Solution.

1. We have

\[ X(t) = \frac{e^{-\frac{W^2_t}{2(1 + T - t)}}}{\sqrt{1 + T - t}}. \]

Applying Ito’s formula to \( X \) we obtain

\[
dX(t) = e^{-\frac{W^2_t}{2(1 + T - t)}} \left( -\frac{W^2_t}{2(1 + T - t)} + \frac{1}{2(1 + T - t)} - \frac{1}{2(1 + T - t)} + \frac{W^2_t}{2(1 + T - t)^2} \right) dt
- e^{-\frac{W^2_t}{2(1 + T - t)}} \frac{W_t}{\sqrt{1 + T - t}} \frac{1 + T - t}{1 + T - t} dW_t
= -X(t) \frac{W_t}{1 + T - t} dW_t.
\]

So \( X \) is a MG for \((0 \leq t \leq T)\) if (using the Ito isometry)

\[
E \left[ \left( \int_0^T X(t) \frac{W_t}{1 + T - t} dW_t \right)^2 \right] = E \left[ \int_0^T X(t)^2 \frac{W^2_t}{(1 + T - t)^2} dt \right] < \infty.
\]

Looking at \( X(t) \) we see that

\[
\frac{X(t)^2}{(1 + T - t)^2} = \frac{e^{-\frac{W^2_t}{2(1 + T - t)}}}{(1 + T - t)^3} \leq \frac{1}{(1 + T - t)^3} \leq 1, \text{ for } 0 \leq t \leq T.
\]

So therefore we get that

\[
E \left[ \int_0^T X(t)^2 \frac{W^2_t}{(1 + T - t)^2} dt \right] \leq E \left[ \int_0^T W^2_t dt \right] = \int_0^T \mathbb{E}[W^2_t] dt = \int_0^T t dt = T^2/2 < \infty
\]

So we have shown that \( X(t) \) is a martingale for \( 0 \leq t \leq T \).

2. We have the following model for the forward rate

\[
df(t, u) = \sigma(t)^2(u - t) dt + \sigma(t) dW(t)
\]

\[
f(0, u) = c.
\]

We obtain by integrating up the dynamics that

\[
f(t, u) = c + \int_0^t \sigma(s)^2(u - s) ds + \int_0^t \sigma(s) dW(s).
\]

Now since \( r(t) = f(t, t) \) we get

\[
r(t) = c + \int_0^t \sigma(s)^2(t - s) ds + \int_0^t \sigma(s) dW(s).
\]
By applying Ito's formula to \( r(t) \) we obtain the following dynamics for \( r \)
\[
    dr(t) = d\epsilon + \int_0^t \sigma(s)(t - s) \, ds + d\int_0^t \sigma(s) \, dW(s)
    \]
\[
    = (0 + 0 + \int_0^t \sigma(s)^2 \, ds) \, dt + \sigma(t) \, dW(t)
    \]
\[
    = \int_0^t \sigma(s)^2 \, ds \, dt + \sigma(t) \, dW(t)
    \]

\[\blacksquare\]

3. Let \( X(T_0) \) denote the value of the floating rate bond at time \( T_0 \). We start by noting that the value of the floating rate bond is just all expected cash flows discounted back to time \( T_0 \). This gives that

\[
    X(T_0) = \mathbb{E}^Q \left[ \frac{B(T_n)}{B(T_0)} A + \sum_{i=1}^n \frac{B(T_i)}{B(T_{i-1})} A(T_i - T_{i-1}) L_{T_{i-1}}(T_i, T_i) \, | \mathcal{F}_{T_0} \right]
    \]
\[
    = \mathbb{E}^Q \left[ \frac{B(T_n)}{B(T_0)} A \right] \mathbb{E}^Q \left[ \sum_{i=1}^n \frac{B(T_i)}{B(T_{i-1})} A(T_i - T_{i-1}) L_{T_{i-1}}(T_i, T_i) \, | \mathcal{F}_{T_0} \right]
    \]
\[
    = \mathbb{E}^Q \left[ \frac{B(T_n)}{B(T_0)} A \right] \mathbb{E}^Q \left[ \sum_{i=1}^n \left( \frac{B(T_i)}{B(T_{i-1})} A \left( \frac{1}{p(T_{i-1}, T_i)} - 1 \right) \right) \, | \mathcal{F}_{T_0} \right]
    \]

