Lunds universitet
Matematisk statistik

Written exam in probability (5p) MAS203
1999–15–11 kl 9-14.00

Complete answers are required for all problems at the same level of detail. Only paper delivered by the department is allowed. In particular, your own paper is not allowed as scratch paper. Each sheet of paper should contain your full name. Each solution should start on top a new page. Pens or pencils in red colour are not allowed. Allowed tools: the sheet delivered by the department containing a table of distributions, and a hand calculator.

"VG" requires at least 75p and "G" requires at least 50p.

1. (15p) A number X is chosen uniformly on the interval [0,1]. Then another number Y is chosen uniformly among those positive numbers which are smaller then X. Compute $Cov\,(X,Y)$

2. (15p) Suppose that X and Y have a joint moment generating function given by
$$\psi_{X,Y}(t,u) = \exp\{t^2 + 2tu + 2u^2\}.$$
Compute the distribution of Y given that $X < 0$. (The answer can be in an integral form.)

3. (15p) Assume that N_k, $k = 1, 2, \ldots$ are independent random variables distributed as $Po(k)$, respectively. Let $X_1, X_2, \ldots$ be independent $Exp(1)$-distributed random variables independent of N_k, $k = 1, 2, \ldots$. Find the limit in distribution of
$$Y_k \; = \frac{\sum_{i=1}^{k}(X_i - 1)}{\sqrt{N_k}} \quad \text{as } k \to \infty.$$

4. (15p) Assume that some organisms live for 12 hours only and within their life-time they produce offspring independently and according to the Poisson process with intensity 1/24 per hour. Given that initially there were 6 organisms only, what is the probability of the total extinction of their population?

5. (20p) Assume that for an exhibition people arrive independently and their arrivals form a Poisson processes with an intensity $\lambda > 0$ per hour. Given that in the end of the first hour there are 10 people in the exhibition hall, what is the probability that exactly 5 of them came within first 30 min?

6. (20p) Assume we have 2 different dice. One is "fair", i. e. it has numbers from "1" to "6" on its sides. Another one has numbers "1" to "5" as a usual one, but instead of "6" it has one more "1" . We choose one of the dice (with equal probabilities) and call X the number of the throwings of this dice until "1" is obtained, and call Y the sum of the points we get in these X throwings. Find $\mathbf{E}Y$.