

Kernel estimators for the second order parameter in extreme value statistics

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Abstract

We develop and study in the framework of Pareto-type distributions a general class of kernel estimators for the second order parameter ρ , a parameter related to the rate of convergence of a sequence of maximum values, linearly normalized, towards its limit. Inspired by the kernel goodness-of-fit statistics introduced in Goegebeur *et al.* (2008), for which the mean of the normal limiting distribution is a function of ρ , we construct estimators for ρ using ratios of ratios of differences of such goodness-of-fit statistics, involving different kernel functions as well as power transformations. The consistency of this class of ρ estimators is established under some mild regularity conditions on the kernel function, a second order condition on the tail function $1 - F$ of the underlying model, and for suitably chosen intermediate order statistics. Asymptotic normality is achieved under a further condition on the tail function, the so-called third order condition. Two specific examples of kernel statistics are studied in greater depth, and their asymptotic behavior illustrated numerically. The finite sample properties are examined by means of a simulation study.

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