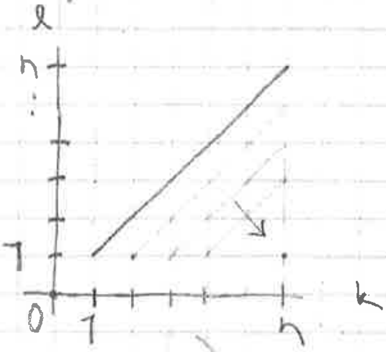
[With modified condition]

Lemma

$$\le \frac{1}{n^2 \varepsilon^2} \mathbb{E}\left[\left(\sum_{k=1}^{n} \phi(X_k) - \int \phi(x)\pi(x)\,dx\right)^2\right]$$

$$= \frac{1}{n^2 \varepsilon^2} \sum_{k=1}^{n} \sum_{\ell=1}^{n} \mathbb{E}\left[\left(\phi(X_k) - \int \phi(x)\pi(x)\,dx\right)\left(\phi(X_\ell) - \int \phi(x)\pi(x)\,dx\right)\right]$$

$$= \frac{1}{n^2 \varepsilon^2} \sum_{k=1}^{n} \sum_{\ell=1}^{n} \mathrm{Cov}\left(\phi(X_k), \phi(X_\ell)\right)$$



symm.

$$= \frac{1}{n^2 \varepsilon^2} \left(\underbrace{\sum_{k=1}^{n} \mathrm{Var}(\phi(X_k))}_{= n \cdot \mathrm{Var}(\phi(X_1))} + 2 \sum_{n \ge k > \ell \ge 1} \mathrm{cov}(\phi(X_k), \phi(X_\ell)) \right)$$

stat.

$$= \frac{1}{n^2 \varepsilon^2} \left(n \cdot \mathrm{Var}(\phi(X_1)) + 2 \cdot \sum_{m=1}^{n-1} \underbrace{(n-m)}_{< n} \cdot \mathrm{Cov}(\phi(X_1), \phi(X_{1+m})) \right)$$

take mod.

$$\le \frac{1}{n^2 \varepsilon^2} \left(n \cdot \mathrm{Var}(\phi(X_1)) + 2n \sum_{m=1}^{n-1} \left| \mathrm{cov}(\phi(X_1), \phi(X_{m+1})) \right| \right)$$

(*)

$$\le \frac{1}{n^{\cancel{2}} \cdot \varepsilon^2} \cdot \left(2\cancel{n} \cdot \sum_{m=1}^{n} \left| \mathrm{cov}(\phi(X_1), \phi(X_m)) \right| \right)$$

$$\le \frac{1}{n \cdot \varepsilon^2} \cdot 2 \underbrace{\sum_{m=1}^{\infty} -''-}_{< \infty} \to 0, \text{ as } n \to \infty.$$