Extremes, wave asymmetry, and other stochastic properties of a ocean wave energy system

aspects on model choice in an engineering application

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Basic models versus EVT

Basic models:

- (G): Water waves have been modeled as a Gaussian process (field) for more than 50 years
- (L): A Lagrange model can produce more realistic asymmetric waves
- (S): Non-linear Schrödinger equations can accurately reproduce real waves

EVT:

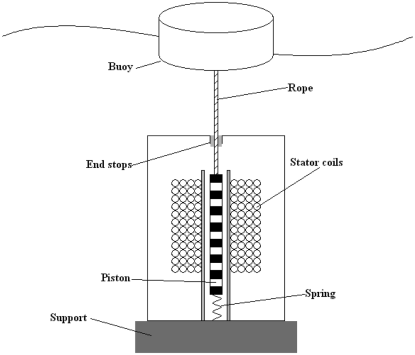
- EVT useful to model extreme wave events regardless of (G), (L), (S)
- EVT also useful for wave climate modeling

A wave energy example

- "340 patents between 1855 and the 1973 oilcrisis"
- Several configurations of wave energy converters have been designed – cf. early flying machines
- Some ideas have survived – even at sea!
- Lab testing of design needs realistic wave models
- Early ideas – deterministic cosines
- Now – irregular, random, Gaussian, wave models
- Why randomness – control, efficiency, reliability, durability

Two modern models of linear wave energy converters

Oregon model and Scandinavian model:



Theoretical calculations with different wave models

Monte Carlo simulation with synthetic waves gives theoretical figures for energy production. The theoretical effect of a wave energy converter depends on the wave model! Compare deterministic (sine) waves with Gaussian waves for four buoy sizes:

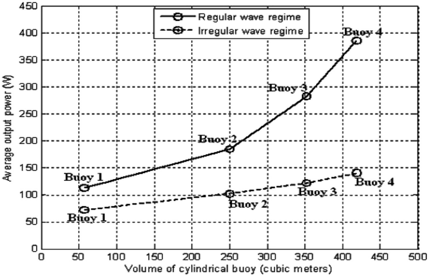


FIG. 6. Calculated electrical output power from the liner generator for each buoy listed in Table I.

Why is this important?

Design, control, reliability:

- Efficiency of different designs - size, dimensions
- Performance of control mechanism - adjust period, control ascending and descending speed
- Safety analysis - protect ceiling and floor

What if waves are asymmetric?

01-Vågkraftskoncept, boj-generator

Motivating questions

- Is the Gaussian wave model good enough in order to describe the extreme movements in a wave energy converter
- or should one use the more complicated wave model that permits statistically asymmetric waves
- For example a Laplace model: $X(t) = \int K(t-u)\, d\Lambda(u)$ with a non-Gaussian spectral measure $\Lambda(u)$ ...
- or the Lagrange wave model – physical motivation exists – ...
- or some other non-Gaussian process ?
- Or perhaps rely only on measurements and EVT ?

The Gaussian wave model (1952)

The height $W(t, s)$ of the water surface at location s at time t is a Gaussian stationary (homogeneous) random process, expressed as a sum

$$W(t, s) = \sum_k A_k \cos(\kappa_k s - \omega_k t + \phi_k)$$

of moving cosines with

- random amplitudes A_k and random phases ϕ_k
- fixed frequencies ω_k (1/wave period)
- fixed wave numbers κ_k (1/wave length).

Gaussian characteristics

A Gaussian sea is statistically symmetric:

- the sea surface can be turned upside down
- a wave movie can be run backwards

and you don't see any difference

It needs to be combined with physics/hydrodynamics – gives the Lagrange model

Gaussian generator and the orbital spectrum

In the Gaussian model the vertical height $W(t,x)$ of a particle at the free surface at time t and location x is an integral of harmonics with random phases and amplitudes:

$$W(t,x) = \int_{\omega=-\infty}^{\infty} e^{i(\kappa x - \omega t)}\, \mathrm{d}\zeta(\omega)$$
$$\omega^2 = g\kappa \tanh \kappa h$$

with $S(\omega)$ = the "orbital spectrum" and $\zeta(\omega)$ is a Gaussian complex "spectral process".

