Extremes, wave asymmetry, and other stochastic properties of a ocean wave energy system aspects on model choice in an engineering application

Georg Lindgren

Lund University

Vimeiro, 8 – 11 September 2013
Workshop EVT2013 - in honour of Ivette Gomes
Basic models versus EVT

Basic models:

- (G): Water waves have been modeled as a Gaussian process (field) for more than 50 years
- (L): A Lagrange model can produce more realistic asymmetric waves
- (S): Non-linear Schrödinger equations can accurately reproduce real waves

EVT:

- EVT useful to model extreme wave events regardless of (G), (L), (S)
- EVT also useful for wave climate modeling
A wave energy example

- "340 patents between 1855 and the 1973 oil crisis"
- Several configurations of wave energy converters have been designed – cf. early flying machines
- Some ideas have survived – even at sea!
- Lab testing of design needs realistic wave models
- Early ideas – deterministic cosines
- Now – irregular, random, Gaussian, wave models
- Why randomness – control, efficiency, reliability, durability
Two modern models of linear wave energy converters

Oregon model and Scandinavian model:

The piston-driven permanent magnet (attached to the buoy) to induce voltages as shown schematically in Fig.1. The rotor could be constructed of an alternating assembly of Neodymium–Iron–Boron (Nd–Fe–B) permanent magnets, interspersed with soft iron pole pieces mounted on a threaded aluminum shaft. The magnets would be stacked in pairs to force the opposing fluxes of the magnetomotive forces through the iron pole pieces and across the air gap to the stator. Regardless of the dimensional details, the induced electromotive force $E(t)$ for such a configuration would be

$$E(t) = \frac{dk_f}{dt},$$

with $k_f$ being the flux linkage between the rotor and the stator coils. For the LG, with $z$ denoting the vertical displacement, and $U_p$ the peak flux, one has

$$k_f = \frac{N U_p}{s},$$

and hence

$$E(t) = \frac{C_0 N U_p (p/s)}{sin \left( \frac{p z}{s} \right)} dz/dt,$$

where $N$ represents the number of turns for the stator coils, and $s$ the pole pitch. For a load impedance $Z_L$, the output voltage ($V(t)$) is related to the generator voltage $E(t)$ as

$$V(t) = \frac{iZ_L}{C_0 L_{COIL}} = \frac{E(t)}{C_0 r_i},$$

where $L_{COIL}$ represents the inductances of the stator coils and $r_i$ is the stator coil resistance.

III. RESULTS AND DISCUSSION

The above model was used to calculate the buoy motion and couple it with the linear generator to obtain the output voltages and currents. For concreteness, the sea state was taken to be characterized by the parameters reported off of the Oregon coast. For example, in Eq.(4) for the summer time, the values were taken as $H_s = 1.5$ m and $\omega = 1.047$ rad/s. The buoy is assumed to be cylindrical in shape and floating vertically in the water with the draft equal to $0.6$ m.
Theoretical calculations with different wave models

Monte Carlo simulation with synthetic waves gives theoretical figures for energy production. The theoretical effect of a wave energy converter depends on the wave model! Compare deterministic (sine) waves with Gaussian waves for four buoy sizes:

![Graph showing average output power vs. volume of cylindrical buoy for different buoy sizes in regular and irregular wave regimes.]

FIG. 6. Calculated electrical output power from the liner generator for each buoy listed in Table I.
Why is this important?

Design, control, reliability:

- Efficiency of different designs - size, dimensions
- Performance of control mechanism - adjust period, control ascending and descending speed
- Safety analysis - protect ceiling and floor

What if waves are asymmetric?
Motivating questions

- Is the Gaussian wave model good enough in order to describe the extreme movements in a wave energy converter
- or should one use the more complicated wave model that permits statistically asymmetric waves
- For example a Laplace model: $X(t) = \int K(t - u) d\Lambda(u)$ with a non-Gaussian spectral measure $\Lambda(u)$ ...
- or the Lagrange wave model – physical motivation exists – ...
- or some other non-Gaussian process?
- Or perhaps rely only on measurements and EVT?
The Gaussian wave model (1952)

The height $W(t, s)$ of the water surface at location $s$ at time $t$ is a Gaussian stationary (homogeneous) random process, expressed as a sum

$$W(t, s) = \sum_k A_k \cos(\kappa_k s - \omega_k t + \phi_k)$$

of moving cosines with

- random amplitudes $A_k$ and random phases $\phi_k$
- fixed frequencies $\omega_k$ (1/wave period)
- fixed wave numbers $\kappa_k$ (1/wave length).
Gaussian characteristics

A Gaussian sea is statistically symmetric:
- the sea surface can be turned upside down
- a wave movie can be run backwards

and you don’t see any difference

It needs to be combined with physics/hydrodynamics – gives the Lagrange model
Gaussian generator and the orbital spectrum