Now we change numeraires so that each cash flow is evaluated under the corresponding forward measure, that is a cash flow at time \( T_i \) is evaluated using the measure \( \mathbb{Q}^{T_i} \) with numeraire \( p(t, T_i) \). This then gives

\[
    X(T_0) = p(T_0, T_n) \mathbb{E}^{Q^{T_n}} \left[ A \right] \mathbb{E}^{Q^{T_i}} \left[ A \left( \frac{1}{p(T_{i-1}, T_i)} - 1 \right) \right] \, | \mathcal{F}_{T_0} \right].
    \]

Now using that \( \frac{1}{p(T_{i-1}, T_i)} = \frac{p(T_{i-1}, T_i)}{p(T_{i-1}, T_i)} \) is martingale under \( \mathbb{Q}^{T_i} \) we get

\[
    X(T_0) = A p(T_0, T_n) + A \sum_{i=1}^n p(T_0, T_i) \left( \frac{p(T_0, T_{i-1})}{p(T_0, T_i)} - 1 \right)
    \]
\[
    = A p(T_0, T_n) + A \sum_{i=1}^n p(T_0, T_{i-1}) - p(T_0, T_i)
    \]
\[
    \text{telescoping sum}
    \]
\[
    = A p(T_0, T_n) + A(p(T_0, T_0) - p(T_0, T_n)) = A p(T_0, T_0) = A.
    \]

So this floating rate bond always has a value equal to the face value \( A \) at time \( T_0 \) for all arbitrage free models. \[\blacksquare\]

