Extreme Value Statistics and Wind Storm Losses: A Case Study

HOLGER ROOTZÉN and NADER TAJVIDI


Statistical extreme value theory provides a flexible and theoretically well motivated approach to the study of large losses in insurance. We give a brief review of the modern version of this theory and a “step by step” example of how to use it in large claims insurance. The discussion is based on a detailed investigation of a wind storm insurance problem. New results include a simulation study of estimators in the peaks over thresholds method with Generalised Pareto excesses, a discussion of Pareto and lognormal modelling and of methods to detect trends. Further results concern the use of meteorological information in the wind storm insurance and, of course, the results of the study of the wind storm claims. Key words: Wind storm claims, large claims insurance, extreme value statistics, generalised Pareto distribution.

1. INTRODUCTION

Developments in statistical extreme value theory during the last decade provide a flexible and theoretically well motivated approach to the study of extremes caused by random events. The aim of the present paper is (i) to provide a review of modern statistical extreme value theory (ii) to give a detailed “step by step” example of how it can be used in large claims insurance and (iii) to make a case study of a concrete windstorm insurance problem. A further aim is (iv) to point out needs for research. While most of the methods and theoretical results in this paper are from published papers or from the statistical folklore, a few are new. In particular this is true of the results of Section 3 and the discussion in Section 4. In addition we believe that the concrete results of the analysis of the windstorm loss data are of independent interest.


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Windstorm insurance is a notoriously difficult problem for insurance companies. The problem is illustrated in Fig. 1 which shows the relative sizes of the accumulated loss in the most severe storms encountered by the Swedish insurance group Länsförsäkringar in a 12-year period. It can be seen that the most costly storm contributes about 25% of the total amount for the period, that it is 2.7 times bigger than the second worst storm, and that four storms together make up about half of the claims (cf Aebi et al., 1992). Further, the second and third largest storms occurred in 1983 and 1984, and then there is a big gap until 1993, when the biggest one came. An illustration of the size of the problem worldwide is given by the hurricane Andrew in August 1992, with an insured damage of about $16 \times 10^9$ according to Catastrophe Reinsurance Newsletter (1993, No. 8).

The problem for the insurance companies then is to predict the size of the next very severe storm—e.g. will it again be 2.7 times bigger than the worst one so far? It is also to keep up vigilance in building up enough reserves, by charging sufficient rates, and in buying enough reinsurance during the many years that may (or may not) pass until the big one strikes.

The traditional approach in the insurance industry is to think in terms of the worst possible event. Companies then have tried to put an upper limit to aggregate exposure in any one event by accumulation control by zone, summing up the sums insured, and by not underwriting beyond this limit. However, this may be hard to achieve in e.g. windstorm insurance where one typically has only partial losses. Today's development in assessing storm exposure in the large insurance and reinsurance companies seems to be to build computer simulation models connecting loss ratios to wind velocities, with the connecting function estimated from a few historical events (cf Geographic Analysis Project, Greig Fester, 1995). Worst case scenarios are then obtained by altering the path and windspeeds of some severe historical storm to a hypothetical worst case, and then using the connecting

Fig. 1. Windstorm losses 1982–1993.
function to compute the loss caused by this hypothetical storm. A problem with this "engineering" approach, as it is often used, is that it doesn't specify the uncertainty, neither in the choice of the worst case storm, nor in the connecting function. We discuss this further in Section 7 below.

Wilkinson (1982) and Kremer (1990, 1994) attempt a formalisation in statistical terms of the idea of a worst possible event through concepts of Probable Maximum Loss (PML). In Kremer's papers, the quantile PML for the risk level \( r \) and a specified time period is simply the \( r \)-th quantile of the distribution of the maximum loss over the time period. In the present paper we will use this PML concept, even though it is some deviation from common usage in insurance; e.g. in wind storm insurance PML often refers to aggregate sum insured by zone.

In reality the distribution of this maximum loss isn't known, but has to be estimated from loss data. Since extreme events by definition are rare, there is only little data, and this inevitably involves an added large statistical uncertainty.

Further, since it is a question of predicting worse events than those already encountered, suitable model assumptions have to be made. Statistical extreme value theory provides a possible set of such model assumptions. Below we apply this theory to the wind storm loss experience of the Swedish insurance group Länsförsäkringar during the period 1982 to 1993. This database contains the individual amounts of all claims, the place and time of the claims, and the type of the claim, with farm insurance comprising of approximately 65\% of the total amount. Claims due to boat insurance are not included. The claims were collected into 46 storm events, with a total claimed amount of 510 million Swedish crowns (MSEK). The storm events were simply defined by calculating a moving sum of length three days, and choosing those sums which exceeded MSEK 0.9. All values were corrected for inflation, but since the portfolio was relatively stable, we made no adjustments for portfolio changes.

