Asymmetric waves in wave energy systems analysed by the Gauss-Lagrange wave model

aspects on model choice in an engineering application

Georg Lindgren

Lund University

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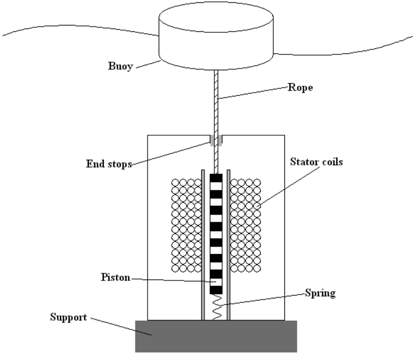
The origin of the study

Two wave energy converters: Oregon model and Scandinavian model:



Theoretical calculations with different wave models

Monte Carlo simulation with synthetic waves gives theoretical figures for energy production. The theoretical effect of a wave energy converter depends on the wave model! Compare deterministic (sine) waves with Gaussian waves for four buoy sizes:

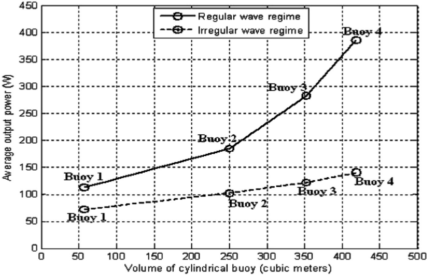


FIG. 6. Calculated electrical output power from the liner generator for each buoy listed in Table I.

Why is this important?

Design, control, reliability:

- Efficiency of different designs - size, dimensions
- Performance of control mechanism - adjust period, control ascending and descending speed
- Safety analysis - protect ceiling and floor

What if waves are asymmetric?

01-Vågkraftskoncept, boj-generator

Motivating questions

- Is the Gaussian wave model good enough in order to describe the exciting forces in the converter
- or should one use the more complicated wave model that permits asymmetric waves
- For example a Laplace model: $X(t) = \int K(t-u)\,d\Lambda(u)$ with a non-Gaussian spectral measure $\Lambda(u)$ ...
- or the Lagrange wave model – physical motivation exists – ...
- or some other non-Gaussian process ?
- Or perhaps rely only on measurements ?

The Gaussian wave model (1952)

The height $W(t, s)$ of the water surface at location s at time t is a Gaussian stationary (homogeneous) random process, expressed as a sum (integral!)

$$W(t, s) = \sum_k A_k \cos(\kappa_k s - \omega_k t + \phi_k)$$

of moving cosines with

- random amplitudes A_k
- random phases ϕ_k
- fixed frequencies ω_k (1/wave period)
- fixed wave numbers κ_k (1/wave length).

Gaussian generator and the orbital spectrum

In the Gaussian model the vertical height $W(t, x)$ of a particle at the free surface at time t and location x is an integral of harmonics with random phases and amplitudes:

$$W(t, x) = \int_{\omega=-\infty}^{\infty} e^{i(\kappa x - \omega t)}\, \mathrm{d}\zeta(\omega)$$

with

$$\omega^2 = g\kappa \tanh \kappa h$$

with $\zeta(\omega)$ is a Gaussian complex "spectral process" with spectrum

$$S(\omega) = \text{the "orbital spectrum"}$$

The stochastic Lagrange model –

Describes joint horizontal and vertical movements of individual surface water particles. Use

$$W(t, u) = \int e^{i(\kappa u - \omega t)} \, \mathrm{d}\zeta(\omega)$$

for the vertical movement of a particle with (intitial) reference coordinate u and write $X(t, u)$ for its horizontal location at time t ...

– with horizontal Gaussian movements

... and the same (vertical) Gaussian spectral process $\zeta(\omega)$ as in $W(t, u)$ to generate also the horizontal variation.

Pierson (1961), Fouques, Krogstad,Myrhaug, Socquet-Juglard (2004), Gjøsund (2000,2003)
Aberg, Lindgren, Lindgren (2006,2007,2008,2009,2010,2011), Guerrin (2009)

$$X_M(t, u) = u + \int \boldsymbol{H}_M(\kappa)\, e^{i(\kappa u - \omega t)}\, \mathrm{d}\zeta(\omega)$$

where the filter function $\boldsymbol{H}_M$ depends on water depth h:

$$\boldsymbol{H}_M(\kappa) = i\,\frac{\cosh \kappa h}{\sinh \kappa h}$$

The stochastic Lagrange model

The 2D stochastic first order free Lagrange wave model is the pair of Gaussian processes

$$(W(t,u), X_M(t,u))$$

All covariance functions and auto-spectral and cross-spectral density functions for $\mathbf{\Sigma}(t, \boldsymbol{s})$ follow from the orbital spectrum $S(\omega, \theta)$ and the filter equation.

