VALUATION OF DERIVATIVE ASSETS, FMS170/MASM19
HOME ASSIGNMENT 1 VT-09

The solutions should be handed in within a week. They shall be well written and properly motivated, so that no misunderstandings can occur. You may work together but *everyone should hand in their own solution*.
To pass you need correct solution to *all* exercises, but you may correct wrong solutions afterwards.

A: Pure Arbitrage relations for European options. Here we will consider some classical pure arbitrage relations for European options. In the following exercises let $\varphi_c(t)$ be the price at time t of a European Call option with strike price K and time to maturity T, $\varphi_p(t)$ the price of the corresponding European Put with the same strike price and date and $\varphi_f(t)$ the price of the corresponding Forward (payer's position forward) with the same strike price and date. Note that we assume throughout this home assignment that the stock is not paying dividends.

1. **Arbitrage bounds for the Put option.** We can not price the European Put with a pure Arbitrage, i.e. without a model for the underlying. However, the price (φ_p) of a put option must lie between the following bounds:

$$\max\{0, -\varphi_f(t)\} \leqslant \varphi_p(t) \leqslant S_t - \varphi_f(t)$$

   Show this with a pure arbitrage argument.

2. **Put/Call parity.** Use a pure arbitrage argument to show that the following relation must hold between the prices of the European Put (φ_p), Call (φ_c) option and Forward (φ_f) with the same strike price (K) and date (T).

$$\varphi_c(t) = \varphi_p(t) + \varphi_f(t)$$

B: The Binomial Model. Consider a three period (one year) binomial model for the stock price S_t. The volatility of the stock is $\sigma = 0.4$, the interest rate is 3% per year and $S_0 = 100$.

1. Compute the arbitrage free price at $t = 0$ of a European Call option on the stock S_t with strike price $K = 110$ and time to maturity one year.
2. Do the same thing for a European Put.
3. What is the price of the American options with the same data.
4. What is the replicating portfolio of the European Call at $t = 0$.
5. Assume the stock goes up the first period. Do you have to rebalance the hedge? If yes, what does the new portfolio look like?