A basic tenant of this paper is that it is unrealistic to hope to be able to summarise all information and uncertainty about future large losses in a single number giving "the size of the largest loss". Instead we describe uncertainty by presenting several different quantiles of the distribution of the maximum loss over several different periods of time. Thus, equivalently, we present the quantile PML for a number of different risk levels and time periods. We also compute the distribution of the excess loss, i.e., the conditional distribution of the loss given that it exceeds the upper reinsurance limit, as a further support to the assessment of the need for reinsurance. Finally, these measures are complemented by a discussion of the statistical uncertainty.

Section 2 contains a review of the extreme value theory approach to modelling of extreme losses, and the result of fitting the basic peaks over thresholds model to the windstorm loss data. The statistical properties of the quantile PML estimators are studied in Section 3 in a small simulation experiment. In Section 4 we discuss lognormal and Pareto modelling. The number of claims and the sizes of the individual claims together determine the total amount of loss caused by a windstorm. In Section 5 these are studied, in particular with the aim of detecting possible
trends. Reliable and representative claim databases often cover substantially shorter time periods than the meteorological records and it is natural to try to use meteorological data to improve the predictions. Section 6 contains an investigation of this possibility for the Länsförsäkringar loss data. Finally, Section 7 contains a discussion, our personal views on some issues encountered in the paper, and a list of areas for further research.

2. EXTREME VALUE MODELLING OF WINDSTORM CLAIMS

Fig. 2 illustrates the basic extreme value model for large values. In the figure the windstorm losses which exceed a suitable high level, \( u \), are plotted. The model is that losses which are larger than this level come as a Poisson process with intensity \( \lambda \), and that excesses, i.e. the amounts by which the losses exceed the level, follow a Generalised Pareto (GP) distribution, with distribution function (d.f.)

\[
H(x) = 1 - \left(1 + \frac{x}{\sigma} \right)^{-1/\gamma}_.
\]

Here \( \sigma > 0 \) is a scale parameter and \( \gamma \) is a shape parameter. The “\( + \)” signifies “positive part”, so that for \( \gamma \) negative, \( H(x) = 1 \) for \( x \geq -\sigma/\gamma \), i.e. the distribution has the finite (positive) right endpoint \( -\sigma/\gamma \). For \( \gamma = 0 \) the expression is interpreted as the limit as \( \gamma \to 0 \), i.e. as the exponential distribution

\[
H(x) = 1 - \exp\left\{-x/\sigma\right\}.
\]

![Graph showing windstorm losses exceeding level \( u = 0.9 \) MSEK, for 1982-1993.](image)

Fig. 2. Windstorm losses which exceed the level \( u = 0.9 \) MSEK, for 1982–1993.
This model is often called the "peaks over thresholds model with GP excesses" or, in hydrology "the partial duration series model with GP excesses". In the sequel we will for brevity omit "with GP excesses", and just write "the peaks over thresholds model".

For large losses, $\gamma$ is always positive, and often between .5 and 1. E.g. if the peaks over thresholds model is fitted to the data from Fig. 2 by Maximum Likelihood, the parameter estimates are $\hat{\gamma} = .70$, $\hat{\delta} = 3.87$ MSEK and $\hat{\lambda} = 3.8$ storms per year. The quality of the fit is illustrated in Fig. 3 by a QQ-plot, where the empirical quantiles of the losses are plotted against the quantiles of the fitted GP distribution. A dispersion test for the Poisson distribution of the number of storms with losses exceeding 0.9 MSEK per year gave the (approximate) p-value 0.82.

For $\gamma$ positive, the GP distribution is closely related to the ordinary Pareto distribution. This is discussed in Section 4 which also contains a comment on the lognormal distribution. However, the full range of the GP model is of interest also in insurance. E.g. the extreme windspeeds which cause the damage are often well modelled by the exponential distribution, i.e. by $\gamma = 0$ or even by $\gamma$ slightly negative.

This model has a number of useful properties which we will discuss in some detail in the rest of this section. The first two characterise the GP distributions, i.e., no other class of distributions have these properties, and together with 6.) and 7.) they provide the basic motivation for the peaks over thresholds model. The remaining

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![Fig. 3. QQ-plot of the windstorm loss data against a GP distribution with $\hat{\gamma} = 0.70$ and $\hat{\delta} = 3.87$ MSEK.](image-url)
items discuss a number of useful properties of the model, including easy formulas for the d.f. and quantiles of the distribution of the largest loss.

