Asymmetric waves in wave energy systems analysed by the Gauss-Lagrange wave model aspects on model choice in an engineering application

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The origin of the study

Two wave energy converters: Oregon model and Scandinavian model:
Monte Carlo simulation with synthetic waves gives theoretical figures for energy production. The theoretical effect of a wave energy converter depends on the wave model! Compare deterministic (sine) waves with Gaussian waves for four buoy sizes:

![Graph showing average output power vs. volume of cylindrical buoy]

**FIG. 6.** Calculated electrical output power from the linear generator for each buoy listed in Table I.
Why is this important?

Design, control, reliability:

- Efficiency of different designs - size, dimensions
- Performance of control mechanism - adjust period, control ascending and descending speed
- Safety analysis - protect ceiling and floor

What if waves are asymmetric?
Motivating questions

- Is the Gaussian wave model good enough in order to describe the exciting forces in the converter
- or should one use the more complicated wave model that permits asymmetric waves
- For example a Laplace model: \( X(t) = \int K(t - u) d\Lambda(u) \) with a non-Gaussian spectral measure \( \Lambda(u) \)...
- or the Lagrange wave model – physical motivation exists – ...
- or some other non-Gaussian process?
- Or perhaps rely only on measurements?
The Gaussian wave model (1952)

The height $W(t, s)$ of the water surface at location $s$ at time $t$ is a Gaussian stationary (homogeneous) random process, expressed as a sum (integral!)

$$W(t, s) = \sum_k A_k \cos(\kappa_k s - \omega_k t + \phi_k)$$

of moving cosines with
- random amplitudes $A_k$
- random phases $\phi_k$
- fixed frequencies $\omega_k$ (1/wave period)
- fixed wave numbers $\kappa_k$ (1/wave length).
Gaussian generator and the orbital spectrum

In the Gaussian model the vertical height $W(t, x)$ of a particle at the free surface at time $t$ and location $x$ is an integral of harmonics with random phases and amplitudes:

$$W(t, x) = \int_{\omega=-\infty}^{\omega=\infty} e^{i(\kappa x - \omega t)} d\zeta(\omega)$$

with

$$\omega^2 = g \kappa \tanh \kappa h$$

with $\zeta(\omega)$ is a Gaussian complex “spectral process” with spectrum

$$S(\omega) = \text{the “orbital spectrum”}$$
The stochastic Lagrange model –

Describes joint horizontal and vertical movements of individual surface water particles. Use

$$W(t, u) = \int e^{i(\kappa u - \omega t)} d\zeta(\omega)$$

for the vertical movement of a particle with (initial) reference coordinate $u$ and write $X(t, u)$ for its horizontal location at time $t$ …
... and the same (vertical) Gaussian spectral process $\zeta(\omega)$ as in $W(t, u)$ to generate also the horizontal variation.


$$X_M(t, u) = u + \int H_M(\kappa) e^{i(\kappa u - \omega t)} d\zeta(\omega)$$

where the filter function $H_M$ depends on water depth $h$:

$$H_M(\kappa) = i \frac{\cosh \kappa h}{\sinh \kappa h}$$
The stochastic Lagrange model

The 2D stochastic first order free Lagrange wave model is the pair of Gaussian processes

\[(W(t, u), X_M(t, u))\]

All covariance functions and auto-spectral and cross-spectral density functions for \(\Sigma(t, s)\) follow from the orbital spectrum \(S(\omega, \theta)\) and the filter equation.

Space wave: keep time coordinate fixed

Time wave: keep space coordinate \(- X_M(t, u)\) fixed
Front-back asymmetry

The model $H_M(\kappa) = i \frac{\cosh \kappa h}{\sinh \kappa h}$ gives front-back statistically symmetric waves.

Adding a slope-dependent term gives asymmetric waves. For example,

$$\partial^2 X(t, u)/\partial t^2 = \partial^2 X_M(t, u)/\partial t^2 + \alpha W(t, u),$$

$$H(\kappa) = i \frac{\cosh \kappa h}{\sinh \kappa h} + \frac{\alpha}{(-i\omega)^2} = \rho(\omega) e^{i\theta(\omega)}$$

Implies an extra phase shift (to the phase $\theta = \pi/2$ in the free model).

A general form for $X(t, u)$ is

$$X(t, u) = u + \int e^{i(\kappa u - \omega t + \theta(\omega))} \rho(\omega) \, d\zeta(\omega)$$
2D Lagrange waves

Asymmetric Lagrange 2D time waves (top) and space wave (bottom)
Is the Lagrange model useful?