4. Using Feynman-Kačs representation formula we obtain
\[
    f(t, x) = e^{-r(T-t)} \mathbb{E}[I(a \leq X(T) \leq b) | X_t = x]
    \]
where \( X \) has the following dynamics for \( t \leq s \leq T \)
\[
    dX_t = rX_t \, ds + \alpha X_t \, dW_t, \ X_t = x.
    \]
So the solution is the price of a derivative with pay-off \( I(a \leq X(T) \leq b) \) at maturity \( T \) for the case where the underlying asset follows the standard Black-Scholes model. Using this we see that
\[
X(T) = xe^{(r-\sigma^2/2)(T-t)+\sigma(W_T-W_t)} \overset{d}{=} xe^{(r-\sigma^2/2)(T-t)+\sigma\sqrt{T-t}G},
\]
where \( G \) is standard Gaussian random variable. We thus obtain that
\[
f(t,x) = e^{-r(T-t)} \int_{-\infty}^{\infty} I(a \leq xe^{(r-\sigma^2/2)(T-t)+\sigma\sqrt{T-t}y} \leq b) \frac{e^{-y^2}}{\sqrt{2\pi}} dy
\]
\[
= e^{-r(T-t)} \int_{\ln(a/x) - (r-\sigma^2/2)(T-t) \sigma\sqrt{T-t}}^{\ln(b/x) - (r-\sigma^2/2)(T-t) \sigma\sqrt{T-t}} \frac{e^{-y^2}}{\sqrt{2\pi}} dy
\]
\[
= e^{-r(T-t)} \left( N \left( \frac{\ln(b/x) - (r-\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \right) - N \left( \frac{\ln(a/x) - (r-\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \right) \right),
\]
Where \( N \) is the distribution function of a standard Gaussian random variable. So we have that
\[
f(t,x) = e^{-r(T-t)} \left( N \left( \frac{\ln(b/x) - (r-\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \right) - N \left( \frac{\ln(a/x) - (r-\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \right) \right)
\]
We start by checking the boundary condition. Now
\[
\lim_{t \uparrow T} \frac{\ln(b/x) - (r-\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} = \begin{cases} 
\infty & x < b \\
-\infty & x > b 
\end{cases}
\]
\[
\lim_{t \uparrow T} \frac{\ln(a/x) - (r-\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} = \begin{cases} 
\infty & x < a \\
-\infty & x > a 
\end{cases}
\]
\[
f(T,x) = \begin{cases} 
N(\infty) - N(\infty) = 0 & x < a \\
N(\infty) - N(-\infty) = 1 & a < x < b \\
N(-\infty) - N(-\infty) = 0 & b < x 
\end{cases}
\]
So the boundary condition is satisfied.
However checking all the partial derivatives leads to long and complicated calculations which are much harder than calculating the expectation in the Feynman-Kač representation formula. To check that the solution satisfies the PDE we start by calculating the partial derivatives:
\[
\frac{\partial}{\partial t} f(t,x) = rf(t,x) + e^{-r(T-t)} \frac{e^{-d_1^2/2}}{\sqrt{2\pi}} \left( \frac{r - \sigma^2/2}{\sigma\sqrt{T-t}} + \frac{d_1}{2(T-t)} \right)
\]
\[
- e^{-r(T-t)} \frac{e^{-d_2^2/2}}{\sqrt{2\pi}} \left( \frac{r - \sigma^2/2}{\sigma\sqrt{T-t}} + \frac{d_2}{2(T-t)} \right)
\]
\[
r \frac{\partial}{\partial x} f(t,x) = - e^{-r(T-t)} \frac{r}{\sigma\sqrt{T-t}} \left( \frac{e^{-d_1^2/2}}{\sqrt{2\pi}} - \frac{e^{-d_2^2/2}}{\sqrt{2\pi}} \right)
\]
\[
\frac{\sigma^2 x^2}{2} \frac{\partial^2}{\partial x^2} f(t, x) = e^{-r(T-t)} \frac{e^{-d_1/2}}{\sqrt{2\pi}} \left( \frac{\sigma^2}{2} - \frac{d_1}{2(T-t)} \right) - e^{-r(T-t)} \frac{e^{-d_2/2}}{\sqrt{2\pi}} \left( \frac{\sigma^2}{2} - \frac{d_2}{2(T-t)} \right).
\]

Putting all this together we get that
\[
\frac{\partial}{\partial t} f(t, x) + r x \frac{\partial}{\partial x} f(t, x) + \left( \sigma^2 x^2 / 2 \right) \frac{\partial^2}{\partial x^2} f(t, x) = rf(t, x) + e^{-r(T-t)} \frac{e^{-d_1/2}}{\sqrt{2\pi}} \left( r - \frac{\sigma^2}{2} \frac{d_1}{2(T-t)} - \frac{r}{\sigma \sqrt{T-t}} + \frac{\sigma^2}{2 \sigma \sqrt{T-t}} - \frac{d_1}{2(T-t)} \right) - e^{-r(T-t)} \frac{e^{-d_2/2}}{\sqrt{2\pi}} \left( r - \frac{\sigma^2}{2} \frac{d_2}{2(T-t)} - \frac{r}{\sigma \sqrt{T-t}} + \frac{\sigma^2}{2 \sigma \sqrt{T-t}} - \frac{d_2}{2(T-t)} \right) = rf(t, x).
\]

Thus we have that
\[
f(t, x) = e^{-r(T-t)} \left( N \left( \frac{\ln(b/x) - (r - \sigma^2/2)(T-t)}{\sigma \sqrt{T-t}} \right) - N \left( \frac{\ln(a/x) - (r - \sigma^2/2)(T-t)}{\sigma \sqrt{T-t}} \right) \right)
\]
solves the PDE. 