1.) The peaks over thresholds model is stable under an increase of the level: If the excesses of a level \( u \) come as a Poisson process and the sizes of the excesses are GP-distributed and independent, then the excesses of a higher level \( u + v \) (for \( v > 0 \)) have the same properties. In addition, the parameters for the higher level are easily obtained from the ones for the lower level.

This can be seen as follows. Suppose the intensity of the Poisson process of excesses of the level \( u \) is \( \lambda \), and that the GP parameters are \( \gamma \) and \( \sigma \). The point process of excess times for the higher level is obtained from the point process of the lower level by deleting those points for which the excesses are lower than \( v \). This happens independently, and with probability \( 1 - (1 + \gamma v / \sigma)^{-1/y} \). By a standard result on thinning of Poisson processes it follows that the point process for the level \( u + v \) also is a Poisson process, with intensity \( \lambda (1 + \gamma v / \sigma)^{-1/y} \).

Further, the distribution of an excess of \( u + v \) is just the conditional distribution of an excess of \( u \) given that it is larger than \( v \), renormalised by subtracting \( v \), i.e. it has the d.f.

\[
1 - \frac{(1 + \gamma \frac{v + x}{\sigma})^{-1/y}}{(1 + \gamma \frac{v}{\sigma})^{-1/y}} = 1 - \left( 1 + \gamma \frac{x}{(\sigma + v\gamma)} \right)^{-1/y}.
\]

(2.2)

Clearly this is a GP distribution, with the same shape parameter \( \gamma \) and with scale parameter \( \sigma + v\gamma \).

2.) The peaks over thresholds model is motivated by extreme value theory: The basic result of statistical extreme value theory is that if the distribution of the maximum \( M_n = \max \{ \xi_1, \ldots, \xi_n \} \) of i.i.d variables converges under a linear normalisation, i.e. if

\[
P((M_n - u_n) / \sigma_n \leq x) \to G(x), \quad n \to \infty
\]

(2.3)

for some constants \( u_n \) and \( \sigma_n > 0 \), then \( G(x) \) has to be an extreme value (EV) distribution, i.e. it has the form

\[
G(x) = \exp \left\{ - \left( 1 + \gamma \frac{x - \mu}{\sigma} \right)^{-1/y} \right\},
\]

(2.4)

for some location parameter \( \mu \) and scale parameter \( \sigma \).

Now, it can be seen that this holds if and only if the peaks over thresholds model applies, see Pickands (1975) and Tajvidi (1995). Specifically, if (2.3) and (2.4) hold, then the distribution of the excesses of a high level converge to a limiting GP form, i.e. there exist constants \( \sigma(u); u \geq 0 \) such that

\[
P((\xi_1 - u) / \sigma(u) \leq x | \xi_1 > u) \to 1 - \left( 1 + \gamma \frac{x}{\sigma} \right)^{-1/y}, \quad u \to \infty,
\]

(2.5)

for \( x \geq 0 \) and some \( \sigma > 0 \) and \( \gamma \). Further, by the Poisson process limit for a
binomial process, the (normalised) times of excesses of a high level \( u \) asymptotically occur as a Poisson process. Thus, asymptotically the peaks over thresholds model holds.

It may be remarked that the \( \gamma \)'s in (2.4) and (2.5) are the same. However, if \( u_n, e_n \) in (2.3) are replaced by \( u', \sigma' \) with \( \sigma_n \to a > 0 \) and \( (u_n - u)_{\sigma_n} \to b \), then (2.3) still holds, but with \( \sigma \) replaced by \( a \sigma \) and \( \mu \) replaced by \( b + \mu \) in (2.4). Hence there is an arbitrariness in the choice of parameters in the limiting distribution. Similarly, if \( \sigma(u) \) in (2.5) is replaced by \( \sigma'(u) \) with \( \sigma(u)/\sigma'(u) \to a > 0 \) then (2.5) still holds, but with \( \sigma \) replaced by \( a \sigma \). Hence there is the same arbitrariness in (2.5), and there is no immediate connection between the \( \sigma \)-s in (2.4) and (2.5) (although the connection can be worked out). However, in statistical analysis this arbitrariness doesn't come into play; e.g. (2.5) means that the d.f. of the excess \( \xi - u \) (approximately, for large \( u \)) is \( 1 - (1 + \gamma(x/\sigma\sigma(u))^+)^{1/\gamma} \), and hence the \( \sigma \) to be estimated in (2.1) corresponds to \( \sigma\sigma(u) \) in the asymptotic theory.

