All solutions should be well motivated. The total amount of points is 90. Grades are assigned as follows. **FMS180**: 3 (40p), 4 (57p), 5 (73p). **MAS204**: G (40p), VG (65p).

Use the department’s papers only. Begin the solution of each assignment on top of a new paper. Do not use red pencil. Write your full name on each paper.

You are allowed to use a pocket calculator, the Markov processes table of formulas, a table of formulas for a basic course in mathematical statistics, and a reasonable table of mathematical formulas (e.g. TEFYMA).

1. Consider a service system to which two streams of customers arrive: one Poisson process with intensity $2 \text{s}^{-1}$ in which each event corresponds to the arrival of one customer, and one Poisson process with intensity $1 \text{s}^{-1}$ in which each event corresponds to the arrival of two customers.

The two Poisson processes are independent of each other.

Compute the probability that during 1 s, exactly four customers arrive. (10p)

2. Five nodes are connected in a ring:

![Ring diagram]

Two particles are placed at two adjacent nodes, and then move according to two independent random walks on this graph; at each step each particle moves either clockwise or counterclockwise with equal probabilities, and independently of the other particle.

Compute the average time it takes before the particles meet. (10p)

3. A building has five floors, including the ground floor, and one elevator. When the elevator is at the ground floor its next stop is any of the other floors, with equal probabilities. When the elevator is not at the ground floor its next stop is the ground floor with probability 0.7, and any of the other floors—except the current one—with equal probabilities.

(a) Compute the probability that if the elevator currently is at the ground floor, it will be there also three stops later. (5p)

(b) Compute the probability that after a long time, the elevator is at the ground floor. (5p)

4. Consider a queueing system to which customers arrive as a Poisson process with intensity $\lambda$. The system has two servers, A and B. Each customer needs service from one of the servers, but which one is unimportant. Service times are exponentially distributed, with means $1/\mu_A$ and $1/\mu_B$ for servers A and B respectively. Here $1/\mu_A < 1/\mu_B$, so server A is the faster one. If an arriving customer finds both servers empty, he/she will therefore choose server A. If an arriving customer finds both servers busy, he/she will join a queue, waiting for service. This queue is common to both servers, and the number of waiting spaces is infinite. When a server becomes available and there are customers in the queue, the customer first in line occupies the available server. Customers do no interrupt their service to change server. Service times are independent of each other and of the arrival process.

Please turn over!
(a) Draw a model graph of a Markov process that describes the system. Indicate all transition intensities. (10p)

(b) Assuming that the system admits a stationary distribution, compute the stationary probability that server A is busy. (10p)

5. Anne and Birger play a ball game. If the one who serves wins the ball, he/she gets one point. If the server loses the ball, nobody gets a point and the other player serves the next ball. A ball is won by the server with probability 2/3. The outcome of different balls, given the servers, are independent.

Anne starts to serve. Compute the average number of points she gets before Birger gets his first one. (20p)

6. Consider a Markov process with two states 0 and 1. The transition intensity from 0 to 1 is denoted by \( \gamma \), and \( \delta \) is the intensity of the reverse transition. When this process is in state 1, events (which you may think of as arrivals if you wish) occur according to an independent Poisson process with rate \( \lambda \). When the Markov process is in state 0, no Poisson events occur.

The Markov process can thus be said to interrupt the Poisson process (when it is in state 0), and the sequence of events is often called an interrupted Poisson process.

(a) The interrupted Poisson process is a renewal process. Motivate this. (5p)

(b) Compute the intensity of this renewal process. (5p)

(c) Compute the inter-event time distribution of this renewal process. (10p)

Lycka till!