5. So we want to price a geometric Asian option with contract function
\[
\Phi(S) = (e^{\frac{1}{T} \int_0^T \ln(S(u)) \, du} - K)^+, 
\]
with maturity $T$ using the standard Black-Scholes model at time $t = 0$. So we have using the standard solution for the Black-Scholes model
\[
e^{\frac{1}{T} \int_0^T \ln(S(u)) \, du} \overset{d}{=} e^{\frac{1}{T} \int_0^T \ln(S(u)) + (r - \sigma^2/2)u + \sigma W(u) \, du} e^{\sigma \sqrt{T} G},
\]
where $G \in N(0, 1)$ and
\[
a = E^Q \left[ \frac{1}{T} \int_0^T \ln(S(0)) + (r - \sigma^2/2)u + \sigma W(u) \, du \bigg| \mathcal{F}_0 \right], \\
b = V^Q \left[ \frac{1}{T} \int_0^T \ln(S(0)) + (r - \sigma^2/2)u + \sigma W(u) \, du \bigg| \mathcal{F}_0 \right].
\]

We can now express the price of the derivative at time zero $\Pi(0)$ as a function of $a$ and $b$ using the RNVF
\[
\Pi(0) = e^{-rT} E^Q \left[ (e^{\frac{1}{T} \int_0^T \ln(S(u)) \, du} - K)^+ \bigg| \mathcal{F}_0 \right] = e^{-rT} E^Q \left[ e^{\sigma \sqrt{T} G} - K)^+ \bigg| \mathcal{F}_0 \right] = e^{-rT} \int_{-\infty}^{\infty} \left( e^{\sigma \sqrt{T} x} - K \right)^+ e^{-\frac{x^2}{2}} \, dx = e^{-rT} \int_{\ln(b)/\sigma}^{\infty} \left( e^{\sigma \sqrt{T} x} - K \right) e^{-\frac{x^2}{2}} \, dx.
\]
We now derive the values of $a$ and $b$.

\[
\begin{align*}
  a &= E^Q \left[ \frac{1}{T} \int_0^T \ln(S(0)) + (r - \sigma^2/2) u + \sigma W(u) \, du \big| \mathcal{F}_0 \right] \\
  &= \frac{1}{T} \int_0^T \ln(S(0)) + (r - \sigma^2/2) u \, du \\
  &= \ln(S(0)) + \frac{1}{2} (r - \sigma^2/2) \frac{T^2}{2} = \ln(S(0)) + (r - \sigma^2/2) \frac{T}{2} \\
  b &= V^Q \left[ \frac{1}{T} \int_0^T \ln(S(0)) + (r - \sigma^2/2) u + \sigma W(u) \, du \big| \mathcal{F}_0 \right] \\
  &= V^Q \left[ \frac{1}{T} \int_0^T \sigma W(u) \, du \big| \mathcal{F}_0 \right] \\
  &= \frac{\sigma^2}{T^2} \left[ \int_0^T W(u) \, du \big| \mathcal{F}_0 \right] \\
  &= \frac{\sigma^2}{T^2} \left[ \int_0^T \int_0^T \int_0^T dW(u) \, du \big| \mathcal{F}_0 \right] \\
  &= \frac{\sigma^2}{T^2} \left[ \int_0^T (T - s) \, dW(s) \big| \mathcal{F}_0 \right] \\
  &= \frac{\sigma^2}{T^2} \left[ \int_0^T (T - s)^3 \, dW(s) \big| \mathcal{F}_0 \right] \\
  &= \frac{\sigma^2}{T^2} \left[ \int_0^T (T - s)^3 \, ds \big| \mathcal{F}_0 \right] \\
  &= \frac{\sigma^2}{T^2} \left[ \left( \frac{T^2}{2} - \frac{T^4}{3} \right) \right] \\
  &= \frac{\sigma^2}{T^2} \left[ \frac{T^3}{3} \right] = \frac{\sigma^2 T}{3}.
\end{align*}
\]
We can then finally write up the price at time zero $\Pi(0)$

$$
\Pi(0) = e^{-rT + a + \frac{1}{2} b} N \left( \frac{- \ln(K) + a + b}{\sqrt{b}} \right) - Ke^{-rT} N \left( \frac{- \ln(K) + a}{\sqrt{b}} \right),
$$

where

$$a = \ln(S(0)) + (r - \sigma^2/2) \frac{T}{2}, \quad b = \frac{\sigma^2 T}{2}$$

and $N$ is the distribution function of the standard Gaussian distribution.