The stochastic Lagrange model –

Describes horizontal and vertical movements of individual surface water particles. Use

$$W(t,u) = \int e^{i(\kappa u - \omega t)}\,\mathrm{d}\zeta(\omega)$$

for the vertical movement of a particle with (intitial) reference coordinate u and write $X(t,u)$ for its horizontal location at time t

– with horizontal Gaussian movements

Use the same (vertical) Gaussian spectral process as in $W(t, x)$ to generate also the horizontal variation as

$$X(t, u) = u + \int \boldsymbol{H}(\kappa)\, e^{i(\kappa u - \omega t)}\, \mathrm{d}\zeta(\omega)$$

where the filter function $\boldsymbol{H}$ depends on water depth h and on an asymmetry parameter α:

$$\boldsymbol{H}(\kappa) = i\,\frac{\cosh \kappa h}{\sinh \kappa h} + \frac{\alpha}{(-i\omega)^2} = \rho(\omega)\, e^{i\theta(\omega)}$$

The stochastic Lagrange model

The 2D stochastic first order Lagrange wave model is the pair of Gaussian processes

$$(W(t,u), X(t,u))$$

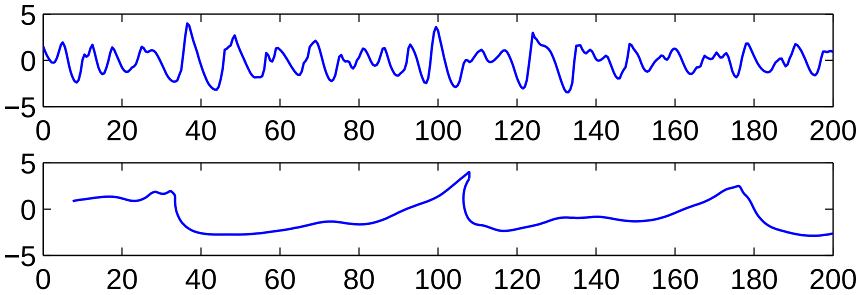
All covariance functions and auto-spectral and cross-spectral density functions for $\mathbf{\Sigma}(t, \boldsymbol{s})$ follow from the orbital spectrum $S(\omega, \theta)$ and the filter equation.

Space wave : keep time coordinate fixed

Time wave : keep space coordinate – $X(t, u)$ – fixed

2D Lagrange waves

Asymmetric Lagrange 2D time waves (top) and space wave (bottom)



Explicit definitions – I

The model

$$(W(t,u), X(t,u))$$

is an implicitly defined model. The space and time models can be made explicit by

$$u = X^{-1}(t,x)$$

equal to the reference point that is mapped to the observation point x.

Note: There may be many solutions to $X(t,u) = x$.

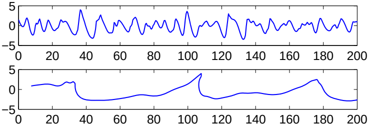
Explicit definitions – II

Space wave $L(t_0, x) = W(t_0, X^{-1}(t_0, x))$ = photo of the surface

Time wave $L(t, x_0)$ is the parametric curve:

$$t \mapsto W(t, X^{-1}(t, x_0))$$

= measured at a wave pole

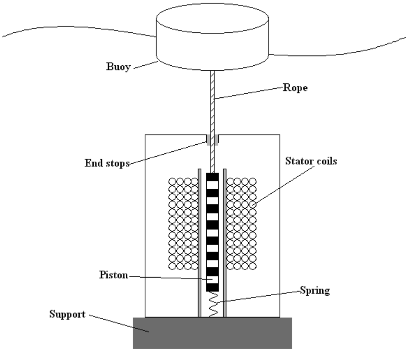


The linear wave power extractor as linear filter

The linear wave power extractor of the Scandinavian model is a linear filter

$$mY''(t) + zY'(t) + kY(t) = L(t)$$

where m is buoy+piston mass, z is the damping from the magnet/coil, and k depends on the buoy shape and the anchoring spring. $L(t)$ is the the sea surface height and $Y(t)$ is the buoy/piston displacement



Filter frequency function

The piston displacement $Y(t)$ is obtained by Fourier transformation:

$$\mathcal{Y}(\omega) = \mathcal{L}(\omega)\, H_{filter}(\omega)$$

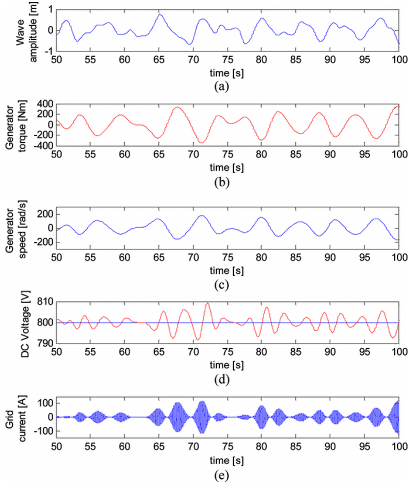
where the filter has frequency function

$$H_{filter}(\omega) = \frac{1}{m\,(i\omega)^2 + z\,(i\omega) + k}$$

Inverse Fourier transformation gives $Y(t) = \mathcal{F}^{-1}\mathcal{Y}(t)$

Quantities of interest

- Compare asymmetry of Gaussian and Lagrangian waves as input to linear filter . . .
- and asymmetry after passage through a linear filter
- Figure shows an experiment from NTNU with Gaussian waves – reduction in power by 25% compared to sinuoidal waves

