Exercise 6.4. Usually the Asian option is defined using an arithmetic average, which in B&S market is hard to compute. However, using a geometric it is quite simple task if the formula in a) can be used. This formula is not very important outside this exercise and may be skipped if it is too elaborate.

a) Show that;
\[ \sum_{i=1}^{N} \sum_{j=1}^{N} \min\{i, j\} = \frac{N(N + 1)(2N + 1)}{6} \]

b) Find the value at \( t = 0 \) of the Asian option with the following payoff function;
\[ \Phi(S) = \left( \prod_{i=1}^{N} (S_{t_i})^{\frac{1}{N}} - K \right)^+ \]

where \( \{t_i\} \) are \( N \) equally distributed dates in the interval \([0, T]\).
\[ \Pi_0 = S_0e^{\mu_z + \sigma_z^2 r T} N[d_1] - Ke^{-rT} N[d_2] \]
\[ d_2 = \frac{\ln(S_0/K) + \mu_z}{\sigma_z} \quad d_1 = d_2 + \sigma_z \]
\[ \mu_z = (r - \frac{\sigma^2}{2}) \frac{TN(N+1)}{2N^2} \quad \sigma_z^2 = \frac{\sigma^2}{6N^3} \frac{TN(N+1)(2N+1)}{6N^3} \]