Steve O. Rice (1907 – 1986)
useful noise inspires statistics research

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Statistics afternoon
Risk, Noise, and Extreme events
–
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Extended version of invited talk at
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1 Steve (Stephen) Oswald Rice
   - His person
   - His profession and his work

2 Mathematical analysis of random noise
   - The paper
   - Why is it important

3 Rice’s formula
   - Number of level crossings
   - Early, and some late, applications of Rice’s formula
   - Application theme 1: Level crossings and extremes
   - Application theme 2: Random fields

4 Summary and references
Steve (Stephen) Oswald Rice, 1907–1986

Steve Rice was one of three “most influential American researchers” during the 1940s on statistical radio communication theory – the others were Claude Shannon and Norbert Wiener.
Some details on Steve Rice

- Born November 29, 1907, in Shedds, Oregon, USA – a small town of about 800 people
- Father (Stephen Rice) a butter maker, mother Selma Bergren
- Selma’s father was born in Sweden, mother of Swedish origin
- Degree in Electrical engineering from Oregon State University (1929)
- Started a PhD program in Physics at CalTech but never finished
- Continued PhD work at Columbia University, NY – Submitted PhD thesis in the early 1940s – rejected!
- Honorary doctor with OSU 1961
Rice, Bell Telephone and Bell Labs

- Joined Bell Telephone Laboratories, New York, 1930
- to do independent research in communication theory
- and to be a resource in communication engineering
- Obtained encyclopedic knowledge in applied mathematics (special functions) in the Bateman project
- Retired from Bell Labs, Murry Hill, NJ, 1972 as “Head, Communication Theory Department”
- Continued research at University of California, La Jolla, until his death, 1986
What did he do for statistics?

- gave communication theory a firm statistical ground
- inspired a world of engineering to use stochastic processes on real problems –
  - random vibration in aviation
  - stochastic control
  - random fatigue in mechanical engineering
  - random ocean waves
  - reliability in structural engineering
- problems and solutions were all formulated in a stochastic language within 10 years
- presented the statistical community a wealth of research problems – it took a little longer
What became of the rejected PhD thesis? MARN 1944-1945

Mathematical Analysis of Random Noise
By S. O. Rice

INTRODUCTION

This paper deals with the mathematical analysis of noise obtained by passing random noise through physical devices. The random noise considered is that which arises from shot effect in vacuum tubes or from thermal agitation of electrons in resistors. Our main interest is in the statistical properties of such noise and we leave to one side many physical results of which Nyquist's law may be given as an example.1

About half of the work given here is believed to be new, the bulk of the new results appearing in Parts III and IV. In order to provide a suitable introduction to these results and also to bring out their relation to the work of others, this paper is written as an exposition of the subject indicated in the title.

When a broad band of random noise is applied to some physical device, such as an electrical network, the statistical properties of the output are often of interest. For example, when the noise is due to shot effect, its mean and standard deviations are given by Campbell's theorem (Part I) when the physical device is linear. Additional information of this sort is given by the (auto) correlation function which is a rough measure of the dependence of values of the output separated by a fixed time interval.

The paper consists of four main parts. The first part is concerned with shot effect. The shot effect is important not only in its own right but also because it is a typical source of noise. The Fourier series representation of a noise current, which is used extensively in the following parts, may be obtained from the relatively simple concepts inherent in the shot effect.

The second part is devoted principally to the fundamental result that the power spectrum of a noise current is the Fourier transform of its correlation function. This result is used again and again in Parts III and IV. A rather thorough discussion of the statistics of random noise currents is given in Part III. Probability distributions associated with the maxima of the current and the maxima of its envelope are developed. Formulas for the expected number of zeros and maxima per second are given, and a start is made towards obtaining the probability distribution of the zeros.

When a noise voltage or a noise voltage plus a signal is applied to a non-

1 An account of this field is given by E. B. Moulin, “Spontaneous Fluctuations of Voltage,” Oxford (1938).

Part II: “... the fundamental result that the power spectrum of the noise current is the Fourier transform of its correlation function.”

Part III: “... Formulas for the expected number of zeros and maxima per second are given, and a start is made towards obtaining the probability distribution of the zeros.”
“In 1944 and 1945, S.O. Rice published a monumental study of noise, generally regarded to be the single most useful source of information about Gaussian noise.” *(Bell Labs history)*

“Mathematical analysis of random noise” (162 pages, Dover) is now cited about 100 times per year in journals on

- reliability of structures, materials
- ocean safety
- communication engineering
- statistics
- physics
“Random noise” in the 1930s and 40s

Why is MARN so important in statistical science history?

Three key contributions from MARN:

- The “environment” (the electronic device, the atmospheric disturbances, the ocean waves) is a stationary/homogeneous random process – often Gaussian, so covariance and mean are the single characteristics.

  Khintchine outlined the properties of a stationary process 1934.

  Cramér gave more mathematical details 1940 –

  and Blanc-Lapierre & Forté and Lévy did so later – but they wrote in French – and so did Emile Borel (Borel-Cantelli) earlier

- Describe the random mechanism in nature – different from “fitting a model to time series data”
It stressed the Fourier relation between the correlation properties and the spectrum and made room for the use of linear (and non-linear) filters to describe the deterministic system response to a random environment, example: ship response to ocean waves. Norbert Wiener 1930

Rice comments in MARN that Khinchine and Cramér lacked contact with important concrete problems (but Cramér’s seminars in Stockholm did!)