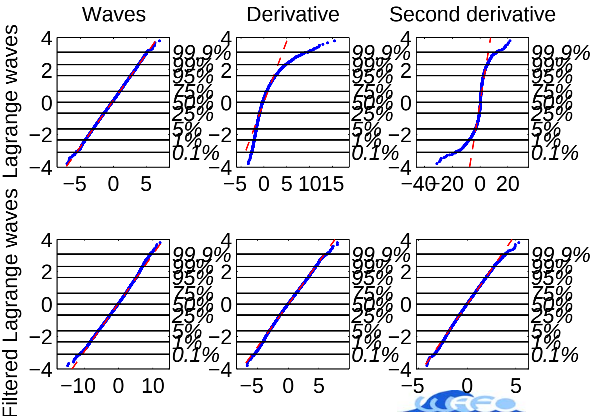


Experimental setup for a Lagrange experiment

- Jonswap orbital spectrum for $W(t, u_0)$
- Water depth $h = 25$m (Lysekil)
- Degree of asymmetry: $\alpha = 1$ strong asymmetry
- Damping z depends on the loading on the generator
- Significant wave height: $H_s = 7$m, $H_s = 1.41$m (Lysekil)
- Peak period: 11s, 6s (Lysekil)

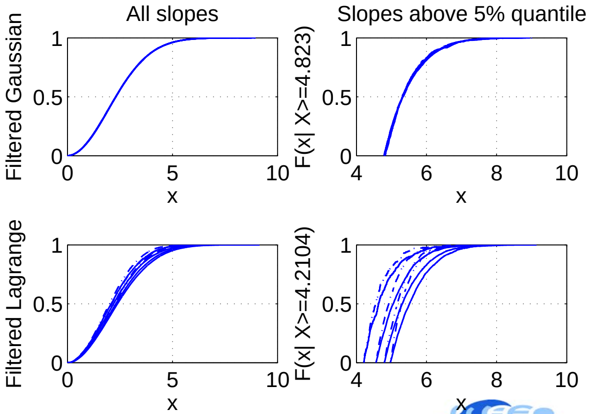
The linear filter removes part of the asymmetry . . .

Big waves: $H_s = 7\text{m}$, Soft spring: $z = 0.2$, Strong wave asymmetry: $\alpha = 1$

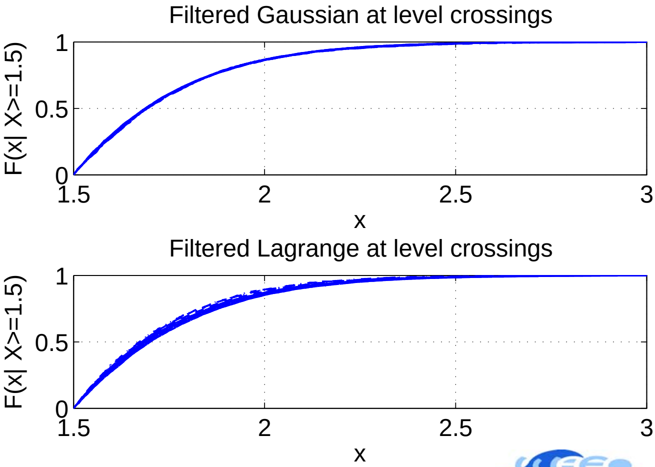


. . . but not in the extremes

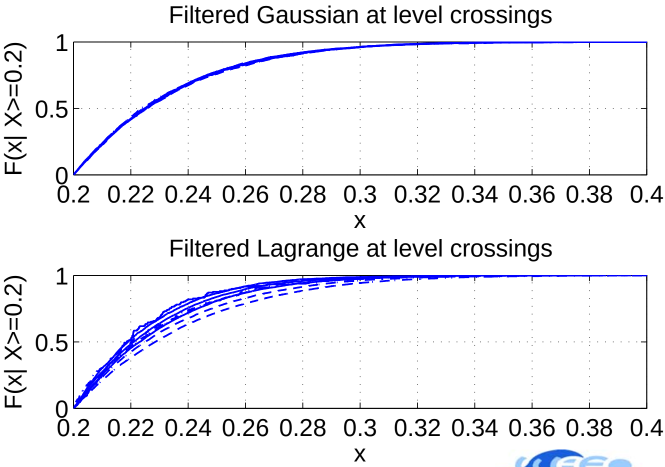
Still asymmetry in slopes at level crossings



Base data from the Lysekil station - resonant conditions



Base data from the Lysekil station - non-resonant conditions



Conclusions

- A Lagrange wave model gives realistic asymmetric slope distributions
- Lagrange waves become more "Gaussian" after linear filtration like in a wave power station
- Still considerable asymmetry at high levels
- Model simulations with irregular sea should take asymmetry into account
- Full scale experiment + EVT still needed