Stochastic models in marine sciences

Georg Lindgren[1] Jörg Wegener[1]

[1]Centre for Mathematical Sciences
Lund University

IWAP 2008
Compiègne, July 10, 2008

Overview

Emerging statistics
- Emerging statistics
- Confused seas
- Wind-waves-safety

Wind/waves
- Climate and weather
- Wave climate

Wave models
- Gaussian model
- non-Gaussian models
- Rogue waves

Ocean waves vs regular waves

"In early days, the major difficulties in the study of ocean surface waves were their random properties and the complex mechanisms of their evolution. These properties of ocean surface waves are quite different from those of regular water waves and due to this difference the fundamental studies of ocean water waves were much delayed.

. . .

Modern studies of ocean surface waves started only in the 1940s with the outstanding study by Sverdrup and Munk (1947).

. . . "Significant waves" are characterized by a kind of mean wave height and mean wave period."

Hisashi Mitsuyasu, J. Oceanography, 2002

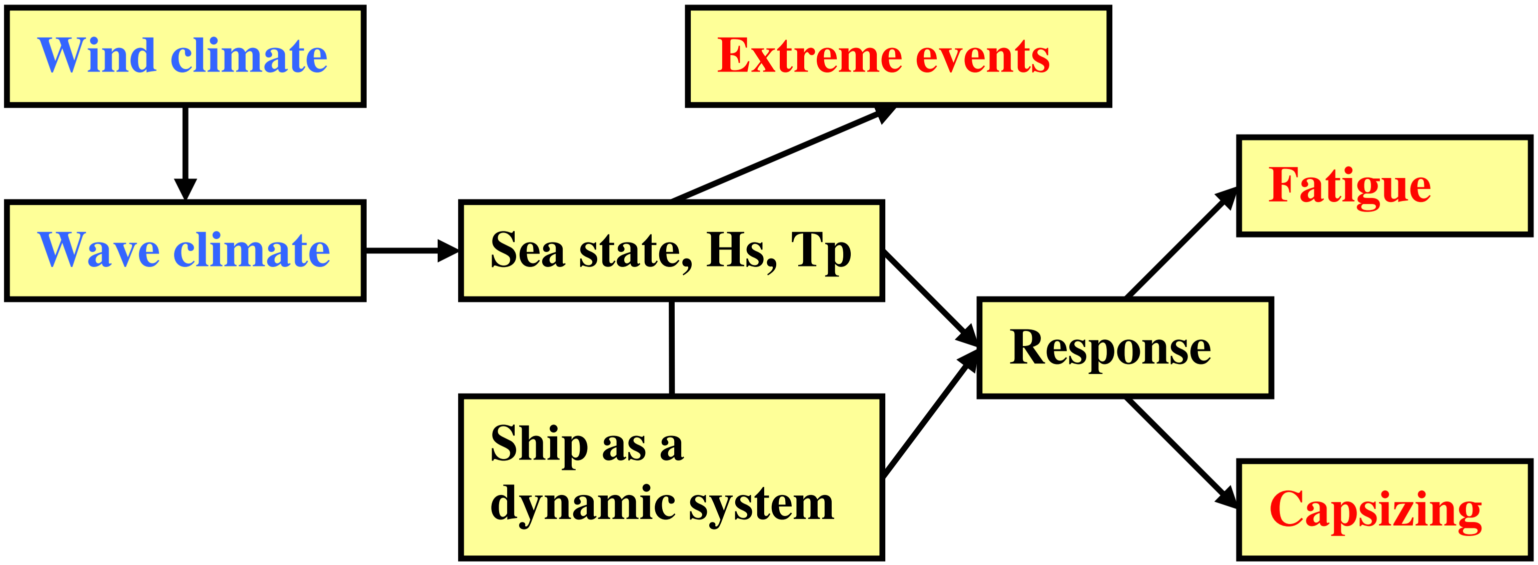
"Confused seas"

- Lord Rayleigh: The basic law of the seaway is the apparent lack of any law ???
- 50th Anniversary of Motions of Ships in Confused Seas marked at WMTC 2003:
- "The tremendous impact that the 1953 SNAME paper *On the Motions of Ships in Confused Seas* by Manley St. Denis and Willard J. Pierson, which established the foundation for rationally predicting seakeeping characteristics of ships, will be again acknowledged at the upcoming World Maritime Technology Conference in San Francisco."
- Based on S.O. Rice: *On random noise*, 1944
- Longuet-Higgins, 1952 –

Statistics everywhere

Statistics in all steps in the chain

- Wind and wave climate
- Wave and ship models
- Safety consequences



Stochastic models in meteorology and climate

Slow start: Hasselmann, 1976: Stochastic climate models

Problem 1: From climate models to regional weather (wind) models – "Statistics and climatology"

- Ensemble forecasts – next fig

Problem 2: Data assimilation in numerical weather models – IWAP2008: Monbet, Ailliot, Cuzol