Space wave : keep time coordinate fixed

Time wave : keep space coordinate – $X_M(t,u)$ – fixed

Front-back asymmetry

The model $\boldsymbol{H}_M(\kappa) = i\,\frac{\cosh \kappa h}{\sinh \kappa h}$ gives front-back statistically symmetric waves.

Adding a slope-dependent term gives asymmetric waves. For example,

$$\partial^2 X(t,u)/\partial t^2 = \partial^2 X_M(t,u)/\partial t^2 + \alpha W(t,u),$$

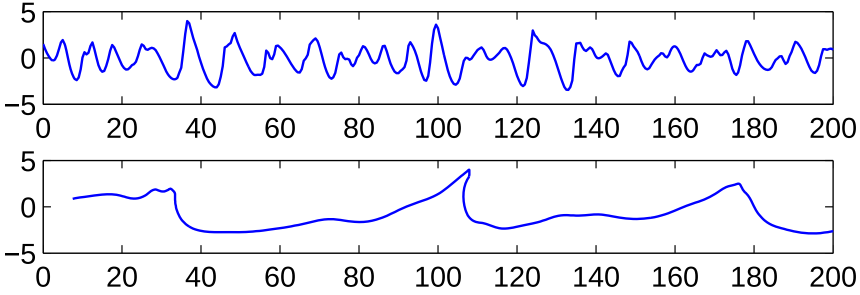
$$\boldsymbol{H}(\kappa) = i\,\frac{\cosh \kappa h}{\sinh \kappa h} + \frac{\alpha}{(-i\omega)^2} = \rho(\omega)\, e^{i\theta(\omega)}$$

Implies an extra phase shift (to the phase $\theta = \pi/2$ in the free model). A general form for $X(t,u)$ is

$$\boxed{X(t,u) = u + \int e^{i(\kappa u - \omega t + \theta(\omega))} \rho(\omega)\,\mathrm{d}\zeta(\omega)}$$

2D Lagrange waves

Asymmetric Lagrange 2D time waves (top) and space wave (bottom)



Is the Lagrange model useful?

The joint Gaussian character of the vertical, $W(t,u)$, and horizontal, $X(t,u)$, component makes it possible to compute exact statistical distributions of wave characteristics in 2D and 3D:

- wave steepness
- wave asymmetry in time and space
- wave front velocity

Is the Lagrange model realistic

when it comes to:

- Particle movements - particle orbits
- Wave geometry - front-back and crest-trough asymmetry
- 3D properties - horseshoe-like patterns

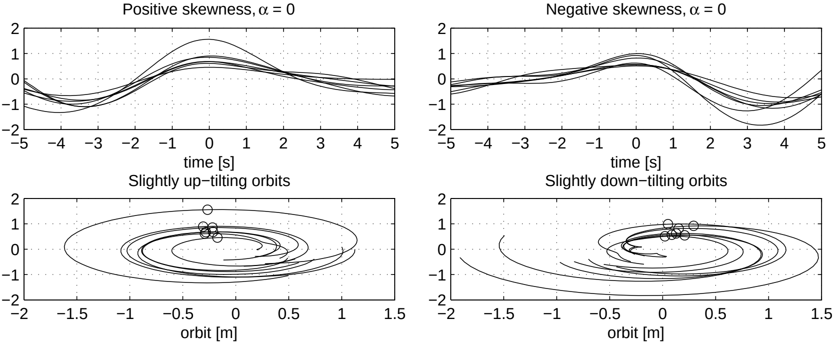
?

Particle orbits

- Miche model: particle orbits are ellipses parallell to the surface/bottom.
- Standard Lagrange model: orbits are irregular with elliptic shape
- Coupled Lagrange model: orbits are irregular tilted ellipses

Orbit shape and orientation depend on wave asymmetry - I

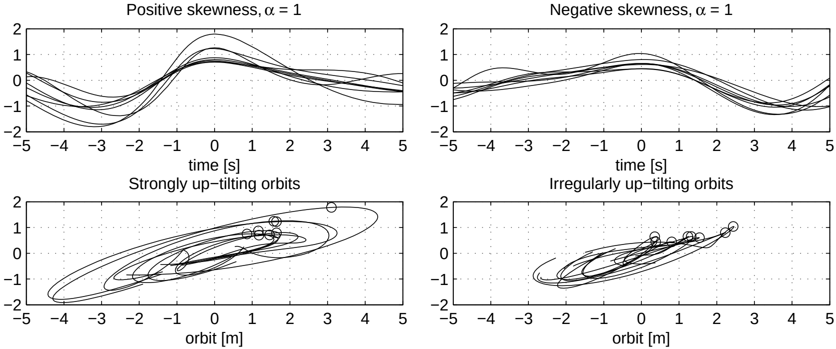
The free model with $\alpha = 0$: Wave asymmetry has small, but noticeable relation to orbit orientation.



