Crest/trough and front/back asymmetric waves in wave energy systems

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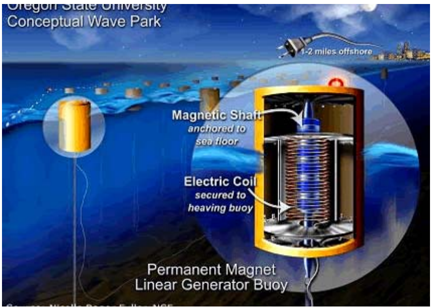
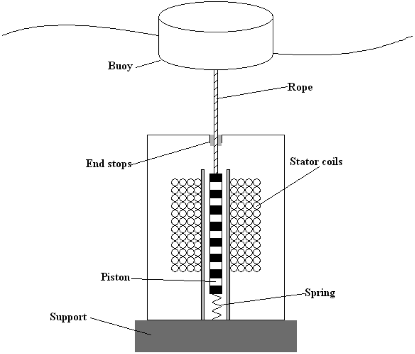
Workshop on
"Can Stochastic Geometry handle
Dynamics of Risk Management?"
Lund, April 18-20, 2018

LUND UNIVERSITY

The origin of the study

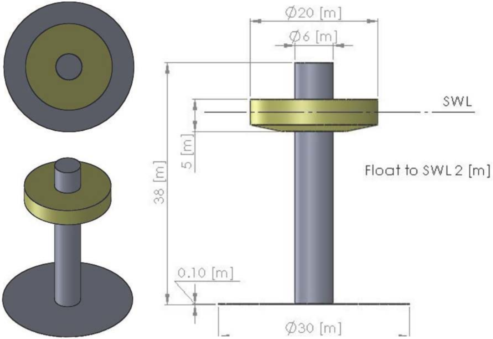
Two wave energy converters: Oregon model and Scandinavian model:





A standard simulation model

From WEC-Sim (Wave Energy Converter SIMulator), an open-source wave energy converter simulation tool in Matlab.



Theoretical calculations with different wave models

Monte Carlo simulation with synthetic waves gives theoretical numbers for energy production. The theoretical effect of a wave energy converter depends on the wave model! Compare deterministic (sine) waves with Gaussian waves for four buoy sizes:

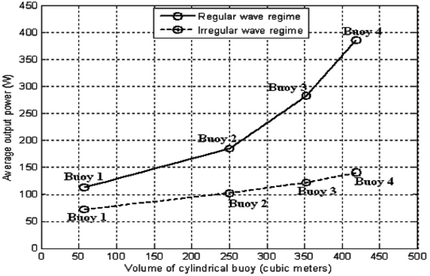


FIG. 6. Calculated electrical output power from the liner generator for each buoy listed in Table I.

Motivating questions

- Is the Gaussian wave model good enough in order to describe the exciting forces in the converter
- or should one use the more complicated wave model that permits asymmetric waves
- For example a Laplace model: $X(t) = \int K(t-u)\, d\Lambda(u)$ with a non-Gaussian spectral measure $\Lambda(u)$ ...
- or the Lagrange wave model – physical motivation exists – ...
- or some other non-Gaussian process ?
- Or perhaps rely only on measurements ?

Gaussian generator for the Lagrange model

In the Gaussian model the vertical height $W(t, x)$ of a particle at the free surface at time t and location x is an integral of harmonics with random phases and amplitudes:

$$W(t, x) = \int_{\omega=-\infty}^{\infty} e^{i(\kappa x - \omega t)} \, \mathrm{d}\zeta(\omega),$$

approximated by a real sum

$$W(t, x) = \sum_k A_k \cos(\kappa_k x - \omega_k t + \phi_k)$$

with random A_k and ϕ_k.

The stochastic Lagrange model –

Describes joint horizontal and vertical movements of individual surface water particles. Use

$$W(t,u) = \int e^{i(\kappa u - \omega t)}\,\mathrm{d}\zeta(\omega)$$

for the vertical movement of a particle with (intitial) reference coordinate u and write $X(t,u)$ for its horizontal location at time t ...

– with horizontal Gaussian movements

... and the same (vertical) Gaussian spectral process $\zeta(\omega)$ as in $W(t,u)$ to generate also the horizontal variation.

$$X_M(t,u) = u + \int \boldsymbol{H}_M(\kappa)\, e^{i(\kappa u - \omega t)}\, \mathrm{d}\zeta(\omega)$$

where the filter function $\boldsymbol{H}_M$ depends on water depth h:

$$\boldsymbol{H}_M(\kappa) = i\, \frac{\cosh \kappa h}{\sinh \kappa h}$$

The stochastic Lagrange model

The 2D stochastic first order free Lagrange wave model is the pair of Gaussian processes

$$(W(t,u), X_M(t,u))$$

All covariance functions and auto-spectral and cross-spectral density functions for $\boldsymbol{\Sigma}(t, \boldsymbol{s})$ follow from the orbital spectrum $S(\omega, \theta)$ and the filter equation.

