Stochastic models in marine sciences

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Overview

Emerging statistics
   Emerging statistics
   Confused seas
   Wind-waves-safety

Wind/waves
   Climate and weather
   Wave climate

Wave models
   Gaussian model
   non-Gaussian models
   Rogue waves
Ocean waves vs regular waves

“In early days, the major difficulties in the study of ocean surface waves were their random properties and the complex mechanisms of their evolution. These properties of ocean surface waves are quite different from those of regular water waves and due to this difference the fundamental studies of ocean water waves were much delayed.

... Modern studies of ocean surface waves started only in the 1940s with the outstanding study by Sverdrup and Munk (1947). ...“Significant waves” are characterized by a kind of mean wave height and mean wave period.”

Hisashi Mitsuyasu, J. Oceanography, 2002
“Confused seas”

► Lord Rayleigh: The basic law of the seaway is the apparent lack of any law ???

► 50th Anniversary of Motions of Ships in Confused Seas marked at WMTC 2003:

► “The tremendous impact that the 1953 SNAME paper On the Motions of Ships in Confused Seas by Manley St. Denis and Willard J. Pierson, which established the foundation for rationally predicting seakeeping characteristics of ships, will be again acknowledged at the upcoming World Maritime Technology Conference in San Francisco.”

► Based on S.O. Rice: On random noise, 1944

► Longuet-Higgins, 1952 –
Statistics everywhere

Statistics in all steps in the chain
- Wind and wave climate
- Wave and ship models
- Safety consequences

Wind climate

Wave climate

Sea state, Hs, Tp

Ship as a dynamic system

Response

Extreme events

Fatigue

Capsizing
Stochastic models in meteorology and climate

**Slow start:** Hasselmann, 1976: Stochastic climate models

**Problem 1:** From climate models to regional weather (wind) models – “Statistics and climatology”
- Ensemble forecasts – next fig

**Problem 2:** Data assimilation in numerical weather models – **IWAP2008:** Monbet, Ailliot, Cuzol
Ensemble prediction systems - EPS

Perturbed initial values
\[ \downarrow \]
Numerical weather prediction systems
\[ \downarrow \]
need for continuous correction of prediction from observed data = data assimilation
Ensemble Prediction Systems (EPS)

Motivation for EPS

![Graph showing PDF(0) and PDF(T) over forecast time]
General difficulties

- High dimensionallity: $202 \times 178 \times 31 \times 8 \approx 9$ mio. per time step and EPS-member (2002, now more)
- NWP, highly non-linear
- Uncertainties enter at different stages, initial conditions, observations, model itself, ...
- Need for simplifications
- Calibration: Global climate models and regional models give not only general pattern but also the statistical variability e.g. in temperature.
- Calibrated against historical data – Northern Atlantic Oscillation index
From wind climate to wave climate

Problem 3: From wind to waves:

- WAM: physically based methods to go from wind fields to significant wave height and wave energy directional spectra

\[ H_s = 4 \times \text{sea surface standard deviation} \]

- Ongoing development - calibration against buoy data

- Summary of “Sea state” characteristics
  - Wave spectrum for wind driven waves
  - Significant wave height, peak period, directional spreading, multi-directionality
$H_s$: time series of buoy data and WAM model values

![Graph showing the comparison between NEW and WAM model predictions for buoy 62081 in the NE Atlantic.](image)
The Gaussian wave model

Introduced by StDenis & Pierson and Longuet-Higgins:
Water surface elevation at time $t$ at location $s = (s_1, s_2)$ is a time and space dependent Gaussian field

$$W(t, s) = \int e^{i(\kappa_1 s_1 + \kappa_2 s_2 - \omega t)} d\zeta(\kappa) \approx \sum A_k \cos(\kappa_1 s_1 + \kappa_2 s_2 - \omega_k t)$$
Wave cycles that can be handled

- max-min cycles $M^+$, $M^-$
- crest-trough cycles $A_c, A_t$
- mean-separated crest-trough cycles $A_c^0, A_t^0$
- rainfall cycles $M^+, M_{Rfc}^-$
- cycle periods $T_{Mm}$, $T_{CT}$, $T_{CT}^0$, $T_{Rfc}$, $T_C$
Gaussian wave models +++

- Fits in linear filters
- All wave distributions from spectrum (Palm!)
- Encountered waves and the stochastic Doppler effect
- Space and time formulations
- Envelope and wave propagation – IWAP2008: Podgorski & Rychlik
- “All” cycle distributions can be computed – depending on the full spectral density
- Wafo – a Matlab programs simple to use
- version from December 2007 available at http://www.maths.lth.se/matstat/wafo/
Where is the difficulty?