Ensemble prediction systems - EPS

Perturbed initial values

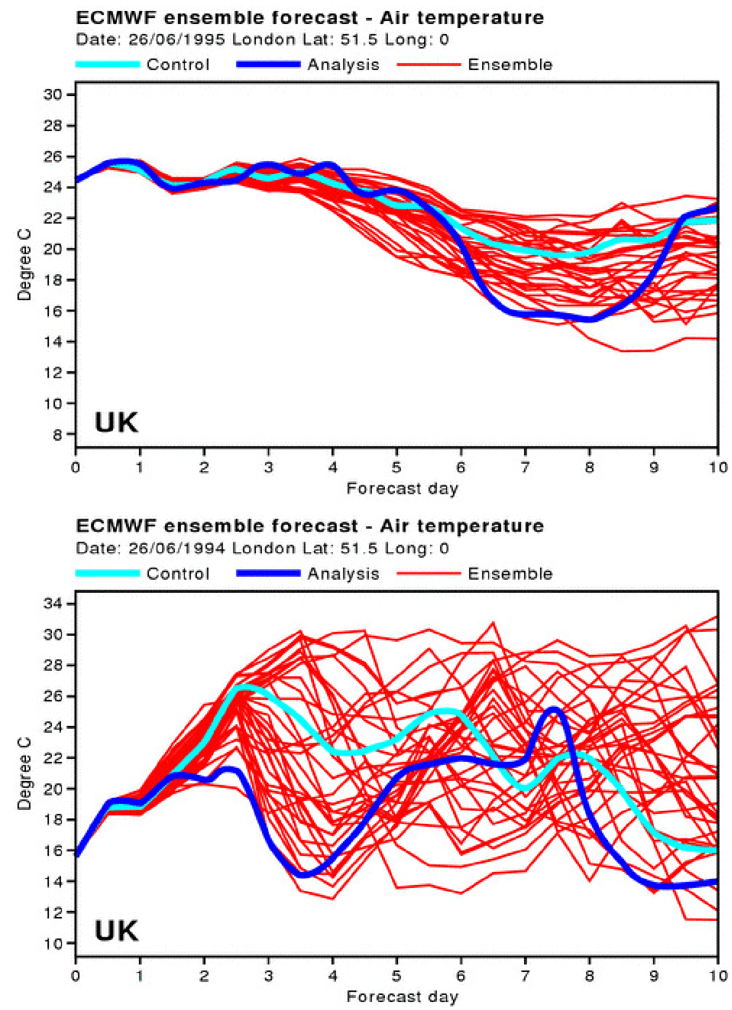
↓

Numerical weather prediction systems

↓

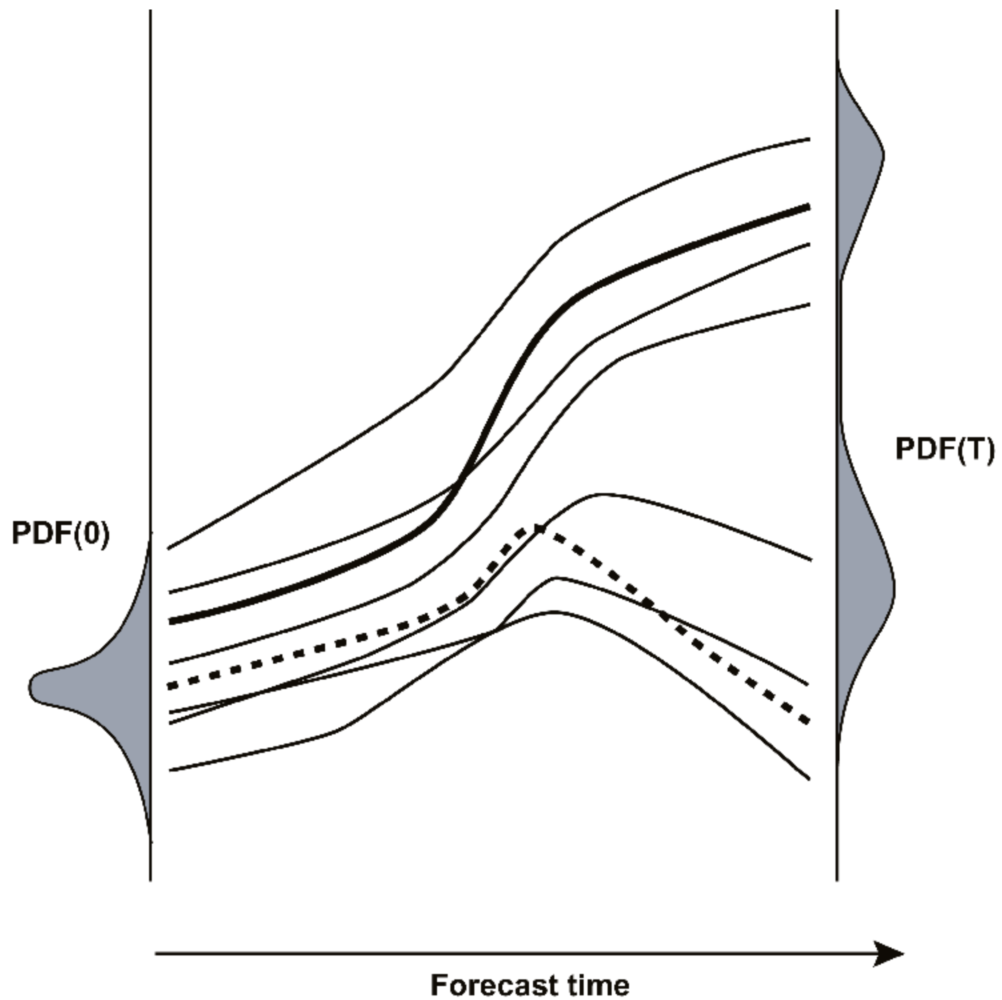
need for continuous correction of prediction from observed data = data assimilation

→



Ensemple Prediction Systems (EPS)

Motivation for EPS



General difficulties

- High dimensionallity: $202 * 178 * 31 * 8 \approx 9$ mio. per time step and EPS-member (2002, now more)
- NWP, highly non-linear
- Uncertaities enter at different stages, initial conditions, observations, model itself, ...
- Need for simplifications
- Calibration: Global climate models and regional models give not only general pattern but also the statistical variability e.g. in temperature.
- Calibrated against historical data – Northern Atlantic Oscillation index

From wind climate to wave climate

Problem 3: From wind to waves:

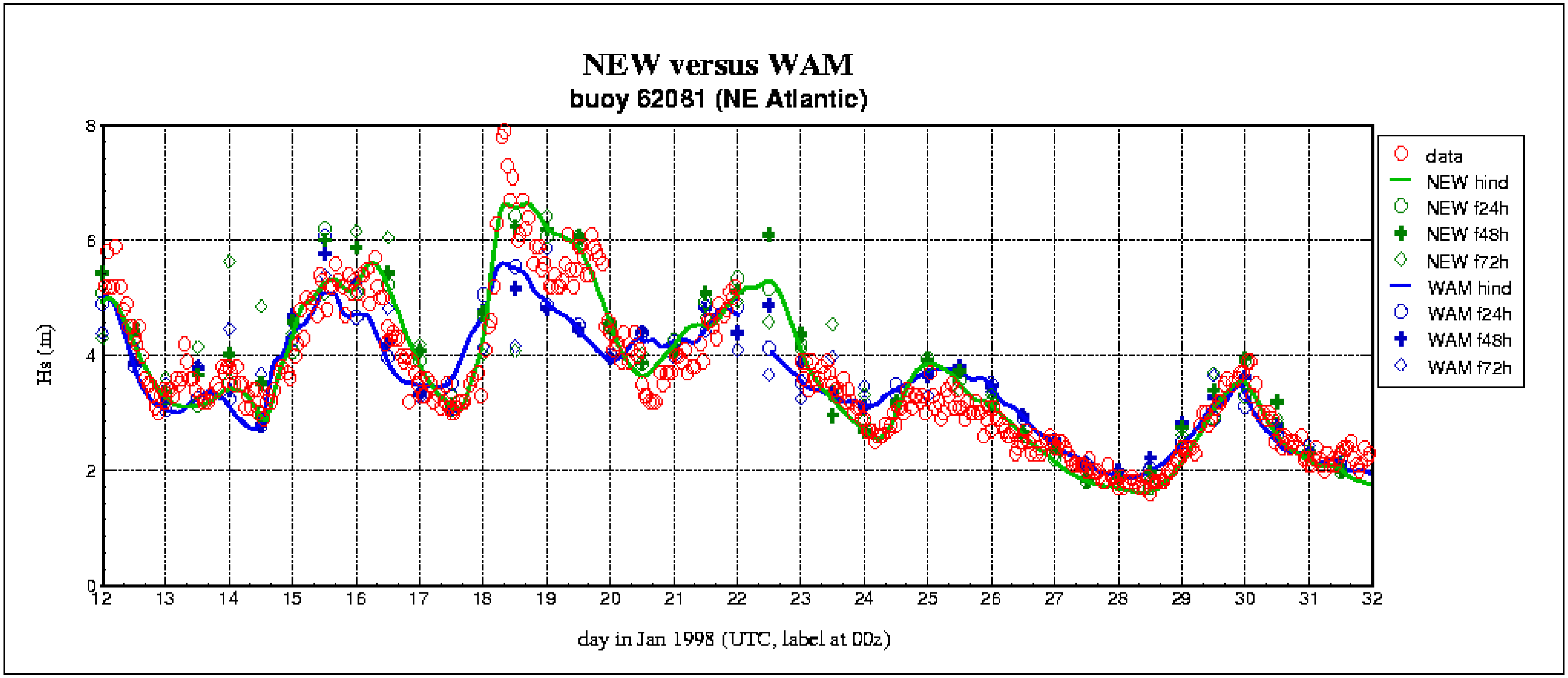
- WAM: physically based methods to go from wind fields to significant wave height and wave energy directional spectra

$$H_s = 4 \times \text{ sea surface standard deviation}$$

- Ongoing development - calibration agaings buoy data

- Summary of "Sea state" characteristics
  - Wave spectrum for wind driven waves
  - Significant wave height, peak period, directional spreading, multi-directionality

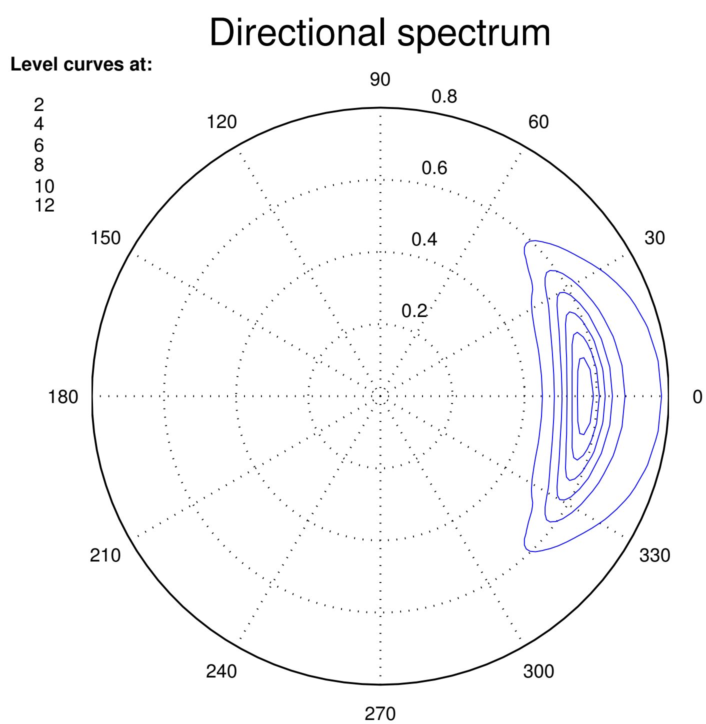
H_s: time series of buoy data and WAM model values