Space wave : keep time coordinate fixed

Time wave : keep space coordinate – $X_M(t,u)$ – fixed

Front-back asymmetry

The model $\boldsymbol{H}_M(\kappa) = i\,\frac{\cosh \kappa h}{\sinh \kappa h}$ gives front-back statistically symmetric waves.

Adding a slope-dependent term gives asymmetric waves. For example,

$$\partial^2 X(t,u)/\partial t^2 = \partial^2 X_M(t,u)/\partial t^2 + \alpha W(t,u),$$

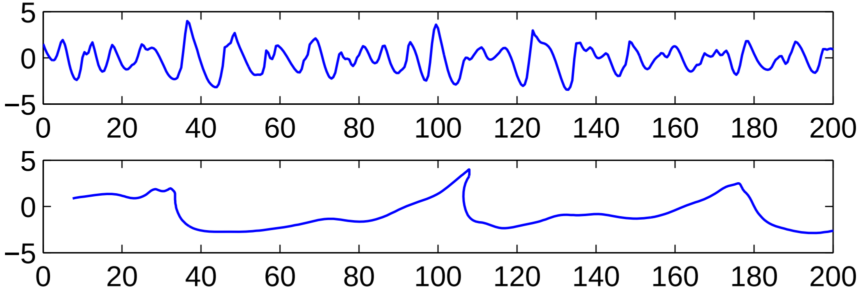
$$\boldsymbol{H}(\kappa) = i\,\frac{\cosh \kappa h}{\sinh \kappa h} + \frac{\alpha}{(-i\omega)^2} = \rho(\omega)\, e^{i\theta(\omega)}$$

Implies an extra phase shift (to the phase $\theta = \pi/2$ in the free model).
A general form for $X(t,u)$ is

$$\boxed{X(t,u) = u + \int e^{i(\kappa u - \omega t + \theta(\omega))} \rho(\omega)\, \mathrm{d}\zeta(\omega)}$$

2D Lagrange waves

Asymmetric Lagrange 2D time waves (top) and space wave (bottom)



Is the Lagrange model useful?

The joint Gaussian character of the vertical, $W(t,u)$, and horizontal, $X(t,u)$, component makes it possible to compute exact statistical distributions of wave characteristics in 2D and 3D:

- wave steepness
- wave asymmetry in time and space
- wave front velocity

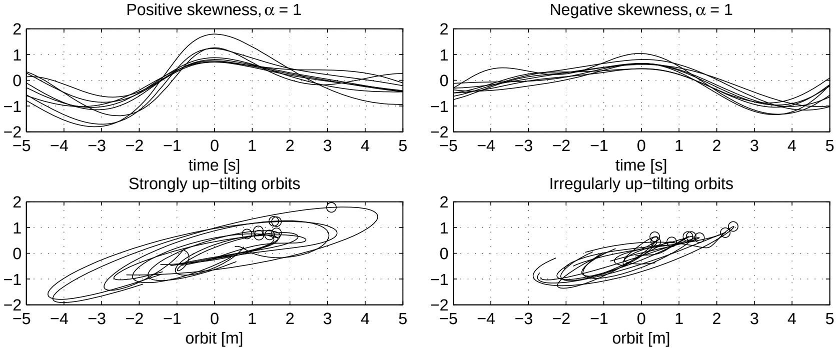
Is the Lagrange model realistic

when it comes to:

- Particle movements - particle orbits Unclear?
- Wave geometry - front-back and crest-trough asymmetry Yes!
- 3D properties - horseshoe-like patterns In theory, yes!

Orbit shape and orientation depend on wave asymmetry

The coupled model with $\alpha = 1$: Wave asymmetry has strong relation to orbit orientation.



Back to wave energy

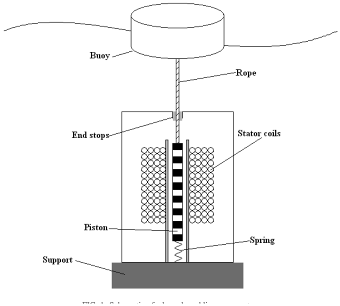
Does wave asymmetry matter in the wave energy example?

The linear wave power extractor as linear filter

Simplistic model: The linear wave power extractor as a linear filter

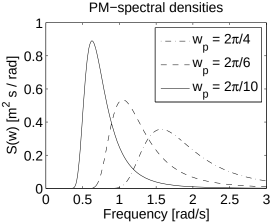
$$mZ''(t) + zZ'(t) + kZ(t) = L(t)$$

where m is total mass, z is total damping, and k depends on the buoy shape and the anchoring spring. $L(t)$ is the sea surface variation and $Z(t)$ is the buoy/piston displacement from equilibrium.
Neglects hydrodynamic forces!