6. (a) We start by examining the pay-off:

$$
\max(S_A(T), S_B(T)) = S_A(T) I(S_A(T) \geq S_B(T)) + S_B(T) I(S_B(T) > S_A(T))
$$

$$
= S_A(T) I \left( \frac{S_B(T)}{S_A(T)} \leq 1 \right) + S_B(T) I \left( \frac{S_A(T)}{S_B(T)} < 1 \right).
$$

After this rewriting we are ready to attack the problem with RNVF. Let $\Pi(t, S_A(t), S_B(t))$ be the value of the contract at time $t$. Thus we have

$$
\Pi(t, S_A(t), S_B(t)) = \mathbb{E}^Q \left[ \frac{B(t)}{B(T)} \max(S_A(T), S_B(T)) | \mathcal{F}_t \right]
$$

$$
= \mathbb{E}^Q \left[ \frac{B(t)}{B(T)} S_A(T) I \left( \frac{S_B(T)}{S_A(T)} \leq 1 \right) | \mathcal{F}_t \right] + \mathbb{E}^Q \left[ \frac{B(t)}{B(T)} S_B(T) I \left( \frac{S_A(T)}{S_B(T)} < 1 \right) | \mathcal{F}_t \right].
$$

Now we apply the change of numeraire technique to simply the calculations:

$$
\Pi(t, S_A(t), S_B(t)) = S_A(t) \mathbb{E}^{Q_{S_A}} \left[ I \left( \frac{S_B(T)}{S_A(T)} \leq 1 \right) | \mathcal{F}_t \right] + S_B(t) \mathbb{E}^{Q_{S_B}} \left[ I \left( \frac{S_A(T)}{S_B(T)} < 1 \right) | \mathcal{F}_t \right]
$$

We can now use that $S_B(u)/S_A(u)$ is a $Q_{S_A}$ martingale and that $S_A(u)/S_B(u)$ is a $Q_{S_B}$ martingale, since they are both ratios of traded assets and numeraires. We then get the following $Q_{S_A}$-dynamics for $S_B(u)/S_A(u)$ (using Ito and the MG-property)

$$
\frac{d}{dt} \left( \frac{S_B(u)}{S_A(u)} \right) = \frac{S_B(u)}{S_A(u)} \left( (\sigma_B - \sigma_A) dW_1^{Q_{S_A}}(t) + \sigma_B \sqrt{1 - \rho^2} dW_2^{Q_{S_A}}(t) \right)
$$

$$
\frac{d}{dt} \left( \frac{S_B(u)}{S_A(u)} \right) = \sqrt{(\sigma_B - \sigma_A)^2 + \sigma_B^2} dW^{Q_{S_A}}(t)
$$

$$
= \frac{S_B(u)}{S_A(u)} \tilde{\sigma} dW^{Q_{S_A}}(t),
$$

where $\tilde{\sigma} = \sqrt{(\sigma_B - \sigma_A)^2 + \sigma_B^2}$ and $W^{Q_{S_A}}$ is a standard $Q_{S_A}$ BM. So then we get that

$$
\frac{S_B(T)}{S_A(T)} = \frac{S_B(t)}{S_A(t)} e^{-\tilde{\sigma}^2(T-t) + \tilde{\sigma} \sqrt{T-t} G},
$$

where $G$ is standard Gaussian random variable.

