Random load models for use in stochastic fatigue life assessment – some unpredicted consequences of Bertram Broberg’s urge to understand

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Bertram Broberg - some not so wellknown impacts of a curious man

▶ “Knut Bertram Broberg (1925 – 2005) was internationally acclaimed for his pioneering research in fracture mechanics, geophysics, biomechanics and mathematics”
▶ What about mathematics?
▶ There is nothing as inspiring for theoretical research as a good practical problem.
An event – 50 years ago

Two gentlemen in June 1961:

Cycle range $H$ distribution

$X(t)$ is a Gaussian, stationary load process
Sjöström and Broberg, 1961

\[ M = \frac{M^+ + M^-}{2} = \text{cycle mean} \]

\[ H/2 = \frac{M^+ - M^-}{2} = \text{cycle (half) range} \]

Sjöström: “One can conclude that \( H/2 \) has a Rayleigh distribution provided \( M \) and \( H/2 \) are independent random variables.”

Broberg: “Yes - that follows from Lévy-Cramér’s characterisation of the normal distribution, plus Rice’s formula for the expected number of crossings!”

BUT – the independence assumption is false
Crossings and extreme value theory for Gaussian processes

- Mathematical theory for crossings and extremes in Gaussian processes
- Anders Lundström: Find the distribution of the Rain Flow Cycles
Rainflow cycle distribution in Gaussian loads

- Is it possible to evaluate the statistical distribution of the Rain Flow cycle range from the energy (frequency) spectrum of a Gaussian random load?
- The Rain Flow cycle range (Endo) is equivalent to (Rychlik, 1986)

\[ H_k^{\text{Rfc}} = M_k - m_k^{\text{Rfc}} \]
An observation from ocean engineering

- Extreme waves often come many in a row – Markov chain?
- In Gaussian loads, the sequence of local maxima and minima are “almost” a Markov chain
- Rychlik – the Rain Flow definition can be reformulated as an absorption problem in a random walk
- The EXACT Rain Flow cycle (RFC) distribution can be derived from the Markov chain transition probabilities
- Transition probabilities in the (approximative) Markov chain can be computed from the power spectrum of a Gaussian or transformed Gaussian load
The role of the random environment

Fatigue life depends on

- Material properties – fracture mechanics
- Structural properties - structural mechanics
- Environmental properties – statistics
  - Random environment – wind loads on wind power mills, road induced loads on cars, wave induced loads on ships
  - Random usage – operation policies for wind power farms, fatigue consequences of operation and driver conditions for heavy trucks, fatigue aspects on route planning under meteorological uncertainties for ocean transports
Industrial impact

- Changing practise in the automotive industry
- Better understanding of the environmental impact on ship hull fatigue
An ocean is more random than a highway

- Statistical models for ocean waves and ocean wave climate have been used since ca 1950
- Sea state is characterized by its directional wave energy spectrum
- Basic parameters
  Significant wave height $H_s = 4 \times$ standard deviation of sea surface
  and Peak period $T_p$
- Predicted from wind forecasts by means of wind-wave models
- Measured by ship masters, buoys and wave recorders, satellites
Fatigue in a marine environment

- Wind climate
- Wave climate
- Sea state, $H_s$, $T_p$
- Extreme events
- Ship as a dynamic system
- Response
- Fatigue
- Capsizing
The ship – from a PhD thesis 2010 by Wengang Mao, Chalmers
The voyages
The ocean is a space and time dependent random surface. During short periods, 30 min, one can assume it is statistically homogeneous and stationary with standard deviation $\sigma$ and with energy distributed over wave frequencies $\omega$ according to a spectral density $S(\omega)$ or, with also wave direction $\theta$ taken into account directional spectral density $S(\omega) \cos^m(\theta, \omega)$
Problems to be dealt with

- Estimate $H_s$ during the whole voyage
  - Measure $H_s$ from the ship
  - Measure $H_s$ from satellites
  - Calculate $H_s$ from wind data

- Estimate $h_s$ ...
  - Measure $h_s$ on the ship
  - Calculate $h_s$ by structural dynamics numerical software

- ... find its distribution ...

- ... and use it for fatigue damage estimation and extreme response prediction
A basic equation

\[ 4\sigma_x = h_s = C \cdot H_s \]
A common approach to random fatigue

- Fatigue is generated by cyclic stress changes in a construction detail
- Cycle range is important – a cycle with range $S$ causes damage
  \[ S^k / \alpha \]
  
  where $k = m = 3$ and $\alpha$ are material dependent parameters
- During a time span $T$, with cycles $\{ S_i \}$ occurring at a rate $f_0$ per time unit, the total damage is
  \[ D(T) = \sum S_i^k / \alpha \approx T f_0 \mathbb{E}[S^k] / \alpha \]
- So, we need $f_0$ and the distribution of $S$
RFC damage computation from spectrum

Two ways to find the RFC damage:

- Calculate as $D(T) = \sum S_i^k / \alpha$
- from direct measurements of the rainflow cycles in the stress signal
- from spectrum + some distribution assumption for the (statistically stationary) stress process assuming the sequence of stress maxima and minima are approximatively a Markov process (Rychlik)
- Gives fatigue damage for each encountered sea state during a voyage
Slamming effect on stress
Non-linear response

Linear response theory will reproduce only the wave induced peak to the left in the estimated spectra. The middle peak with little energy caused by non-linear phenomena cause considerable fatigue.

50 estimated spectra from 30 minutes stress measurements
Some results from the Container ship experiment

- RFC damage observed from total travel compared to the sum of RFC damage from separate (stationary) 30 minutes period agree
- RFC damage on measured stress about 1.5% higher than theoretically calculated RFC damage for Gaussian stress with same spectra
- Numerically computed stress functions considerably overestimates fatigue, compared to observed RFC damage
- $H_s$ from on-board measurements is larger than $H_s$ from satellites
Next steps

- route planning for fatigue
- ship certification rules
Unexpected consequences

Question:
– What came out of the Sjöström – Broberg discussion 1961?

Answer:
– About 60 PhDs in Mathematical statistics and other fields!

Conclusion:
– There is nothing as inspiring for theoretical research as a good practical problem.