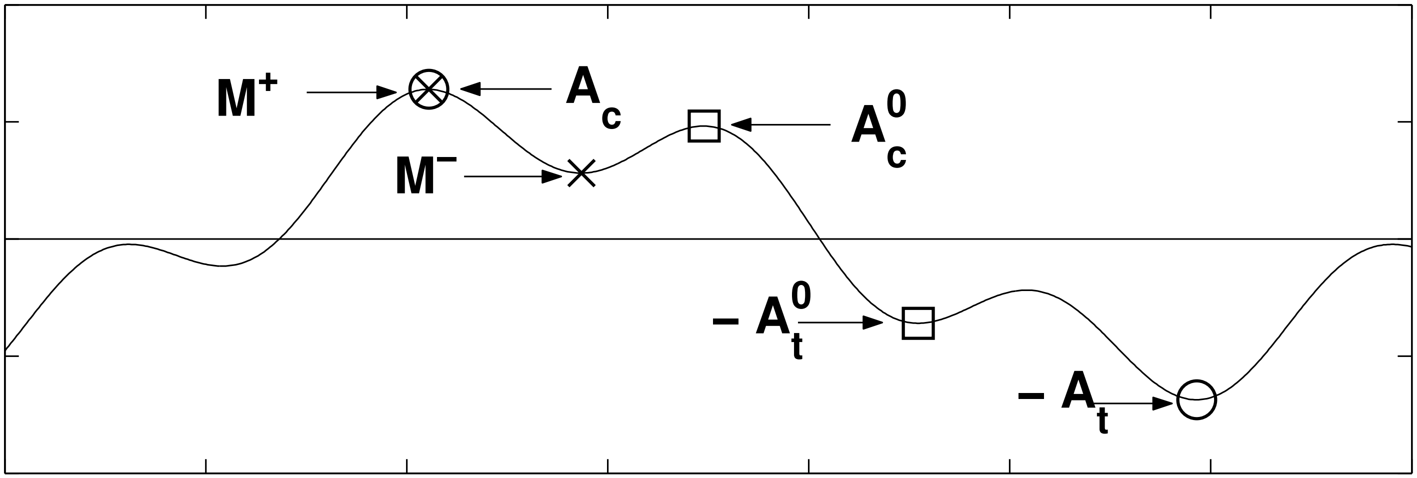
The Gaussian wave model

Introduced by StDenis & Pierson and Longuet-Higgins:
Water surface elevation at time t at location $s = (s_1, s_2)$ is a time and space dependent Gaussian field

$$W(t,s) = \int e^{i(\kappa_1 s_1 + \kappa_2 s_2 - \omega t)}\, d\zeta(\kappa) \approx \sum A_k \cos(\kappa_1^k s_1 + \kappa_2^k s_2 - \omega_k t)$$



Wave cycles that can be handled



- max-min cycles M^+, M^-
- crest-trough cycles A_c, A_t
- mean-separated crest-trough cycles A_c^o, A_t^o
- rainflow cycles M^+, M^-_{Rfc}
- cycle periods T_{Mm}, T_{CT}, T^o_{CT}, T_{Rfc}, T_{C}

Gaussian wave models +++

- Fits in linear filters
- All wave distributions from spectrum (Palm!)
- Encountered waves and the stochastic Doppler effect
- Space and time formulations
- Envelope and wave propagation – IWAP2008: Podgorski & Rychlik
- "All" cycle distributions can be computed – depending on the full spectral density
- Wafo – a Matlab programs simple to use
- version from December 2007 available at `http:/www.maths.lth.se/matstat/wafo/`

Where is the difficulty?

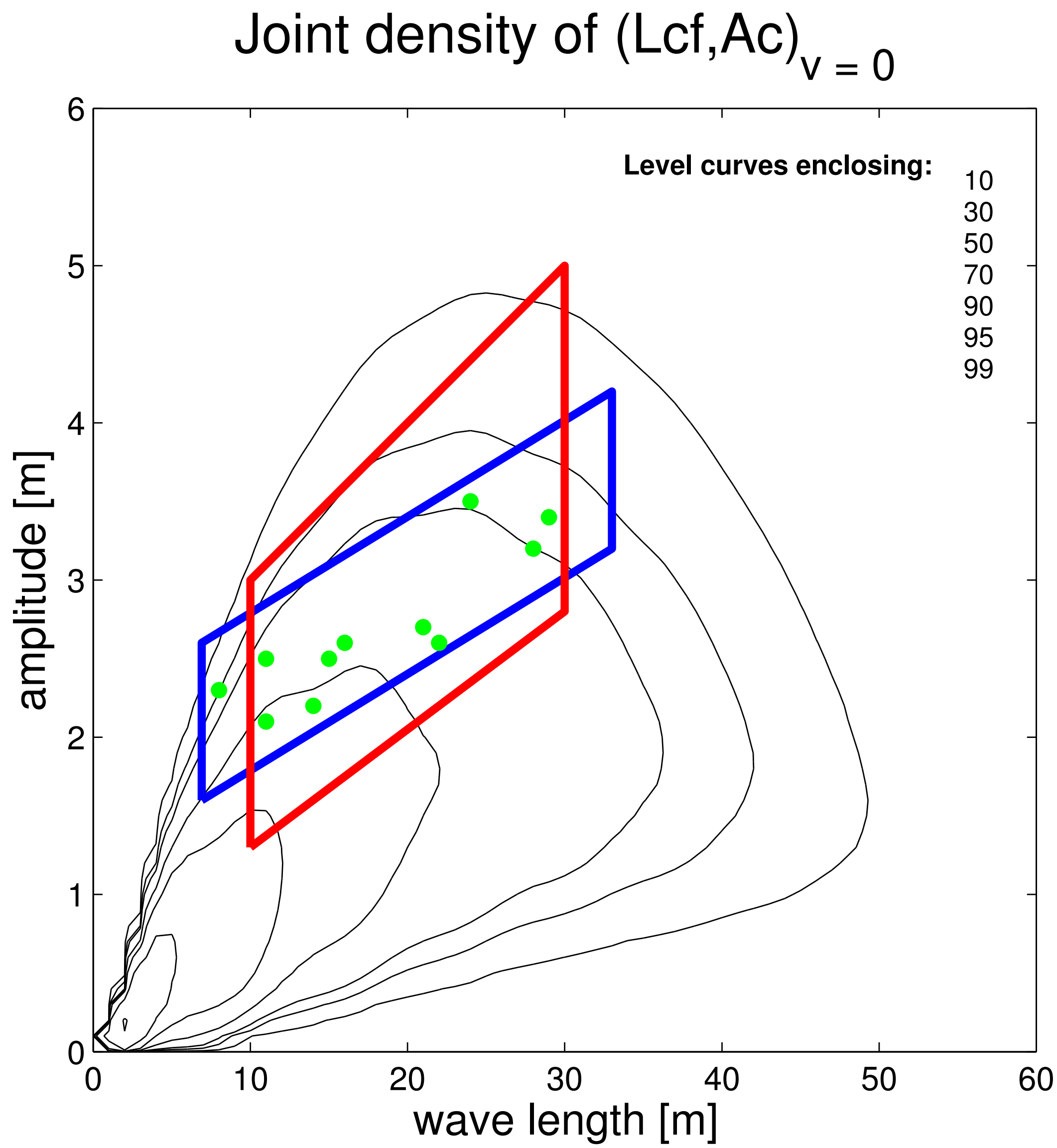
- Wave cycles distributions need computation of "infinite dimensional" probabilities, for example, for time between mean up- and down-crossing, (with $s = 0$)

$$
\begin{aligned}
f_{T_c}(t) &= f_{w(0),w(t)}(0,0) \\
&\quad \cdot E\left(w_1'(0)^+ w_1'(t)^- I(t) \mid w(0) = w(t) = 0\right) \\
I(t) &= I\left\{w(x) > 0, 0 < x < t\right\}
\end{aligned}
$$