3.) The maximum of the observations in the peaks over thresholds model has an EV distribution. This of course is a consequence of the fact that (2.3) and (2.4) are equivalent to (2.5) and the Poisson process limit of the times of excesses. However, it can also be seen directly. Let \( M_T \) be the largest of the observations during a time period of length \( T \). Thus, either \( M_T \leq u \), or there is at least one excess of \( u \), and then \( M_T \) equals \( u + \) the largest excess of \( u \) in \([0, T]\). As noted in 1.), the excesses of the level \( u + v \) for \( v > 0 \) occur as a Poisson process with intensity \( \lambda(1 + \gamma v/\sigma)^+ \).

Since the maximum is smaller than \( u + v \) if and only if this Poisson process has no points in the interval \([0, T]\), it follows that

\[
P(M_T \leq u + v) = \exp \left\{ -\lambda T \left( 1 + \frac{v}{\sigma} \right)^{1/\gamma} \right\}
= \exp \left\{ - \left( 1 + \frac{v}{\sigma} \frac{\left(1 - 1/\gamma \right) \sigma u/v}{\lambda T} \right)^{1/\gamma} \right\}.
\]

(2.6)

4.) By equating (2.6) to \( 1 - p \) and solving for \( v \), the \( p \)-th upper quantile, \( x_{T,p} \), of the distribution of \( M_T \) is obtained as

\[
x_{T,p} = u + \frac{\sigma}{\gamma} \left( \frac{(\lambda T)^\gamma}{-(\log(1 - p)\gamma)} - 1 \right).
\]

(2.7)

In the language used in the introduction, \( x_{T,p} \) is the quantile PML for the risk level \( p \) and a time period of length \( T \). Hence, by inserting the estimated parameters for the peaks over thresholds model into (2.7), it is possible to estimate quantile PML's. Some results of this procedure for the windstorm loss data are given in Table 1. The statistical uncertainty of the estimates is discussed in Section 3 below.

5.) Equation (2.6) directly gives an estimate of the probability that a company experiences a claim which exceeds a given reinsurance limit. This is complemented by (2.2) which gives the distribution of the excess loss. This is illustrated in Fig. 4 for the windstorm loss data, and for the reinsurance limit 850 MSEK.

It may be noted that according to the present model, the probability of exceeding the reinsurance limit may be made arbitrarily small by buying enough reinsurance,
Table 1. *Estimated quantile PML-s in MSEK for various risk levels and time periods for the windstorm portfolio*

<table>
<thead>
<tr>
<th>Risk</th>
<th>Next year</th>
<th>Next 5 years</th>
<th>Next 15 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>66</td>
<td>215</td>
<td>473</td>
</tr>
<tr>
<td>1%</td>
<td>366</td>
<td>1149</td>
<td>2497</td>
</tr>
</tbody>
</table>

but that an excess, if it nevertheless occurs, is predicted to be quite sizeable. One way of illustrating this is by noting that by similar reasoning as for (2.7) the median $m(u + v)$ of an excess of the level $u + v$ is by (2.2) given by the formula

$$m(u + v) = \frac{\sigma + v\gamma}{\gamma} (2^\gamma - 1)$$

$$= \frac{\sigma}{\gamma} (2^\gamma - 1) + v(2^\gamma - 1).$$

(2.8)

For the windstorm losses, expressed in MSEK this function is estimated to be

$$m(u + v) = 3.48 + 0.63v.$$  

(2.9)

Fig. 4. Estimated conditional probability that the excess loss over 850 MSEK is larger than $x$ MSEK, given that the loss is at least 850 MSEK, for the windstorm loss data. The probability that the loss in one year exceeds 850 MSEK is estimated to be 0.077%.
Thus, perhaps contrary to expectation, the median increases with the limit $u$, and is roughly of the same size as $u$.

There is a second interpretation of (2.8). This is that it gives the median of the distribution of the size of the next claim which is large enough to exceed the largest claim so far encountered. In the wind storm data set the largest claim so far is 134 MSEK, and hence the median of the next loss which exceeds this is estimated to be $134 + 3.48 + 0.63 \times (134 - 0.9) = 222$ MSEK.

A third use of (2.8) is as a diagnostic tool to complement the QQ-plot in Fig. 3. For this, the empirical medians of the excesses of $u$ are plotted against $u$, for $u$ in a suitable range. If the peaks over thresholds model holds, this median excess plot should be roughly linear. In addition it is possible to use the plot and (2.8) to obtain preliminary estimates of the parameters. This is shown for the windstorm loss data in Fig. 5. The intercept and slope of the plot from a least squares fit are 1.80 and .91 which gives the preliminary estimates $\hat{\gamma} = .93$ and $\hat{\sigma} = 1.84$. It may be noted that a mean excess plot often is used instead of a median excess plot. However, mean excess plots are very variable in the present heavytailed case and the median excess plot may be preferable.