cf. PIV-experiments by Umeyama et at. 2011–

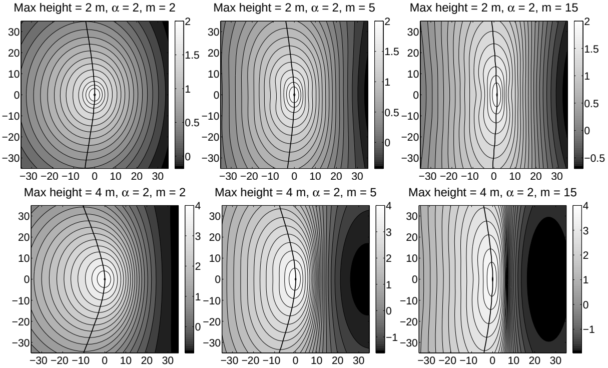
Orbit shape and orientation depend on wave asymmetry - II

The coupled model with $\alpha = 1$: Wave asymmetry has strong relation to orbit orientation.



3D Lagrange waves generate horseshoe-like patterns

3D Lagrange model with broad, medium, small directional spreading.
Average field around local maxima with different height; $H_s = 7m$.



Back to wave energy

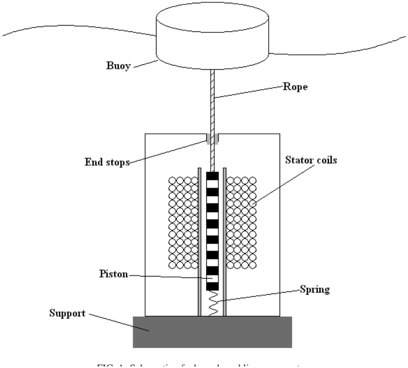
Does wave asymmetry matter in the wave energy example?

The linear wave power extractor as linear filter

The linear wave power extractor of the Scandinavian model is a linear filter

$$mZ''(t) + zZ'(t) + kZ(t) = L(t)$$

where m is total mass, z is total damping, and k depends on the buoy shape and the anchoring spring. $L(t)$ is the sea surface variation and $Z(t)$ is the buoy/piston displacement from equilibrium.



Filter frequency function

The piston displacement $Y(t)$ is obtained by Fourier transformation $\mathcal{Y} = \mathcal{F}(Y)$, etc.,

$$\mathcal{Z}(\omega) = \mathcal{L}(\omega)\, H_{filter}(\omega)$$

where the filter has frequency function

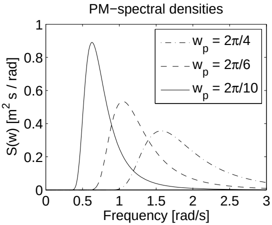
$$H_{filter}(\omega) = \frac{1}{m\,(i\omega)^2 + z\,(i\omega) + k}$$

Inverse Fourier transformation gives $Z(t)$:

$$Z(t) = \mathcal{F}^{-1}\mathcal{Z}(t)$$

Experimental setup for a numerical Lagrange experiment

- Pierson-Moskowitz orbital spectrum for $W(t, u_0)$
- Water depth $h = 20$m
- Degree of asymmetry: $\alpha = 3$
- Damping z depends on the loading on the generator
- Wave height: $H_s = 2.5$m, Peak period: $T_p = 4$s, 6s, 10s



Quantities of interest

- Compare asymmetry of Gaussian and Lagrangian waves as input to linear filter
- and asymmetry after passage through a linear filter
- Figure shows an experiment from NTNU (?) with Gaussian waves – great reduction in power compared to sinuoidal waves