- The infinite dimensional integral can be computed numerically to high accuracy if replaced by a finite-dimensional one – but of high dimension

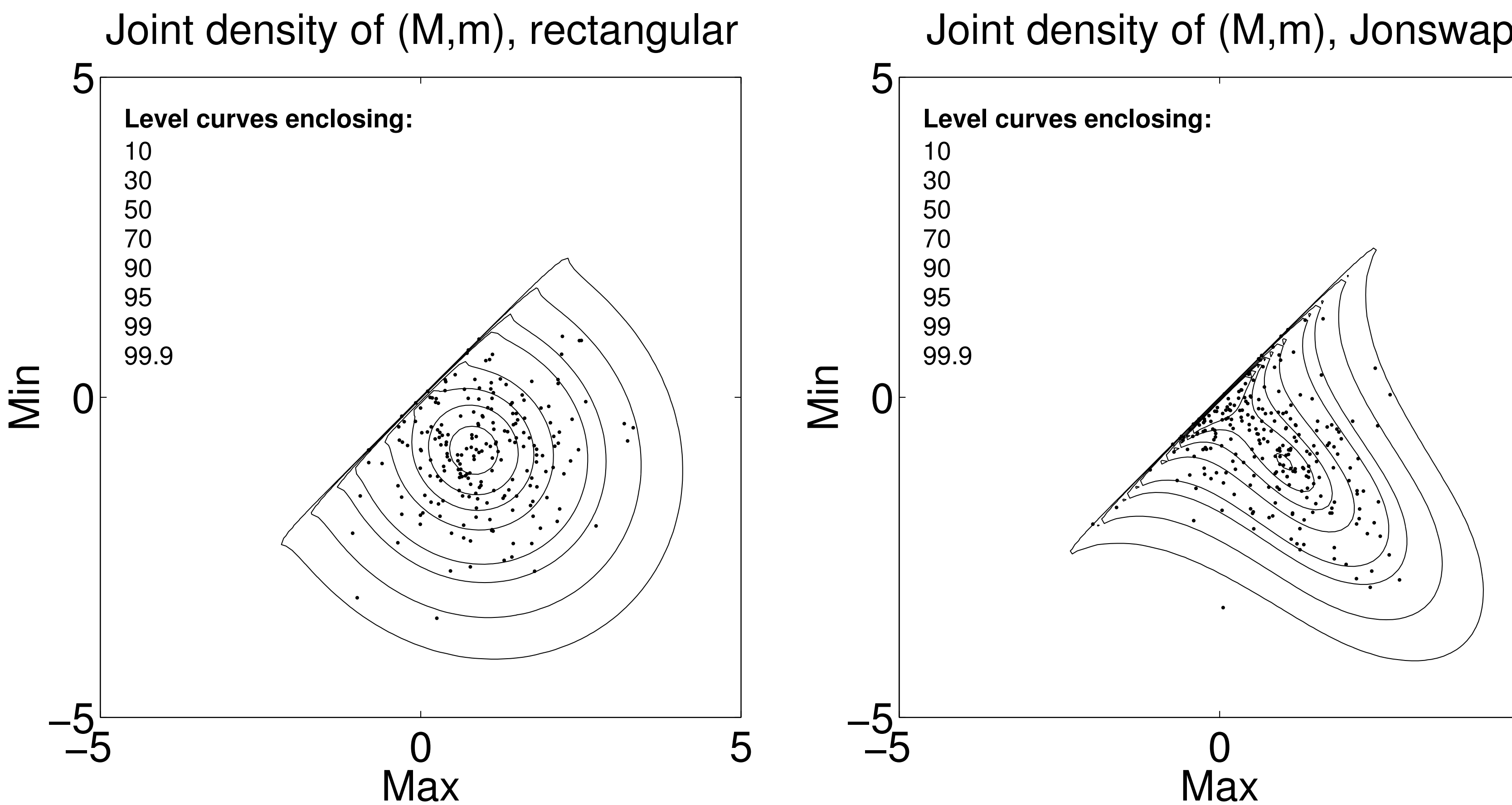
Capsizing probability, encountered sea

- Cf. IWAP2008: Leadbetter on capsizing
- Green dots = observed capsizing
- Blue area = possible danger
- Joint density of (half) wavelength and amplitude