Using the same type of arguments we get the following distribution for $S_A(u)/S_B(u)$ under $Q_{S_B}$.

$$
\frac{S_A(T)}{S_B(T)} = \frac{S_A(t)}{S_B(t)} e^{-\tilde{\sigma}^2(T-t) + \tilde{\sigma} \sqrt{T-t} G},
$$
where \( G \) is standard Gaussian random variable and where
\[
\tilde{\sigma} = \sqrt{b^\sigma - \sigma_A} + \frac{\sigma_B}{\sigma_B}
\] as before. Putting this together we obtain

\[
II(t, S_A(t), S_B(t)) = S_A(t)Q^{S_A(t)} \left( \frac{S_B(t)}{S_A(t)} e^{-\tilde{\sigma}^2(T-t)+\tilde{\sigma}\sqrt{T-t}G} \leq 1 | F_t \right)
\]

\[
+ S_B(t)Q^{S_B(t)} \left( \frac{S_A(t)}{S_B(t)} e^{-\tilde{\sigma}^2(T-t)+\tilde{\sigma}\sqrt{T-t}G} < 1 | F_t \right)
\]

\[
= S_A(t)Q^{S_A(t)} \left( G \leq \frac{\ln \left( \frac{S_A(t)}{S_B(t)} \right) + \frac{\tilde{\sigma}^2}{2} (T-t)}{\tilde{\sigma}\sqrt{T-t}} | F_t \right)
\]

\[
+ S_B(t)Q^{S_B(t)} \left( G < \frac{\ln \left( \frac{S_A(t)}{S_B(t)} \right) + \frac{\tilde{\sigma}^2}{2} (T-t)}{\tilde{\sigma}\sqrt{T-t}} | F_t \right)
\]

\[
= S_A(t)N \left( \frac{\ln \left( \frac{S_A(t)}{S_B(t)} \right) + \frac{\tilde{\sigma}^2}{2} (T-t)}{\tilde{\sigma}\sqrt{T-t}} \right) + S_B(t)N \left( \frac{\ln \left( \frac{S_A(t)}{S_B(t)} \right) + \frac{\tilde{\sigma}^2}{2} (T-t)}{\tilde{\sigma}\sqrt{T-t}} \right)
\]

As a preparation for (d) we also look explicitly at the price at time zero

\[
II(0, S_A(0), S_B(0)) = S_A(0)N \left( \frac{\ln \left( \frac{S_A(0)}{S_B(0)} \right) + \frac{\tilde{\sigma}^2}{2} T}{\tilde{\sigma}\sqrt{T}} \right) + S_B(0)N \left( \frac{\ln \left( \frac{S_A(0)}{S_B(0)} \right) + \frac{\tilde{\sigma}^2}{2} T}{\tilde{\sigma}\sqrt{T}} \right)
\]

(b) The easiest way of showing this is to re-write the value at time \( T \) for (b)

\[
\Phi_b(S_A(T), S_B(T)) = S_A(T) + (S_B(T) - S_A(T))^+
\]

\[
= S_A(T) + (S_B(T) - S_A(T))I(S_A(T) < S_B(T))
\]

\[
= S_A(T)I(S_A(T) \geq S_B(T)) + S_B(T)I(S_A(T) < S_B(T))
\]

\[
= \max(S_A(T), S_B(T)).
\]

This was exactly what we needed to show since now the values in (a) and (b) coincide for all possible outcomes at maturity and thus they must also coincide for all previous times to avoid arbitrage.

(c) The Black-Scholes like market in this problem is complete so we can hedge all contingent claims. To find the hedge we use the standard \( \Delta \)-hedge approach.

\[
b_B(t) = \frac{1}{B(t)} II(t, S_A(t), S_B(t)) - h_{S_A}(t)S_A(t) - h_{S_B}(t)S_B(t)
\]

\[
b_{S_A}(t) = \frac{\partial}{\partial S_A} II(t, S_A(t), S_B(t))
\]

\[
b_{S_B}(t) = \frac{\partial}{\partial S_B} II(t, S_A(t), S_B(t))
\]