- Wave cycles distributions need computation of “infinite dimensional” probabilities, for example, for time between mean up- and down-crossing, (with \( s = 0 \))

\[
f_{T_c}(t) = f_{w(0),w(t)}(0, 0) 
\cdot E \left( w_1'(0)^+ w_1'(t)^- l(t) \mid w(0) = w(t) = 0 \right)
\]

\[
l(t) = l \{ w(x) > 0, 0 < x < t \}
\]

- The infinite dimensional integral can be computed numerically to high accuracy if replaced by a finite-dimensional one – but of high dimension
Capsizing probability, encountered sea

- Cf. IWAP2008: Leadbetter on capsizing
- Green dots = observed capsizing
- Blue area = possible danger
- Joint density of (half) wavelength and amplitude

Joint density of \((L_{cf}, A_c)_{v = 0}\)

Level curves enclosing:

\[
\begin{align*}
&10 & 30 & 50 & 70 & 90 & 95 & 99 \\
&10 & 30 & 50 & 70 & 90 & 95 & 99 \\
&10 & 30 & 50 & 70 & 90 & 95 & 99 \\
&10 & 30 & 50 & 70 & 90 & 95 & 99 \\
&10 & 30 & 50 & 70 & 90 & 95 & 99 \\
&10 & 30 & 50 & 70 & 90 & 95 & 99 \\
&10 & 30 & 50 & 70 & 90 & 95 & 99 \\
&10 & 30 & 50 & 70 & 90 & 95 & 99 \\
\end{align*}
\]
Max-min waves

- simple algorithm gives “exact results” for joint density of height of local maximum \( M \) and following local minimum \( m \):

Joint density of \((M,m)\), rectangular

Joint density of \((M,m)\), Jonswap

Level curves enclosing:
10
30
50
70
90
95
99
99.9
Max-min range

- solid line shows exact density of Max-min range
- dashed line the approximating Rayleigh density

Range density, rectangular spectrum

Range density, Jonswap spectrum
Non-Gaussian models

- Lagrange waves - first order, very promising – second order, better but future work needed

- Second order Stokes waves work quite well - peaked crests and shallow troughs – stochastic properties difficult but possible

- Non-linear Schrödinger equation can describe the Draupner wave – little stochastics known
Lagrange models for particle movement

- In a Gaussian wave model the sea surface at time $t$ and location $s$ is a Gaussian process

$$W(t, s) = \int e^{i(\kappa s - \omega t)} d\zeta(\kappa, \omega)$$

- In a stochastic linear Lagrange model the $W(t, s)$ process is the height of the sea surface at a randomly changing location

$$X_M(t, s) = s + \int H(\kappa) e^{i(\kappa s - \omega t)} d\zeta(\kappa, \omega)$$

which is a linear filtration of the $W$-process ($M =$ Miche-wave)

- gives peaked, crest-trough asymmetric waves

Lagrange space waves, time fixed

Depth = 4m

Depth = 8m

Depth = 16m

Depth = 32m

Depth = ∞
Asymmetric Lagrange waves

To get realistic front-back asymmetry one needs a model with external input from wind, for example:

$$\frac{\partial^2 X(t, s)}{\partial t^2} = \frac{\partial^2 X_M(t, s)}{\partial t^2} + \alpha W(t, s)$$

The slope of the space wave surface at an arbitrary crossing of a level $v$ at time $t_0$ is

$$L = \frac{\partial W(t_0, s) / \partial s}{\partial X(t_0, s) / \partial s}$$

conditioned on a crossing of level $v$ by $W(t_0, s)$. Now, the slope at an upcrossing in a Gaussian process $W$ is Rayleigh. This gives the following distribution of the Lagrange slope – next frame
Slope of asymmetric Lagrange waves

Theorem: The slope of a Lagrange space process observed at (up)crossovers of a level $\nu$ has the same distribution as

$$aR $$

$$1 + b\nu + cR + dU$$

where $a, b, c, d \neq 0$ depend on the correlations between the $W$- and the $X$-process, and $R, U$ are independent standard Rayleigh and normal variables

Aberg and Lindgren: 2008
Modified Lagrange waves with front-back asymmetry

Averaged up- (solid) and down- (dashed) crossings waves

\[ h = \infty, \alpha = 0 \]

\[ h = 32, \alpha = 0 \]

\[ h = 8, \alpha = 0 \]

\[ h = \infty, \alpha = 0.4 \]

\[ h = 32, \alpha = 0.4 \]

\[ h = 8, \alpha = 0.4 \]

\[ h = \infty, \alpha = 0.8 \]

\[ h = 32, \alpha = 0.8 \]

\[ h = 8, \alpha = 0.8 \]
Non-linear waves and response

- A ship on a random wave surface is a non-linear system
- Second order response equation – IWAP2008: Guédé & Prévosto
Rogue waves

First time hard evidence – New years Draupner wave, Jan 1, 1995:
Observations, equations

- In Gaussian model, Crest-trough range $> 2 H_s$ is very unlikely
- Since Draupner, reliable observations from platforms
- Satellite measure individual high waves – still some dispute about how frequent
- Non-linear Schrödinger equation can reproduce, very accurately, observed rogue waves
- But still no generally accepted method to predict under what conditions they are likely to occur
- Crossed seas, current, . . .
References

▫ [http://www.maths.lth.se/matstat/wafo/](http://www.maths.lth.se/matstat/wafo/)