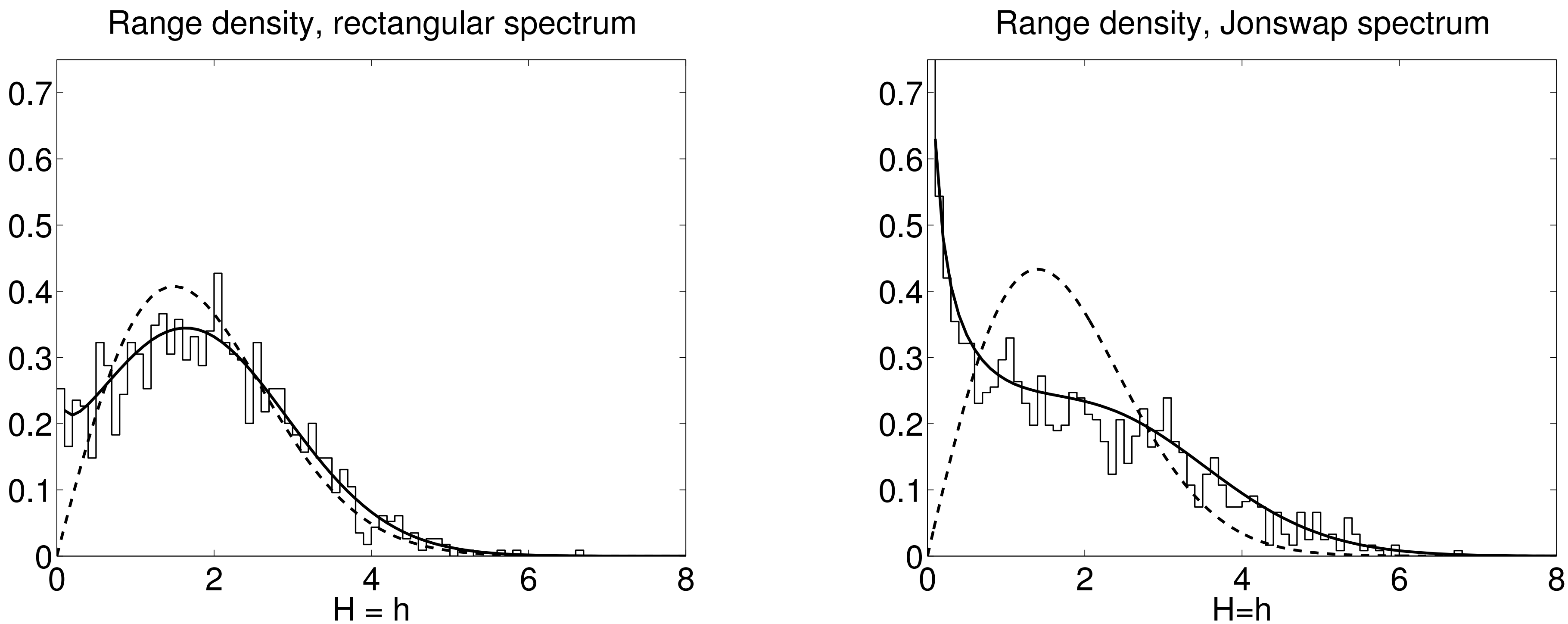
Max-min waves

- simple algorithm gives "exact results" for joint density of height of local maximum M and following local minimum m:



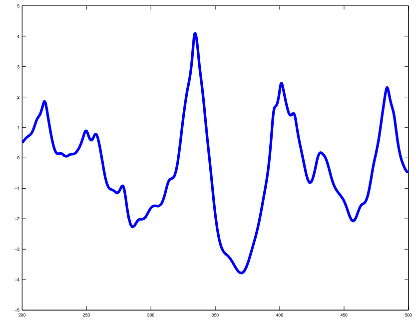
Max-min range

- solid line shows exact density of Max-min range
- dashed line the approximating Rayleigh density

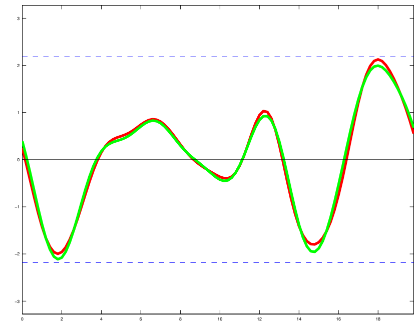


Non-Gaussian models

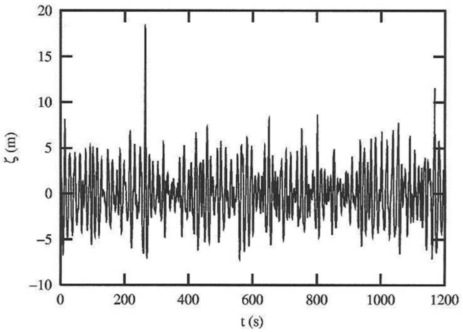
- Lagrange waves - first order, very promising – second order, better but future work needed



- Second order Stokes waves work quite well - peaked crests and shallow troughs – stochastic properties difficult but possible



- Non-linear Schrödinger equation can describe the Draupner wave – little stochastics known



Lagrange models for particle movement

- In a Gaussian wave model the sea surface at time t and location s is a Gaussian process

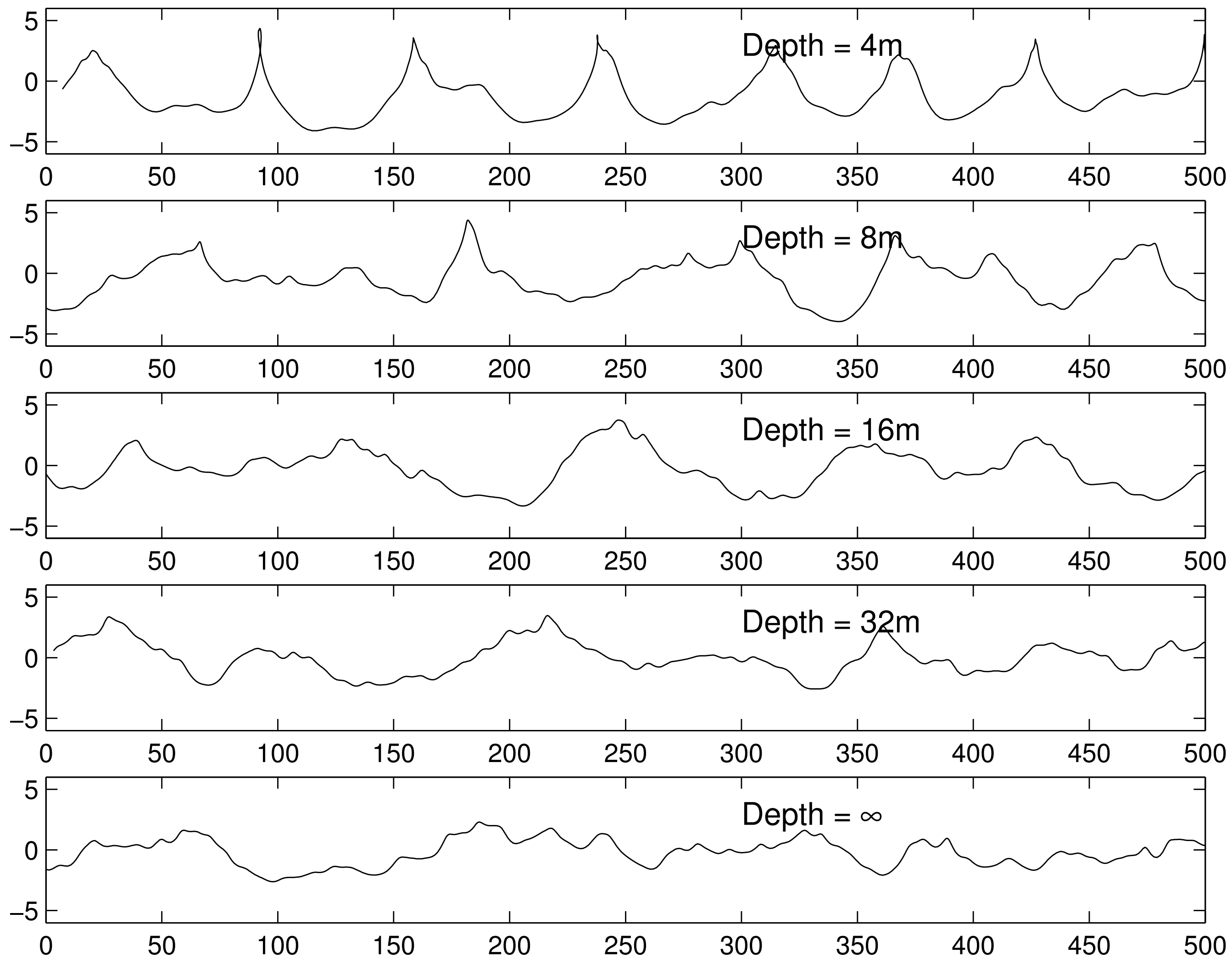
$$W(t,s) = \int e^{i(\kappa s - \omega t)} \, d\zeta(\kappa, \omega)$$