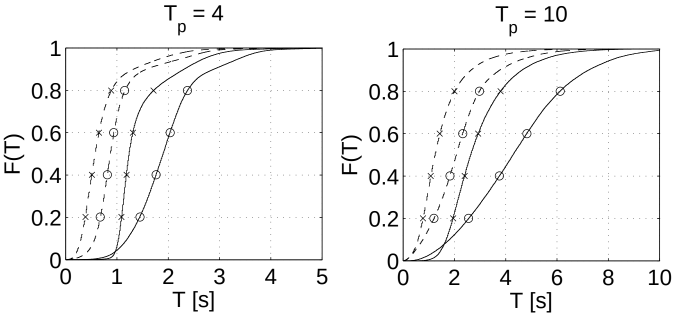
Experimental setup for a numerical Lagrange experiment

- Pierson-Moskowitz orbital spectrum for $W(t, u_0)$
- Water depth $h = 20$m
- Degree of asymmetry: $\alpha = 3$
- Damping z depends on the loading on the generator
- Wave height: $H_s = 2.5$m, Peak period: $T_p = 4$s, 6s, 10s



Front-back asymmetry statistics

Solid lines: CDF's of full crest front (x) and back (o) periods
Dashed curves: CDF's of half crest front (x) and back (o) periods



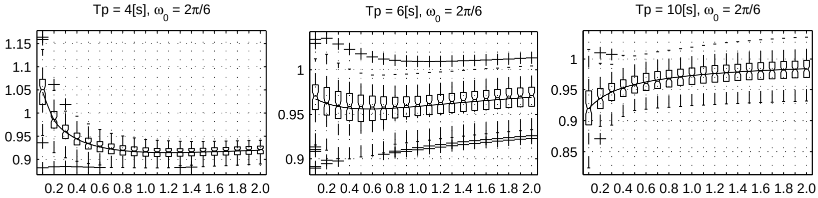
Relative efficiency of converter: Lagrange vs Gaussian

The average power of the converter is

$$P = \gamma \mathsf{E}\left(\left(\frac{\partial Z(t)}{\partial t}\right)^2\right)$$

where $Z(t)$ is the position of the magnet and γ a damping coefficient

Figures show relative efficiency $P_{Lagrange}/P_{Gauss} < 1$ as function of γ



Including hydrodynamic forces - Yingguang Wang, 2018

$$(M+A)Z''(t) + \int_{-\infty}^{t} K(t-\tau)Z'(\tau)\,\mathrm{d}\tau + P_{hs} = P_{wave}(t) + P_{ext}(t) + P_{visc}(t)$$

Here, $Z(t)$ is the height of the buoy as before, and the P_{xx} are all different hydrodynamic forces acting on the buoy.
This can be simulated by WEC-Sim.
It takes the wave time function as input.

Wang uses second order Stokes waves

- Second order Stokes waves are crest/trough asymmetric ...
- ... but front/back statistically symmetric
- With full hydrodynamic equation, the Stokes wave model gives $P_{Stokes}/P_{Gauss} > 1$.
- My question: Does front/back asymmetric Stokes/Lagrange waves + full equation give

$$\frac{P_{Stokes/Lagrange}}{P_{Gauss}} \begin{cases} < 1 \\ > 1 \end{cases}$$

- No answer yet!

Conclusions

- A Lagrange wave model gives realistic asymmetric slope distributions
- Gaussian input in simulations may give too optimistic/pessimistic efficiency
- Model simulations with irregular sea should take asymmetry into account
- Full scale experiment needed

References

- Pierson, W.J.(1961). Model of random seas based on the Lagrangian equations of motion. New York University
- Lindgren, G. and Aberg, S. (2008). First order stochastic Lagrange models for front-back asymmetric ocean waves. *J. OMAE*, **131**
- Lindgren, G. (2010). Slope distribution in front-back asymmetric stochastic Lagrange time waves. *Adv. Appl. Prob.*, **42**
- Lindgren, G. (2015). Asymmetric waves in wave energy systems analysed by the stochastic Gauss-Lagrange wave model. *Proc. Estonian Acad. Sci.*, **64**
- Yingguang Wang (2018). A novel method for predicting the power outputs of wave energy converters. *Acta Mechanica Sinica*, https://doi.org/10.1007/s10409-018–0755-2
- WEC-Sim – Wave Energy Converter SIMulator (2015-2018). http://wec-sim.github.io/WEC-Sim/