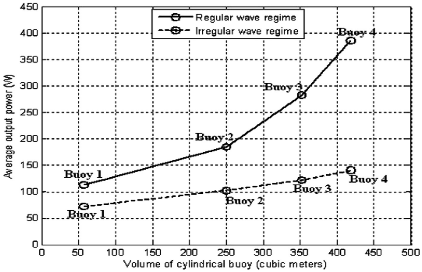
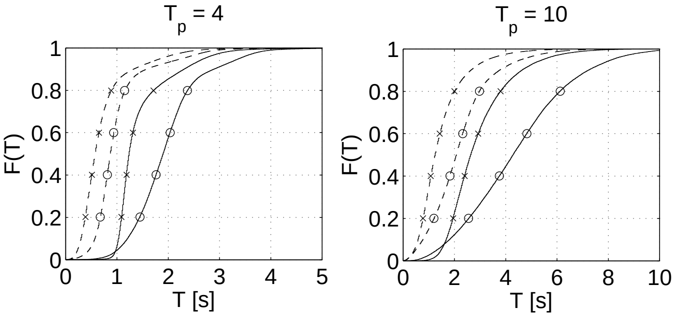


FIG. 6. Calculated electrical output power from the liner generator for each buoy listed in Table I.

Front-back asymmetry statistics

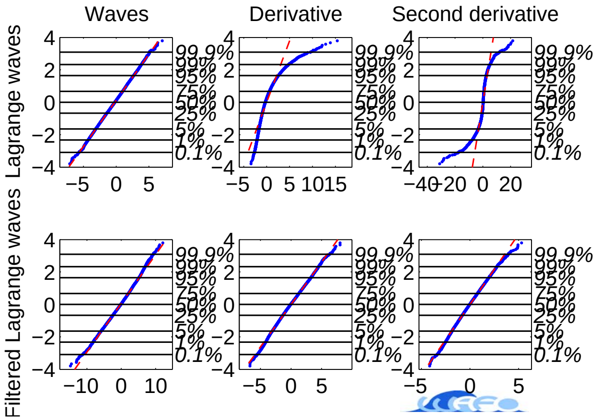
Solid lines: CDF's of full crest front (x) and back (o) periods
Dashed curves: CDF's of half crest front (x) and back (o) periods



See table in paper for numerical measures on asymmetry.

Linear filter removes some asymmetry – but not in extremes

Big waves: $H_s = 7$m, Soft spring: $z = 0.2$, Strong wave asymmetry: $\alpha = 1$



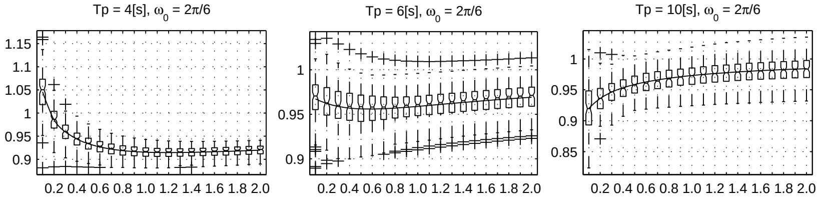
Relative efficiency of converter: Lagrange vs Gaussian

The average power of the converter is

$$P = \gamma \mathsf{E}\left(\left(\frac{\partial Z(t)}{\partial t}\right)^2\right)$$

where $Z(t)$ is the position of the magnet and γ a damping coefficient

Figures show relative efficiency: $P_{Lagrange}/P_{Gauss}$ as function of γ



Conclusions

- A Lagrange wave model gives realistic asymmetric slope distributions
- Lagrange waves become more “Gaussian” after linear filtration like in a wave power station
- Still considerable asymmetry at high levels
- Gaussian input in simulations may give too optimistic efficiency
- Model simulations with irregular sea should take asymmetry into account
- Full scale experiment needed

References

- Pierson, W.J.(1961). Model of random seas based on the Lagrangian equations of motion. New York University
- Lindgren, G. (2006). Slepian models for the stochastic shape of individual Lagrange sea waves. *Adv. Appl. Prob.* **38**
- Aberg, S. (2007). Wave intensities and slopes in Lagrangian seas. *Adv. Appl. Prob.*, **39**
- Lindgren, G. and Aberg, S. (2008). First order stochastic Lagrange models for front-back asymmetric ocean waves. *J. OMAE*, **131**
- Lindgren, G. (2009). Exact asymmetric slope distributions in stochastic Gauss-Lagrange ocean waves. *Appl. Ocean Res*, **31**
- Lindgren, G. (2010). Slope distribution in front-back asymmetric stochastic Lagrange time waves. *Adv. Appl. Prob.*, **42**
- Lindgren, G. and Lindgren, F. (2011). Stochastic asymmetry properties of 3D-Gauss-Lagrange ocean waves with directional spreading. *Stochastic Models*, **27**
- WAFO-group, (2011). *WAFO – a Matlab Toolbox for Analysis of Random Waves and Loads; Tutorial for WAFO Version 2.5*, URL http://www.maths.lth.se/matstat/wafo