It went beyond the simple statistical measures like mean and variance and faced structural properties like local extremes, level crossings, waiting times.
MARN in four parts

I  On shot noise - explains generation of noise in electronic devices

II Power spectra and correlation – Fourier analysis as a tool for stochastic modeling of a random environment

\[ x(t) = \sum_n a_n \cos \omega_n t + b_n \sin \omega_n t = \sum_n c_n \cos(\omega_n t - \phi_n) \]

III Statistical properties of stationary correlated noise
  ▶ height and number of local maxima
  ▶ Rice’s formula for the expected number of level crossings
  ▶ distribution of time between crossings

IV Linear and non-linear filters
Rice’s formula for the expected number of level crossings – 1939

In modern notation:
For a differentiable stationary stochastic process, the expected number of upcrossings per time unit interval $[0, 1]$ is

$$\mu^+_u = f_{X(0)}(u)E(X'(0)^+ \mid X(0) = u)$$

Rice studied maxima of random curves – current reflected by random irregularities along telephone transmission lines.

Rice’s interest was in the height of the local maxima – “Rice distribution” – very practical

NOTE: “Rice distribution” also for the length of a bivariate normal vector
Refinements of Rice’s formula: Variance and conditioning

Variance of the number of crossings:

- **Rice 1945** Implicit in Rice series
- **In physics 1955** For a special class of Gaussian processes
- **Strict 1959** Volkonski & Rozanov; assume $\lambda_6 < \infty$
- **Strict 1965** Leadbetter & Cryer; sufficient
  \[ \int_0^\delta (r''(t) - r''(0))/t \, dt < \infty \]
- **Book 1967** Cramér & Leadbetter: Stationary and related stochastic processes
- **Final 1972** Don Geman; LC condition is also necessary

Conditioning on crossings:

- **Kac & Slepian 1951** Conditioning on crossings by limiting horizontal window
- **Slepian 1963** Representation of process, conditioned on crossings – Slepian model
Two major statistical research themes based on Rice’s formula

Theme 1
Number of and distance between level crossings
Time to first crossing
Applications in reliability and extreme value theory

Theme 2
Geometry of random surfaces
Ocean waves and other moving surfaces
Radio communication beyond the horizon
A Rice series for the distribution of

\[ N_T = \text{number of zero crossings in } [0, T] \]

is an alternating, inclusion-exclusion, series of moments of \( N_T \).

\[
P[N_T = 0] = 1 + \sum_{k=1}^{\infty} \frac{(-1)^k}{k!} E(N_T(N_T - 1) \ldots (N_T - k + 1))
\]

\[
\geq 1 - E(N_T)
\]

\[
\leq 1 - E(N_T) + \frac{1}{2} E(N_T(N_T - 1))
\]

\[
\geq 1 - E(N_T) + \frac{1}{2} E(N_T(N_T - 1)) - \frac{1}{6} E(N_T(N_T - 1)(N_T - 2))
\]

Numerically demanding and slow to converge!
An exemple low-frequency Gaussian noise

Cdf for the time of first exceedance of level $u = 1$. Modern methods solves the problems by very high-dimensional normal integration (RIND).
Have you ever seen such a pdf?

2D pdf of two successive zero crossing intervals for Gaussian process with shifted Gaussian spectrum (left).
red = exact; blue = joint pdf with independent intervals; black = simulated with 10 million pairs
Non-linear extremes – click noise in FM radio

Question June 2003: Wierd FM radio noise

On my FM radio (yes, I am pretty old fashioned), if you tune it to be slightly out of a station or if there is a weak signal or something, you can distinctly here a strange pattern of interference in the background. This consists of a rapid pulsing of white noise at about 100 bpm. As the radio is tuned further from the station, the length and volume of the pulses increase until all the sound is drowned out by static. This is probably reproducible on most radios. Now, I have no idea where this pulsing static comes from, and there seems to be nothing in the visible vicinity that may cause it. (no mobile, computer etc around) So where does it come from?
Noise in FM receivers, explained by Steve Rice 1963

A naive explanation of Frequency Modulated radio:
Sender: Signal $s(t) = A \sin(440 \times t)$ with frequency 440 Hz
Receiver: Complex signal plus noise $\xi(t) = A \exp(i \int s(t)dt) + N(t)$
Differentiate the argument: $s^*(t) = \frac{d}{dt} \arg(\xi(t))$ and smooth it

Trajectories of $\xi(t)$:

Clicks caused by rapid change in $\arg \xi(t)$
A “Slepian model”, named after David Slepian, next office colleague of Steve Rice at Bell Labs, is a regression type explicit random function that describes a Gaussian process conditioned on a level crossing.

Normalized click shape for large signal strength $A^{-1}s^*(t/A)$:

$$R(X + t^2/2) \over (-X + t^2/2 + Nt)^2 + R^2 t^2$$

where $R, X, N$ are random independent exponential, Rayleigh, and normal
Measurement of the Roughness of the Sea Surface from Photographs of the Sun’s Glitter

CHARLES COX AND WALTER MUNK
Scripps Institution of Oceanography,* La Jolla, California
(Received April 28, 1954)
Geometry of random surfaces – theory based on Rice’s formula

After few years “Naval engineering” turned stochastic inspired by Steve Rice.

- Irregular ocean waves – as Gaussian stationary waves from a variety of directions
- Michael S Longuet-Higgins; 20 large papers (1952–1991) on the statistical geometry of random surfaces and waves
- Typical problems: wave height and wave period distributions, wave groups, wave climate over the oceans, extreme (rogue) waves
- Rice himself wrote 1953 on the disturbances on radio communication “far beyond the horizon” – an early example of the earth surface as a random field
Summary: Steve Rice

- Gave telecommunication sharp statistical tools
- Showed an engineering world how to use stochastic processes
- Delivered motivation and basic tools for statistical extreme value theory and applications
References