6) The GP distributions are quite flexible, and the model allows for quite a variety of tail behaviour. Thus, for $\gamma > 0$ the tail of the distribution asymptotically decreases polynomially as a power $-1/\gamma$, for $\gamma = 0$ the tails are exponential, and for $\gamma < 0$ there is a polynomial decrease at a finite endpoint.

*Fig. 5. Median excess plot for the windstorm loss data. $k$ is the number of excesses of $u$. 
7.) The peaks over thresholds model has the potential to become the basic standard which is tried first, and used unless it becomes clear that more sophisticated modelling is needed. One advantage of this would be to give reinsurers and cedents a common ground for dealing with large losses. No other model has the attractive properties discussed above, and we believe that if there is to be a "standard", the only possibility is this model.

8.) There is by now a rather large literature on the peaks over thresholds model and on statistics of extremes in general. A useful reference, aimed directly at insurance is the forthcoming book by Embrechts et al. (1995). Other relevant references are Falk et al. (1994), Gumbel (1958), Leadbetter et al. (1983), Resnick (1987) and Smith (1989). In addition program packages for extreme value analysis, (Castillo et al., 1993 and Haßmann et al., 1993) are available.

One obvious conclusion of this literature is that the peaks over thresholds model is robust against nonstationarities in the Poisson process of excesses—the only thing that has to be changed is to replace $\lambda T$ in the formulas by the integrated intensity $\Lambda(T)$. In fact, in Scandinavia severe wind storms rarely occur during the summer, and hence for the wind storm data, the intensity of excesses is not constant over the year, although it may with good approximation be considered to be constant over the non summer months, cf. Abild (1990). However, this doesn’t affect the conclusions of this paper. Further, dependence of the data can be handled, either by so called declustering, see e.g. Smith (1989), or directly, as in Rootzén et al. (1992). However, the model is not robust against departures from the assumption that the tail of the distribution is approximated by a GP distribution. If there are marked deviations from a GP tail, the results will be misleading. Of course, any reasonable model for values which are larger than what so far has been observed will have this kind of sensitivity.

An important practical problem is the choice of the level $u$ for the excesses. There are theoretical suggestions on how to do it, based on compromises between bias and variance—a higher level can be expected to give less bias, but instead gives fewer excesses, and hence a higher variance. However, we feel that so far these suggestions don’t quite solve the practical problem, and that the choice of level has to be made from subject matter knowledge, from looking at QQ, median excess and other plots, and on experiments with different levels. If the model produces very different results for different choices of $u$, the results obviously should be viewed with more caution.

3. ESTIMATION IN THE PEAKS OVER THRESHOLDS MODEL

In this section we discuss estimation of the parameters and quantiles in the peaks over thresholds model. The discussion is based on a small simulation study where we only considered the parameter values $n = 46$, $\gamma = .4$, .6, .8, 1.0 and $\lambda = 3.8$, which were the values of immediate interest for the windstorm data. The reason for this is that modern computing technology makes simulation straightforward, and that we hence think it is feasible to include a small simulation study with exactly the
parameter values of interest into routine applications of the peaks over thresholds method. Further, $\sigma$ is a scale parameter, and hence it is enough to have $\sigma = 1$ in the simulations.

The main competitors for estimation of the parameters in the peaks over thresholds model are the Method of Moments (MoM), Probability Weighted Moments (PWM), Maximum Likelihood (ML), and a number of ad hoc estimators, see e.g. Pickands (1975) and de Haan et al. (1992). From a large sample standpoint, ML-estimators are known to be optimal in the regular case, i.e. for $\gamma > -.5$. Nevertheless, Hosking et al. (1987) argue that PWM or MoM for some ranges of parameter values have better small sample properties. Since MoM estimators use the second sample moment, they are known to perform less well when second moments do not exist, i.e. for $\gamma \geq .5$. The simulations also confirmed this and we do not discuss MoM estimates further below. The asymptotic theory for the PWM estimators, given in Hosking et al. (1987), assumes the existence of second moments. However, the derivation of the estimators only requires $\gamma \leq 1$. We have not investigated any of the ad hoc estimators.

In the simulation study we, for each value of $\gamma$, generated 500 samples with $n = 46$ and for each sample computed the PWM and ML estimators of $\gamma$ and $\sigma$. Boxplots of the results are shown in Figs 6 and 7, and the bias of the estimators are given in Table 2. We have not included $\gamma = .4$, in the presentation of results since this case is covered by the paper by Hoskins & Wallis (1987) and since our results agreed with theirs. It can be seen that both estimation methods give somewhat skewed distributions for the estimators and that the PWM estimators of $\gamma$ have a substantial bias but instead are less variable than the ML.