In the Gaussian model the vertical height $W(t, x)$ of a particle at the free surface at time $t$ and location $x$ is an integral of harmonics with random phases and amplitudes:

$$W(t, x) = \int_{\omega=-\infty}^{\infty} e^{i(\kappa x - \omega t)} d\zeta(\omega)$$

$$\omega^2 = g\kappa \tanh \kappa h$$

with $S(\omega) =$ the “orbital spectrum” and $\zeta(\omega)$ is a Gaussian complex “spectral process”.
The stochastic Lagrange model –

Describes horizontal and vertical movements of individual surface water particles. Use

\[ W(t, u) = \int e^{i(\kappa u - \omega t)} d\zeta(\omega) \]

for the vertical movement of a particle with (initial) reference coordinate \( u \) and write \( X(t, u) \) for its horizontal location at time \( t \).
– with horizontal Gaussian movements

Use the same (vertical) Gaussian spectral process as in $W(t, x)$ to generate also the horizontal variation as

$$X(t, u) = u + \int H(\kappa) e^{i(\kappa u - \omega t)} d\zeta(\omega)$$

where the filter function $H$ depends on water depth $h$ and on an asymmetry parameter $\alpha$:

$$H(\kappa) = i \frac{\cosh \kappa h}{\sinh \kappa h} + \frac{\alpha}{(-i\omega)^2} = \rho(\omega) e^{i\theta(\omega)}$$
The stochastic Lagrange model

The 2D stochastic first order Lagrange wave model is the pair of Gaussian processes

\[(W(t, u), X(t, u))\]

All covariance functions and auto-spectral and cross-spectral density functions for \(\Sigma(t, s)\) follow from the orbital spectrum \(S(\omega, \theta)\) and the filter equation.

Space wave: keep time coordinate fixed

Time wave: keep space coordinate – \(X(t, u)\) – fixed
2D Lagrange waves

Asymmetric Lagrange 2D time waves (top) and space wave (bottom)
Explicit definitions – I

The model

\[(W(t, u), X(t, u))\]

is an implicitly defined model. The space and time models can be made explicit by

\[u = X^{-1}(t, x)\]

equal to the reference point that is mapped to the observation point \(x\).

Note: There may be many solutions to \(X(t, u) = x\).
Explicit definitions – II

**Space wave**  \( L(t_0, x) = W(t_0, X^{-1}(t_0, x)) \)
\( = \) photo of the surface

**Time wave**  \( L(t, x_0) \) is the parametric curve:
\[ t \mapsto W(t, X^{-1}(t, x_0)) \]
\( = \) measured at a wave pole
The linear wave power extractor of the Scandinavian model is a linear filter

\[ m Y''(t) + z Y'(t) + k Y(t) = L(t) \]

where \( m \) is buoy+piston mass, \( z \) is the damping from the magnet/coil, and \( k \) depends on the buoy shape and the anchoring spring. \( L(t) \) is the sea surface height and \( Y(t) \) is the buoy/piston displacement.
Filter frequency function

The piston displacement $Y(t)$ is obtained by Fourier transformation:

$$
\mathcal{Y}(\omega) = \mathcal{L}(\omega) H_{\text{filter}}(\omega)
$$

where the filter has frequency function

$$
H_{\text{filter}}(\omega) = \frac{1}{m(i\omega)^2 + z(i\omega) + k}
$$

Inverse Fourier transformation gives

$$
Y(t) = \mathcal{F}^{-1}\mathcal{Y}(t)
$$
Quantities of interest

- Compare asymmetry of Gaussian and Lagrangian waves as input to linear filter ... 
- and asymmetry after passage through a linear filter 
- Figure shows an experiment from NTNU with Gaussian waves – reduction in power by 25% compared to sinusoidal waves
Experimental setup for a Lagrange experiment

- Jonswap orbital spectrum for $W(t, u_0)$
- Water depth $h = 25$ m (Lysekil)
- Degree of asymmetry: $\alpha = 1$ strong asymmetry
- Damping $z$ depends on the loading on the generator
- Significant wave height: $H_s = 7$ m, $H_s = 1.41$ m (Lysekil)
- Peak period: 11s, 6s (Lysekil)
The linear filter removes part of the asymmetry . . .

Big waves: $H_s = 7m$, Soft spring: $z = 0.2$, Strong wave asymmetry: $\alpha = 1$
...but not in the extremes

Still asymmetry in slopes at level crossings

\[ F(x|X \geq 4.823) \]

\[ F(x|X \geq 4.2104) \]
Base data from the Lysekil station - resonant conditions

**Filtered Gaussian at level crossings**

**Filtered Lagrange at level crossings**
Base data from the Lysekil station - non-resonant conditions

Filtered Gaussian at level crossings

Filtered Lagrange at level crossings
Conclusions

- A Lagrange wave model gives realistic asymmetric slope distributions.
- Lagrange waves become more "Gaussian" after linear filtration like in a wave power station.
- Still considerable asymmetry at high levels.
- Model simulations with irregular sea should take asymmetry into account.
- Full scale experiment + EVT still needed.