- In a stochastic linear Lagrange model the $W(t,s)$ process is the height of the sea surface at a randomly changing location

$$X_M(t,s) = s + \int H(\kappa) e^{i(\kappa s - \omega t)} \, d\zeta(\kappa, \omega)$$

  which is a linear filtration of the W-process (M = Miche-wave)
- gives peaked, crest-trough asymmetric waves
- Åberg and Lindgren: Adv Appl Probab (2006,2007), Probabilistic Engineering Mechanics (2008)

Lagrange space waves, time fixed



Asymmetric Lagrange waves

To get realistic front-back asymmetry one needs a model with external input from wind, for example:

$$\frac{\partial^2 X(t,s)}{\partial t^2} = \frac{\partial^2 X_M(t,s)}{\partial t^2} + \alpha W(t,s)$$

The slope of the space wave surface at an arbitrary crossing of a level v at time t_0 is

$$L = \frac{\partial W(t_0,s)/\partial s}{\partial X(t_0,s)/\partial s}$$

conditioned on a crossing of level v by $W(t_0,s)$.
Now, the slope at an upcrossing in a Gaussian process W is Rayleigh. This gives the following distribution of the Lagrange slope – next frame

Slope of asymmetric Lagrange waves

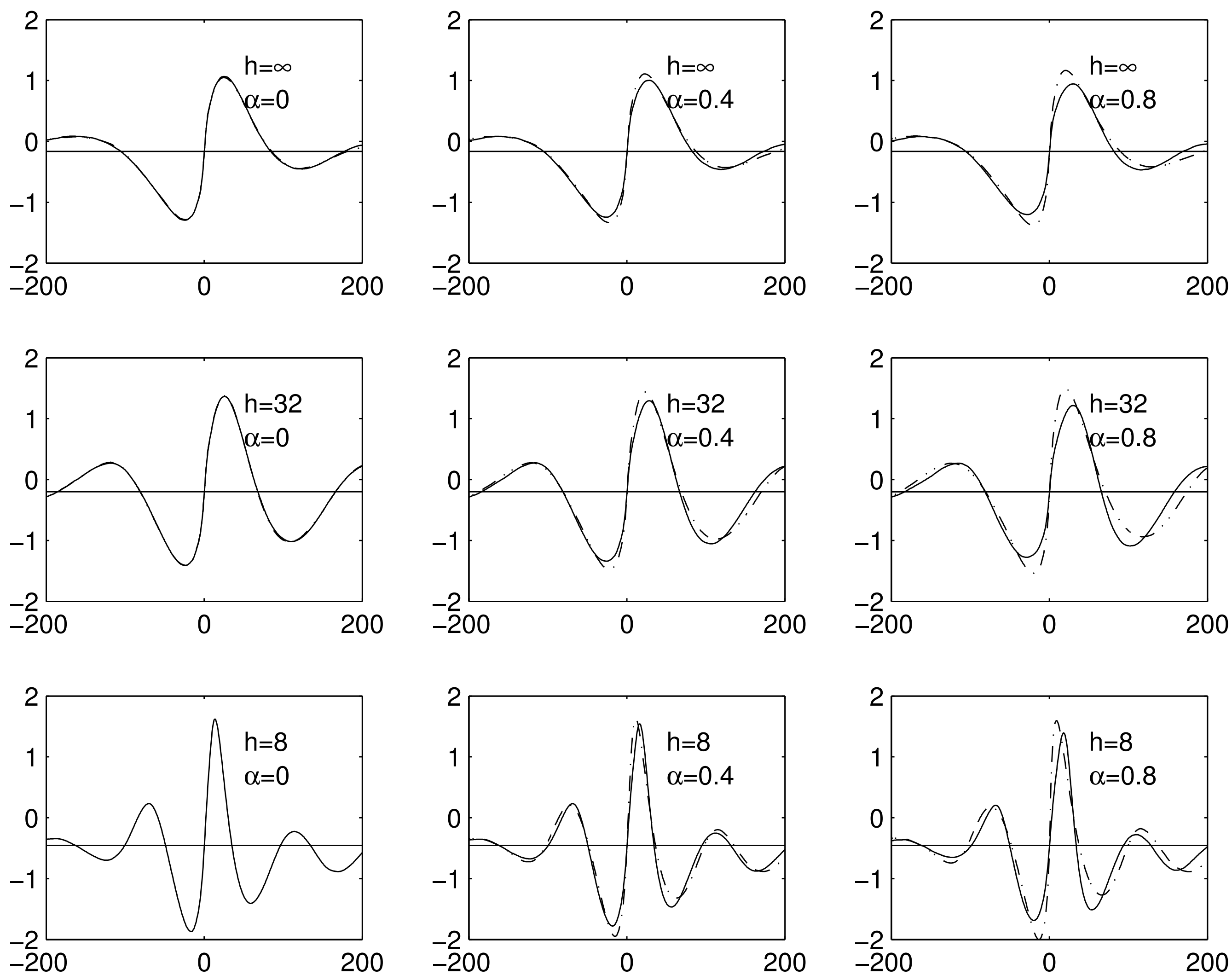
Theorem: The slope of a Lagrange space process observed at (up)crossings of a level v has the same distribution as

