Lunds universitet
Matematisk statistik

Written exam in probability (5p) MAS203
1998–10–26 kl 9-14.00

Complete answers are required for all problems at the same level of detail.

Only paper delivered by the department is allowed. In particular, your own paper is not allowed as scratch paper. Each sheet of paper should contain your full name. Each solution should start on top a new page. Pens or pencils in red colour are not allowed.

Allowed tools: the sheet delivered by the department containing a table of distributions, and a hand calculator.

1. (10p) While driving a car you can see hares or deer along a road. Assume that the moments you see the animals form independent Poisson processes with intensity one per hour for hares, and intensity one per two hours for deer. What is the probability that you do not see any of those animals within 4 hours of drive?

2. (15p) The number of offspring of an organism is a discrete random variable with mean μ and variance σ^2. Each of its offspring reproduces in the same manner. Find the expected number of offspring in the third generation and its variance.

3. (15p) Let $X_1, X_2, \ldots$ be independent $U(0,1)$-distributed random variables, and let $N_k \in Po(k)$ be independent of $X_1, X_2, \ldots$. Find the limiting distribution of

$$Y_k = k \min\{X_1, X_2, \ldots, X_{N_k}\}$$

as $k \to \infty$. (Assume that $Y_k = 0$ if $N_k = 0$.)

4. (20p) Let $\{Y_k, k \geq 1\}$ be independent $U(-a,a)$-distributed random variables ($a > 0$), and set

$$X_n = \frac{\sum_{k=1}^{n} Y_k}{\sqrt{n}\ \max_{1\leq k\leq n} Y_k}.$$

Show that X_n converges in distribution to $N(0, 1/3)$ as $n \to \infty$.

5. (20p) A point P is chosen uniformly on a disk of radius 1. Next a point Q is chosen uniformly within the concentric circle going through P. Let X and Y be the distances of P and Q, respectively, from the center. Find the distribution of $Z = X - Y$.

6. (20p) Assume that the characteristic function of the joint distribution of the vector (X, Y) is

$$\phi_{X,Y}(t_1, t_2) = \exp\{it_2 - a(t_1^2 + t_1 t_2 + \frac{1}{2}t_2^2)\}.$$

Determine the values of a and y such that the conditional distribution of X given $Y = y$ is standard normal.