Fig. 6. Boxplots of 500 replications of the estimators of $\gamma$ computed from 46 simulated values from a GP distribution with $\sigma = 1$. The $\gamma$-values are given in parentheses.
estimates. The numerical values show that the mean square errors are rather similar for the PWM- and ML-estimates. The ML-estimates of $\sigma$ were somewhat better than the PWM estimates.

The main interest in the present paper is not in the parameters themselves, but in the quantiles of the distributions of maxima. These may be estimated by replacing the parameters in (2.7) by their estimates.

Fig. 8 contains boxplots of the simulated 10% quantile estimators. The distributions are very skewed, and it is not meaningful to describe them by the bias and standard deviation. Instead, Table 3 contains the medians and quartiles of the estimators. In the simulations we throughout used 46 excesses, rather than a Poisson number of excesses. This is only expected to have a small influence on the results. It can be seen that the PWM-estimators are less variable. However, they also systematically underestimate the quantiles very severely, and hence we throughout use the maximum likelihood estimators.

Table 2. Bias of estimators of $\gamma$ and $\sigma$, based on 500 replications of 46 simulated values from the GP distribution

<table>
<thead>
<tr>
<th>Method</th>
<th>$\gamma = 0.6$</th>
<th>$\gamma = 0.8$</th>
<th>$\gamma = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma$</td>
<td>$\sigma$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>PWM</td>
<td>$-0.097$</td>
<td>$0.064$</td>
<td>$-0.19$</td>
</tr>
<tr>
<td>MLE</td>
<td>$-0.035$</td>
<td>$0.051$</td>
<td>$-0.059$</td>
</tr>
</tbody>
</table>
Table 3. Distribution of the estimators of the quantile \( x_{T,p} \) for \( T = 1 \) and \( p = 0.1 \), based on 500 replications of 46 simulated values from the GP distribution. \( Q_{25} \) and \( Q_{75} \) are quartiles of the distribution

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>Method</th>
<th>( x_{T,p} )</th>
<th>Median</th>
<th>( Q_{25} )</th>
<th>( Q_{75} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>MLE</td>
<td>12.6</td>
<td>11.5</td>
<td>8.7</td>
<td>16.5</td>
</tr>
<tr>
<td>0.6</td>
<td>PWM</td>
<td>12.6</td>
<td>10.5</td>
<td>8.2</td>
<td>13.4</td>
</tr>
<tr>
<td>0.8</td>
<td>MLE</td>
<td>20.8</td>
<td>18.2</td>
<td>12.6</td>
<td>28.3</td>
</tr>
<tr>
<td>0.8</td>
<td>PWM</td>
<td>20.8</td>
<td>15</td>
<td>10.9</td>
<td>19.9</td>
</tr>
<tr>
<td>1</td>
<td>MLE</td>
<td>35.1</td>
<td>30.1</td>
<td>19.8</td>
<td>50.6</td>
</tr>
<tr>
<td>1</td>
<td>PWM</td>
<td>35.1</td>
<td>20.3</td>
<td>15.5</td>
<td>27.9</td>
</tr>
</tbody>
</table>

Further, it is desirable to be able to construct onesided, upper bounded, confidence intervals for the quantiles. We simulated a number of standard methods for this, viz. the delta method, the bootstrap and the bias corrected and accelerated bootstrap (Efron and Tibshirani, 1993). However, for all these methods the actual coverage probabilities were much lower than the nominal ones, and we believe that more research is needed to find confidence intervals which can be used in practice.

Fig. 8. Boxplots of 500 replications of the estimators of \( x_{T,p} \) computed from 46 simulated values from the GP distribution for \( p = .1, \lambda = 3.8 \) and \( T = 1 \). The \( \gamma \)-values are given in parentheses, and the true quantiles are given in Table 3.
4. LOGNORMAL AND PARETO DISTRIBUTIONS FOR CLAIMS

Quite a number of different distributions have been proposed for large claims, cf. Hogg & Klugman (1984). With the terminology of Johnson and Kotz (1970, p 234), for \( \gamma > 0 \) the generalised Pareto distribution is a reparametrisation of the Pareto distribution of the second kind, and the distribution with d.f.