The joint Gaussian character of the vertical, $W(t, u)$, and horizontal, $X(t, u)$, component makes it possible to compute exact statistical distributions of wave characteristics in 2D and 3D:

- wave steepness
- wave asymmetry in time and space
- wave front velocity
Is the Lagrange model realistic when it comes to:

- Particle movements - particle orbits
- Wave geometry - front-back and crest-trough asymmetry
- 3D properties - horseshoe-like patterns

?
Particle orbits

- Miche model: particle orbits are ellipses parallel to the surface/bottom.
- Standard Lagrange model: orbits are irregular with elliptic shape
- Coupled Lagrange model: orbits are irregular tilted ellipses
Orbit shape and orientation depend on wave asymmetry - I

The free model with $\alpha = 0$: Wave asymmetry has small, but noticeable relation to orbit orientation.

Positive skewness, $\alpha = 0$:
- Slightly up-tilting orbits

Negative skewness, $\alpha = 0$:
- Slightly down-tilting orbits

cf. PIV-experiments by Umeyama et al. 2011–
The coupled model with $\alpha = 1$: Wave asymmetry has strong relation to orbit orientation.

Positive skewness, $\alpha = 1$

Negative skewness, $\alpha = 1$

Strongly up-tilting orbits

Irregularly up-tilting orbits
3D Lagrange waves generate horseshoe-like patterns

3D Lagrange model with broad, medium, small directional spreading. Average field around local maxima with different height; $H_s = 7 \text{m}$.
Back to wave energy

Does wave asymmetry matter in the wave energy example?
The linear wave power extractor of the Scandinavian model is a linear filter

\[ mZ''(t) + zZ'(t) + kZ(t) = L(t) \]

where \( m \) is total mass, \( z \) is total damping, and \( k \) depends on the buoy shape and the anchoring spring. \( L(t) \) is the sea surface variation and \( Z(t) \) is the buoy/piston displacement from equilibrium.
Filter frequency function

The piston displacement $Y(t)$ is obtained by Fourier transformation $Y = \mathcal{F}(Y)$, etc.,

$$Z(\omega) = \mathcal{L}(\omega) H_{\text{filter}}(\omega)$$

where the filter has frequency function

$$H_{\text{filter}}(\omega) = \frac{1}{m(i\omega)^2 + z(i\omega) + k}$$

Inverse Fourier transformation gives $Z(t)$:

$$Z(t) = \mathcal{F}^{-1} Z(t)$$
Experimental setup for a numerical Lagrange experiment

- Pierson-Moskowitz orbital spectrum for $W(t,u_0)$
- Water depth $h = 20m$
- Degree of asymmetry: $\alpha = 3$
- Damping $z$ depends on the loading on the generator
- Wave height: $H_s = 2.5m$, Peak period: $T_p = 4s, 6s, 10s$
Quantities of interest

- Compare asymmetry of Gaussian and Lagrangian waves as input to linear filter
- and asymmetry after passage through a linear filter
- Figure shows an experiment from NTNU (?) with Gaussian waves – great reduction in power compared to sinuoidal waves

FIG. 6. Calculated electrical output power from the linear generator for each buoy listed in Table I.
Front-back asymmetry statistics

Solid lines: CDF’s of full crest front (x) and back (o) periods
Dashed curves: CDF’s of half crest front (x) and back (o) periods

See table in paper for numerical measures on asymmetry.
Linear filter removes some asymmetry – but not in extremes

Big waves: $H_s = 7m$, Soft spring: $z = 0.2$, Strong wave asymmetry: $\alpha = 1$
Relative efficiency of converter: Lagrange vs Gaussian

The average power of the converter is

\[ P = \gamma E \left( \left( \frac{\partial Z(t)}{\partial t} \right)^2 \right) \]

where \( Z(t) \) is the position of the magnet and \( \gamma \) a damping coefficient.

Figures show relative efficiency: \( P_{\text{Lagrange}} / P_{\text{Gauss}} \) as function of \( \gamma \).
Conclusions

- A Lagrange wave model gives realistic asymmetric slope distributions
- Lagrange waves become more “Gaussian” after linear filtration like in a wave power station
- Still considerable asymmetry at high levels
- Gaussian input in simulations may give too optimistic efficiency
- Model simulations with irregular sea should take asymmetry into account
- Full scale experiment needed
References