></td>
<td>0.0053</td>
</tr>
<tr>
<td>WLS</td>
<td>Est.</td>
<td>0.8737</td>
<td></td>
<td>0.0046</td>
</tr>
<tr>
<td></td>
<td>Val.</td>
<td>0.8604</td>
<td></td>
<td>0.0050</td>
</tr>
</tbody>
</table>

Table 6: Model robustness and hedging results for the Heston model calibrated to S&P 500 data. The table contains the global fit error measurements for the calibrated model using yesterday’s estimated parameter values and today’s estimated value of the latent process and today’s stock price.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Set</th>
<th>IS</th>
<th>APE</th>
<th>ARPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>EKF</td>
<td>Est.</td>
<td>0.5757</td>
<td></td>
<td>0.0119</td>
</tr>
<tr>
<td>IEKF</td>
<td>Est.</td>
<td>0.5761</td>
<td></td>
<td>0.0110</td>
</tr>
<tr>
<td>PFIEKF</td>
<td>Est.</td>
<td>0.5648</td>
<td></td>
<td>0.0116</td>
</tr>
<tr>
<td>WLSP</td>
<td>Est.</td>
<td>0.5639</td>
<td></td>
<td>0.0117</td>
</tr>
<tr>
<td>WLS</td>
<td>Est.</td>
<td>0.4467</td>
<td></td>
<td>0.0188</td>
</tr>
</tbody>
</table>

Table 7: Model robustness and hedging results for the Bates model calibrated to S&P 500 data. The table contains the global fit error measurements for the calibrated model using yesterday’s estimated parameter values and today’s estimated value of the latent process and today’s stock price.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Set</th>
<th>IS</th>
<th>APE</th>
<th>ARPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>EKF</td>
<td>Est.</td>
<td>0.5586</td>
<td></td>
<td>0.0119</td>
</tr>
<tr>
<td>IEKF</td>
<td>Est.</td>
<td>0.5925</td>
<td></td>
<td>0.0113</td>
</tr>
<tr>
<td>PFIEKF</td>
<td>Est.</td>
<td>0.5715</td>
<td></td>
<td>0.0120</td>
</tr>
<tr>
<td>WLSP</td>
<td>Est.</td>
<td>0.4704</td>
<td></td>
<td>0.0170</td>
</tr>
<tr>
<td>WLS</td>
<td>Est.</td>
<td>0.3200</td>
<td></td>
<td>0.0299</td>
</tr>
<tr>
<td>Filter</td>
<td>Set</td>
<td>IS</td>
<td>APE</td>
<td>ARPE</td>
</tr>
<tr>
<td>--------</td>
<td>-----</td>
<td>-----</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>EKF</td>
<td>Est.</td>
<td>0.4864</td>
<td>0.0138</td>
<td>0.0299</td>
</tr>
<tr>
<td>IEKF</td>
<td>Est.</td>
<td>0.4975</td>
<td>0.0135</td>
<td>0.0290</td>
</tr>
<tr>
<td>PFIEKF</td>
<td>Est.</td>
<td>0.4558</td>
<td>0.0151</td>
<td>0.0320</td>
</tr>
<tr>
<td>WLSP</td>
<td>Est.</td>
<td>0.4247</td>
<td>0.0219</td>
<td>0.0428</td>
</tr>
<tr>
<td>WLS</td>
<td>Est.</td>
<td>0.3050</td>
<td>0.0370</td>
<td>0.0757</td>
</tr>
</tbody>
</table>

Table 8: Model robustness and hedging results for the NIG-CIR model calibrated to S&P 500 data. The table contains the global fit error measurements for the calibrated model using yesterday’s estimated parameter values and today’s estimated value of the latent process and today’s stock price.

- From a hedging or robustness perspective the IEKF and the PWLS method outperforms the WLS method. This can be seen from a lower global fit of prices, calculated using yesterday’s estimated parameters and today’s estimated latent process, for the WLS method than for the IEKF and the PWLS method. This indicates that the IEKF and the PWLS are better at estimating parameters that are relevant not only for the present day but also for consecutive days.

- The IEKF outperforms the PWLS method in terms of speed.

Using real option data consisting of put and call options on the S&P 500 index we see that:

- The WLS method has the best fit both in and out of sample. However, the WLS method seems to have robustness problems indicated by a higher variation among the results for the different calibration sets, than the other methods. We also find that the WLS method is having problems converging for the NIG-CIR model.

- When pricing options, using yesterday’s estimated parameters and today’s estimated latent process and today’s stock price, we find that:
  - There is a general decrease in global fit over all models and all methods. The decrease for the filtering methods and the PWLS indicates that the assumed constant parameters are actually moving
  - The decrease in global fit is even larger for the WLS method than for the other methods, which indicates that the WLS method overfits the models to noisy measurements. Furthermore, we find that the simplest model (Heston) is the most capable of explaining consecutive day’s market situation.
  - For two of the models there is a noticeable difference between the results for the PWLS method and the IEKF, indicating that there is a need for adaptive calibration methods, such as filtering methods.

These results provide evidence that the self-organizing state space model, combined with an iterated extended Kalman filter is a promising method for option calibration. The iterated extended kalman filter is shown to be both fast and accurate when applied to different market models. We believe that a similar behaviour can be expected from the filter for other models and other datasets.
References