$$\frac{aR}{1 + bv + cR + dU}$$

where $a, b, c, d \neq 0$ depend on the correlations between the W- and the X-process, and R, U are independent standard Rayleigh and normal variables
Aberg and Lindgren: 2008

Modified Lagrange waves with front-back asymmetry

Averaged up- (solid) and down- (dashed) crossings waves

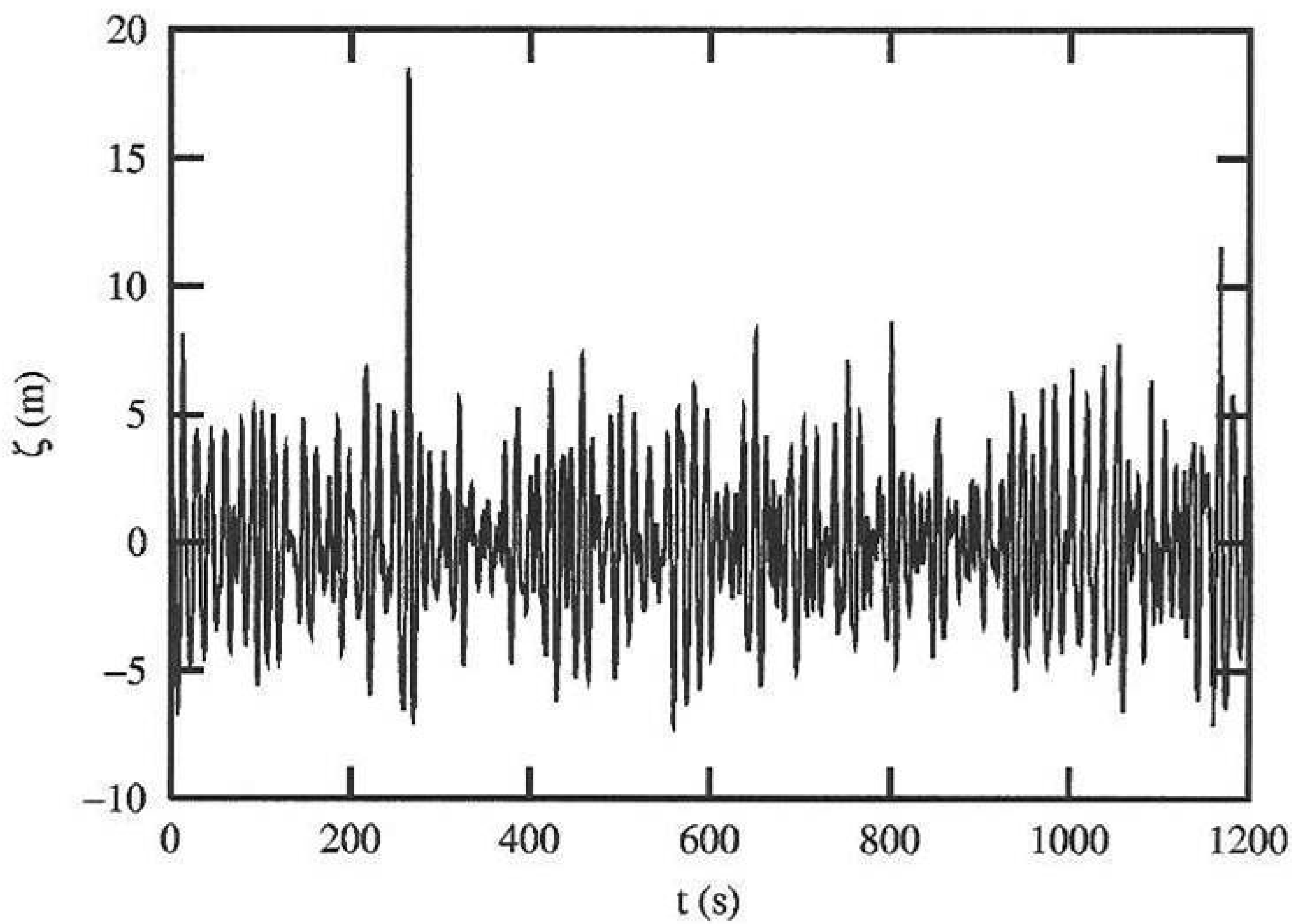


Non-linear waves and response

- A ship on a random wave surface is a non-linear system
- Second order response equation – IWAP2008: Guédé & Prévosto

Rogue waves

First time hard evidence – New years Draupner wave, Jan 1, 1995:



Observations, equations

- In Gaussian model, Crest-trough range $> 2\ H_s$ is very unlikely
- Since Draupner, reliable observations from platforms
- Satellite measure individual high waves – still some dispute about how frequent
- Non-linear Schrödinger equation can reproduce, very accurately, observed rogue waves
- But still no generally accepted method to predict under what conditions they are likely to occur
- Crossed seas, current, ...
- Rogue waves workshops in Brest, 2000, 2004, 2008, ...

References

- Hasselmann (1976): Stochastic climate models. *Tellus*, 28
- Longuet-Higgins (1952): On the statistical distribution of the height of sea waves. *J. Mariner Research*, 80.
- St.Denis and Pierson (1953): On the motion of ships in confused seas. *Soc. Naval Architects and Marine Engineers*. 61.
- `http:/www.maths.lth.se/matstat/wafo/`