Statistical/Mathematical wave models – for what purpose?

Discussion

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Measurement of the Roughness of the Sea Surface from Photographs of the Sun’s Glitter

CHARLES COX AND WALTER MUNK
Scripps Institution of Oceanography,* La Jolla, California
(Received April 28, 1954)

An experiment 60 years ago
The Cox and Munk experiment on wave asymmetry – compared to Lagrange asymmetry:
Deterministic theory for water waves

- Wave "equations"
  - Newton: 1687
  - Euler, Laplace: 1750 – 1776
  - Lagrange: 1781 – 1786
  - Gerstner: 1802 (circles)
  - Miche: 1944 (ellipses)
  - Zakharov: 1968 (non-linear Schrödinger equations)

- Rayleigh: The basic law of the seaway is the apparent lack of any law
A Historical Note on the Study of Ocean Surface Waves

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The modern study of ocean surface waves started with a pioneer study by Sverdrup and Munk (1947). More than half a century has passed since then and the study of ocean surface waves has greatly advanced. The current numerical wave models, supported by many fundamental studies, enable us to compute ocean surface waves on a global scale with sufficient accuracy for practical purposes. However, physical processes controlling the energy balance of ocean surface waves is still not completely understood. The present note is a rough sketch of the historical development of the study of ocean surface waves.
Stochastic wave models

- **Rice:** *On random noise, 1944-45*
- **StDenis and Pierson:** *On the motion of ships in confused seas, 1952*

### Table 1. Advances in the study of ocean surface waves in the latter half of the twentieth century.

<table>
<thead>
<tr>
<th>Generation mechanism</th>
<th>1940s</th>
<th>1950s</th>
<th>1960s</th>
<th>1970s</th>
<th>1980s</th>
<th>1990s</th>
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<tbody>
<tr>
<td></td>
<td>basic studies</td>
<td>new theory</td>
<td>extension</td>
<td>Townsend</td>
<td>Al’Zanaidi and Hui</td>
<td>Belcher</td>
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<td></td>
<td>Wuest, Roll</td>
<td>Phillips, Miles</td>
<td>turbulence</td>
<td>Gent and Taylor</td>
<td>numerical study</td>
<td>Miles (revisit)</td>
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<tr>
<td>Statistical theory</td>
<td>theory of random noise (S. O. Rice)</td>
<td>wave spectrum</td>
<td>spectral form</td>
<td>directional spectra</td>
<td>high frequency wave</td>
<td>wave number-frequency spectra</td>
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<td></td>
<td></td>
<td>wave statistics</td>
<td>nonlinear effect</td>
<td>similarity form</td>
<td>spectrum</td>
<td></td>
</tr>
<tr>
<td>Nonlinear theory</td>
<td>nonlinear theory of regular waves</td>
<td>nonlinear theory of random waves</td>
<td>wave interaction</td>
<td>wave interaction</td>
<td>computation wave</td>
<td>wave breaking and energy dissipation</td>
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<td></td>
<td></td>
<td></td>
<td>wave instability</td>
<td>dispersion relation</td>
<td>breaking</td>
<td></td>
</tr>
<tr>
<td>Experiments (Labo. &amp; Ocean)</td>
<td>basic studies visual observation (instrumental)</td>
<td>wave observation</td>
<td>advanced experiment</td>
<td>local equilibrium</td>
<td>wave dynamics ex.</td>
<td>microwave remote sensing</td>
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<td>ocean experiment</td>
<td>satellite observation</td>
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<td>Air-sea and wave projects</td>
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<td>SWOP</td>
<td>JONSWAPHEXOS</td>
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<td>Sun Glitter Project</td>
<td>ARSLOE SOWEX, RASEX</td>
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<tr>
<td>Wave forecasting</td>
<td>Sverdrup and Munk</td>
<td>SMB method</td>
<td>numerical model</td>
<td>numerical model</td>
<td>WAM</td>
<td>JWA3G data assimilation</td>
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<tr>
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<td>PNJ method</td>
<td>(1st generation)</td>
<td>(2nd generation)</td>
<td>(3rd generation)</td>
<td></td>
</tr>
</tbody>
</table>
Gauss and Gauss-Lagrange models

- The (linear) Gaussian wave model allows exact computation of wave characteristic distributions (the WAFO toolbox)
- produces crest-trough and front-back stochastically symmetric waves
- The modified stochastic Lagrange model can produce
  (2006) crest-trough asymmetric waves (2D)
  (2009) front-back asymmetric waves (2D)
  (2011) front-back asymmetric waves (3D)
- still allows for exact computation of wave characteristic distributions!
- Software exists for calculation of statistical wave characteristic distributions (crest, trough heights, period, steepness etc)
SPDE-fields as continuous field versions of ARMA-processes

Nested SPDE models are solutions to the SPDE

\[
\left( \prod_{i=1}^{n_1} \left( \kappa_i^2 - \Delta \right)^{\alpha_i} \right) X(s) = \left( \prod_{i=1}^{n_2} (b_i + B_i^T \nabla) \right) W(s)
\]

and have spectrum of form

\[
R(k) = \frac{\phi^2}{(2\pi)^2} \frac{(b + k^T BB^T k)^\beta}{(\kappa^2 + k^T k)^\alpha}
\]

Approximate common ocean wave spectra - next slide
SPDE approximations of standard PM ocean wave spectrum
Simulated SPDE waves
Closing the gap between stochastic and deterministic models

- Gaussian model: StDenis and Pierson, Longuet-Higgins, ... , 1950 –
- Stokes waves: 1960 –
- Lagrange models: Gjösund (2003), Fouques et al (2006), ... 
- Laplace models: Podgorski ...
- SPDE-models: Bolin and Lindgren (Finn) 2009, ...

HERE IS THE GAP – HOW CAN THAT BE CLOSED?

- non-linear Schrödinger equation for wave propagation from Gaussian wave boundary conditions
Lagrange versus Laplace versus SPDE

- **Lagrange**
  - pro: physically half-realistic, allows computation of distributions from Gaussian structure
  - con: physically half-realistic, estimation?

- **Laplace**
  - pro: allows linear filtering, estimation by moment method
  - con: no physical interpretation

- **SPDE**
  - pro: close to the physical non-linear wave equations, approximate common wave spectra
  - con: time dimension? estimation?
How close the gap to hydrodynamic models?