Computer Exercise 1
Discrete time models

This computer exercise deals with discrete time models. They are much simpler from a probabilistic point of view than continuous time models but are also necessary for some types of contracts. The number of derivatives that can be priced in a continuous time setting and where the price is given by a closed form expression is limited.

It is preferred if you use Matlab but other languages or program packages are also permitted. However, if some other program or language is used, careful documentation of the code is essential.

1 Preparations before the computer exercise

Read chapter 1 and 2 and section 7.8 (pp. 106–108) in Björk (2004), chapter 2 in Rasmus (2009) and this guide for the computer exercise. You should also solve the exercises B 2.1 - B 2.3 and R 2.1 - R 2.3.

Before the computer exercise starts, the questions below (or a subset of them) will be posed. All of the posed questions must be correctly answered in order for the computer exercise to be approved.

2 Catalogue of Questions

You should be able to answer these questions before the computer exercise.

1. Explain why the put/call paritiy does not apply for American options.
2. What distribution does the binomial model converge to (Hint: look at at the log of the the stockprice.)? Explain why. Is this a good model for stock prices? Discuss the implications.
3. Does it matter if we use objective or riskneutral probabilities?

3 Computer exercise tasks

The computer exercise consists of three assignments. The first assignment is pricing of an European call option, followed by pricing an American put option. Finally, you study the convergence properties of binomial trees.

```
Hint:  Try to write the code as general as
possible since the assignments are similar.
```

3.1 European call option

Consider a European call option, written on a stock, with a strike price 90 which matures in one year. Assume the continuously compounded riskfree interest rate is 3.5%, the current price of the stock is 100 and it's volatility $\sigma = 0.4$.
Assignment: Calculate the price of this option using three periods. Is the answer correct? Compare to the exercises.

3.2 American put option

You already know that American call options are easy to price (why). To price put options, we need more advanced numerical methods. Assume that the parameters are the same as in the previous assignment.
Assignment: Calculate the price of this option using one period, three periods and 10 periods. Comment the result.

3.3 Convergence of European call option

It is obvious that several periods are needed if the algoritm is going to converge. How many? Well, by using the general formula

$$\frac{S(t)}{B(t)} = E^Q\left[\frac{S(T)}{B(T)} \mid \mathcal{F}_t\right],$$

we can calculate the price of a European call option without constructing a tree. This is done by rewriting the tree into a general probabilistic model and calculating the expected value for that model.

Assignment: Calculate the price of a European call option using one period, two periods, three periods, four periods ... up to at least 100 periods. It is important that you calculate all of the prices. Plot the price as a function of the periods and compare the price from the binomial model with the price given by the Black & Scholes formula, see equation (7.48) in Björk (2004) ((6.46) in [Björk, 1998]).

4 Feedback

Comments on the computer exercise (from all kinds of point of view) are welcome. Please send them to Magnus Wiktorsson, `magnusw@maths.lth.se` or phone 046–222 86 25.

5 MATLAB–routines

General MATLAB–routines

`binopdf`	Probability function for the binomial model
`normcdf`	Cumulative distribution function for the normal distribution

References

Björk, T. (2004). *Arbitrage Theory in Continuous Time*, Oxford University Press, New York

Rasmus, S. (2009). *Derivative pricing* Matematikcentrum, Lunds Universitet