\[
G(x) = 1 - \left( \frac{k}{x} \right)^a, \quad a, k > 0, x \geq k
\]

is called a Pareto distribution of the first kind. In this section we comment on excess modelling by the Pareto distribution of the first kind, and by the lognormal distribution. Fitting a Pareto distribution of the first kind is quite standard in large claims insurance, cf Rytaard (1990) and Hesselager (1993). In fact, if the distribution of the excesses \( \xi - u \) has an (approximately) GP form

\[
H(x) = 1 - \left( 1 + \frac{x}{\sigma} \right)^{-1/\gamma},
\]

with \( \gamma > 0 \) then the tails of the \( \xi \)'s are asymptotically Pareto,

\[
P(\xi_1 > x) = P(\xi_1 > u)P(\xi_1 - u > x - u \mid \xi_1 > u)
\]

\[
= P(\xi_1 > u) \left( 1 + \frac{x - u}{\sigma} \right)^{-1/\gamma}
\]

\[
\sim \text{constant} \times x^{-1/\gamma}, \quad x \to \infty.
\]

(4.1)

Conversely, if the \( \xi \)'s are Pareto of the first kind, then for \( u \geq k \)

\[
P(\xi_1 - u > x \mid \xi_1 > u) = \frac{(u + x)^{-1/\gamma}}{(u)^{-1/\gamma}}
\]

\[
= \left( 1 + \frac{x}{u\gamma} \right)^{-1/\gamma}.
\]

(4.2)

Now it should be observed that while (4.1) is asymptotic, (4.2) is exact. Further, as soon as one considers excesses instead of the values themselves, one is brought from the Pareto distribution to the GP distribution, and the GP distribution has the additional advantage of not being dependent on the zero of the scale used. A slightly different facet of this is that the GP distribution has one parameter more, which gives extra flexibility. In the windstorm loss data, this flexibility seems useful: while the QQ-plot in Fig. 3 indicates good fit, a Pareto QQ-plot indicates some deviations from a Pareto distribution, see Fig. 9.

However, the main advantage of the peaks over thresholds model is that it gives a theoretical motivation for the procedures and easy and satisfying ways to find distributions and quantiles for maxima, and to check the model.

Sometimes one instead uses a lognormal distribution for the variables (or for the excesses). This often gives a good fit to the data, and in particular does so for the windstorm loss data. Let \( \Phi \) denote the distribution function of the standardised
normal distribution and let \( \phi(x) = e^{-x^2/2\sqrt{2\pi}} \) be its density function. The approximation \( 1 - \Phi(x) = \phi(x)((1/x) + O(1/x^3)) \) for \( x \to \infty \) gives that if \( \xi \) is lognormally distributed with parameters \( \mu \) and \( \sigma \), then

\[
P(\xi > x) = 1 - \Phi((\log x - \mu) / \sigma) \\
= \exp \left\{ -\frac{(\log x)^2}{2\sigma^2} + \frac{\mu \log x}{\sigma^2} - \frac{\mu^2}{2\sigma^2} \right\} \times \frac{\sigma}{\sqrt{2\pi(\log x - \mu) + O(\log x)^{-3}}} \\
\sim \text{constant} \times x^{-\log x/(2\sigma^2)} \exp^{\mu^2/\sigma^2}(\log x)^{-1}, \quad x \to \infty. \tag{4.3}
\]

This is a quite special tail behaviour, which is determined in a complex way by the parameters of the distribution. To the present authors, it doesn't seem likely that this is sufficiently flexible and direct to adapt well to the way real large claims occur. A further indication of this is that for observations from the lognormal distribution, the ratio of the maximum to the second largest value tends to one as the number of observations increases, as can be seen from (4.3) by straightforward computation. Again, this isn't the way windstorm losses seem to behave. E.g. for the Länsförsäkringar data, the largest claim is 2.7 times bigger than the second largest claim, and in the table over the largest insured catastrophes worldwide in Catastrophe Reinsurance Newsletter (1993, No. 8), the largest claim, due to hurricane Andrew, is 3 times larger than the second largest claim.
A somewhat unfair comparison is as follows. If the largest observation of 134 MSEK is deleted from the wind storm loss data, and the peaks over thresholds model is fitted to the remaining data, then the probability that the next value which is larger than the largest remaining loss (49 MSEK) exceeds 134 MSEK is estimated to be 0.17. If the same calculations are made for the lognormal model, the probability is estimated to be 0.05, which seems unreasonably low.

5. TRENDS
The current position is that the climate is unstable, and may be surprisingly rapidly changing, also without the added effects of human intervention. Further, it is often argued that the individual claim amounts increase with time, due to use of more vulnerable and expensive building methods, and to a larger propensity for building in exposed areas. In this section, we illustrate various methods of investigating if trends are present by applying them to the windstorm loss data.