We start with \( b_{S_A}(t) \)

\[
b_{S_A}(t) = \frac{\partial}{\partial S_A} (S_A(t)N(d_1) + S_B(t)N(d_2))
\]

\[
= N(d_1) + S_A(t)n(d_1) \frac{\partial}{\partial S_A}(d_1) + S_B(t)n(d_2) \frac{\partial}{\partial S_A}(d_2)
\]

\[
= N(d_1) + \frac{\partial}{\partial S_A}(d_1) + S_B(t)n(d_2) \frac{\partial}{\partial S_A}(d_2)
\]

\[
= N(d_1) + \frac{\partial}{\partial S_A}(d_1) + S_B(t)n(d_2) \frac{\partial}{\partial S_A}(d_2)
\]
From (a) we get that the price of derivative at time zero is
\[
\Pi(0, S_A(0), S_B(0)) = S_A(0)N \left( \frac{\ln \left( \frac{S_A(0)}{S_A(t)} \right) + \frac{\sigma_A^2}{2} T}{\sigma_A \sqrt{T}} \right) + S_B(0)N \left( \frac{\ln \left( \frac{S_B(0)}{S_B(t)} \right) + \frac{\sigma_B^2}{2} T}{\sigma_B \sqrt{T}} \right).
\]

Now using that \( S_A(0) = S_B(0) = S \) and that \( \sigma_A = \sigma_B = \sigma \) we get
\[
\Pi(0, S_A(0), S_B(0)) = 2SN \left( \frac{\sigma \sqrt{1 - \rho \sqrt{T}}}{\sqrt{2}} \right).
\]

We can then compare this to buying both stocks which costs 2\( S \). So buying both stocks are always more expensive. Now looking at the potential pay-off we have that the derivative gives the best of the two stocks. We have that the correlation is negative so if one stocks goes up the other goes down compared to the risk free rate. In most cases one of the stocks will be worth considerably more than the other so most of the sum of both stocks will come from the best performing stock. So the derivative will in most cases give almost as high pay off as the sum of the two stocks but with less intial investment. So if our assumption about the correlation is true we should prefer the derivative to the sum of the two stocks. 

(d) From (a) we get that the price of derivative at time zero is
\[
\Pi(0, S_A(t), S_B(t)) = A_N(d_1) + \frac{1}{\sigma \sqrt{T-t}} \left( \frac{S_A(t)}{S_A(t)} n(d_1) - \frac{S_B(t)}{S_B(t)} n(d_2) \right)
\]
\[
= A_N(d_1) + \frac{1}{\sigma \sqrt{T-t}} \left( n(d_1) - \frac{S_B(t)}{S_A(t)} n(d_2) \right)
\]
where \( n(x) = (d/ dx)N(x) = e^{-x^2/2}\sqrt{2\pi}, \)

Using almost similar calculations we obtain that
\[
b_S(t) = A_N(d_2) + \frac{1}{\sigma \sqrt{T-t}} \left( -\frac{S_A(t)}{S_B(t)} n(d_1) + n(d_2) \right).
\]

Using this we finally obtain
\[
b_A(t) = \frac{1}{B(t)} (\Pi(t, S_A(t), S_B(t)) - b_S(t)S_A(t) - b_S(t)S_B(t))
\]
\[
= \frac{1}{B(t)} (\Pi(t, S_A(t), S_B(t)) - S_A(t)N(d_1) - S_B(t)N(d_2))
\]
\[
- \frac{1}{\sigma \sqrt{T-t}} (S_A(t)n(d_1) - S_B(t)n(d_2) - S_A(t)n(d_1) + S_B(t)n(d_2))
\]
\[
= \frac{1}{B(t)} (\Pi(t, S_A(t), S_B(t)) - \Pi(t, S_A(t), S_B(t))) = 0;
\]

\[
b_S(t) = N(d_1) + \frac{1}{\sigma \sqrt{T-t}} \left( n(d_1) - \frac{S_B(t)}{S_A(t)} n(d_2) \right),
\]
\[
b_S(t) = N(d_2) + \frac{1}{\sigma \sqrt{T-t}} \left( -\frac{S_A(t)}{S_B(t)} n(d_1) + n(d_2) \right).
\]