Answers to some of the exercises for week 4

Let throughout the answers the function $d_1$ and $d_2$ be defined as

\[ d_1(K, s, t) = \frac{1}{\sigma \sqrt{T - t}} \left( \ln \left( \frac{s}{K} \right) - \left( r + \frac{\sigma^2}{2} \right) (T - t) \right) \]
\[ d_2(K, s, t) = d_1(K, s, t) - \sigma \sqrt{T - t}, \]

and let $N$ denote the distribution function of the standard Normal distribution. Moreover let $h_B$, $h_S$ and $h_{C_X}$ denote portfolio weights for the bank account, stock and a European call option with strike $X$ and maturity $T$ respectively.

B 8.3
\[ \Pi(t, \chi) = \frac{S_t 1 - e^{-r(T_2 - T_1)}}{T_2 - T_1} \]

B 9.2
\[ \Pi(t, \chi) = S_t (2N(d_1(K, S_t, t)) - 1) - Ke^{-r(T-t)}(2N(d_2(K, S_t, t)) - 1) \]
\[ h_B = Ke^{-rT}, \ h_S = -1, \ h_{C_K} = 2 \]

B 9.3
\[ \Pi(t, \chi) = S_t (N(d_1(A, S_t, t)) - N(d_1(B, S_t, t)) - Ae^{-r(T-t)}(N(d_2(A, S_t, t)) - 1) + Be^{-r(T-t)}N(d_2(B, S_t, t)) \]
\[ h_B = Ae^{-rT}, \ h_{CA} = 1, \ h_{CB} = -1 \]

B 9.4
\[ \Pi(t, \chi) = S_t (N(d_1(A, S_t, t)) - 2N(d_1(B, S_t, t) + N(d_1(C, S_t, t)) - e^{-r(T-t)} \left( AN(d_2(A, S_t, t)) - 2BN(d_2(B, S_t, t)) + CN(d_2(C, S_t, t)) \right) \]
\[ h_{CA} = 1, \ h_{CB} = -2, \ h_{CC} = 1 \]