The obvious way of checking for trends is visual inspection of Fig. 2. A simple quantification of the impression, which does not use the models developed above, may be obtained by fitting an ordinary regression line to the logarithms of the claim amounts. The resulting equation is \( \log(\text{loss}) = 15.1 + 0.0133 \times t \), with \( t \) the time in years from 1982. The standard error of 0.0133 is estimated to be 0.0127 with p-value equal to 0.3028 based on normality and constant variances. Of course, in our situation, \( \log(\text{loss}) \) is not normal, but approximately exponential. However, still neither the visual impression, nor the regression analysis present any evidence of a trend in the loss sizes.

It is also possible to use a model based approach, within the peaks over thresholds framework. For this we apply maximum likelihood to fit a GP distribution with constant shape parameter \( \gamma \) and with \( \sigma = \exp(\alpha + \beta t) \), with \( t \) denoting time in years. The motivation for the particular choice of model is that previous experience of similar situations, and the analysis below indicate that a constant \( \gamma \) should provide a reasonable fit, and that the model for \( \sigma \) corresponds to a constant growth in cost, by \( 100 \times (e^\beta - 1)\% \) per year. Thus, in particular, \( \beta = 0 \) corresponds to an absence of trend. A closely related model was proposed by Smith (1989). Using the quasi-Newton routine of NAG E04JAF we obtain the estimates \( \hat{\gamma} = 0.71 \), \( \hat{\alpha} = 14.75 \), \( \hat{\beta} = 0.017 \). A likelihood ratio test of the hypothesis that \( \beta = 0 \) gives the p-value 0.32 and hence gives the same result as the previous analysis. In addition we fitted a generalised linear model with a Poisson response distribution and with \( \log(\lambda) = \alpha + \beta \times \text{year} \) to the number of storms per year with losses over 0.09 MSEK. The p-value for test of \( \beta = 0 \) was 0.67, which did not indicate a trend in the frequency of large storm losses.

The final analysis in this section concerns the sizes of the individual claims caused by a storm. For this we have again used a peaks over thresholds method, and fitted a separate GP distribution to the individual claims which exceed .04 MSEK for those storms where there are at least 10 such claims. Fig. 10a) contains a plot of the estimated upper 10% quantile of the individual excesses against time. Again there is
little indication of a trend. Fig. 10b) shows the quantiles plotted against storm severity as measured by the number of claims. There does not appear to be any clear relation between the severity of the storms and the size of the quantiles, although the most severe storm out of the 10 storms in Table 4 also had the largest estimated upper 10% quantile.

Fig. 10. Plot of the estimated 10% quantile of the individual excesses in a storm, against a) the time of the storm b) the number of claims over MSEK 0.04, for storms where at least 10 individual claims exceed 0.04 MSEK.
Table 4. 

Table of GP estimates for storms which contain more than 50 claims greater than 0.04 MSEK

<table>
<thead>
<tr>
<th>Storm</th>
<th># of claims over 0.04 MSEK</th>
<th>( \hat{\theta} )</th>
<th>( \hat{\phi} )</th>
<th>estimated 10% quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 83</td>
<td>222</td>
<td>0.68</td>
<td>0.029</td>
<td>0.158</td>
</tr>
<tr>
<td>Dec 83</td>
<td>71</td>
<td>0.63</td>
<td>0.034</td>
<td>0.177</td>
</tr>
<tr>
<td>Jan 84</td>
<td>190</td>
<td>0.45</td>
<td>0.037</td>
<td>0.150</td>
</tr>
<tr>
<td>Dec 86</td>
<td>106</td>
<td>0.41</td>
<td>0.030</td>
<td>0.114</td>
</tr>
<tr>
<td>Dec 88</td>
<td>143</td>
<td>0.70</td>
<td>0.031</td>
<td>0.178</td>
</tr>
<tr>
<td>Jan 90</td>
<td>122</td>
<td>0.41</td>
<td>0.033</td>
<td>0.127</td>
</tr>
<tr>
<td>Feb 90</td>
<td>66</td>
<td>1.03</td>
<td>0.023</td>
<td>0.215</td>
</tr>
<tr>
<td>Feb 92</td>
<td>148</td>
<td>0.74</td>
<td>0.027</td>
<td>0.162</td>
</tr>
<tr>
<td>Jan 93, 13–17</td>
<td>632</td>
<td>0.67</td>
<td>0.043</td>
<td>0.237</td>
</tr>
<tr>
<td>Jan 93, 22–24</td>
<td>67</td>
<td>0.71</td>
<td>0.015</td>
<td>0.088</td>
</tr>
</tbody>
</table>

The estimates for the storms which contain at least 50 excesses of .04 MSEK are shown in Table 4 above. The values are rather stable, although the variation is somewhat larger than what can be attributed to estimation error only.

As a complement Fig. 11 contains the corresponding plots against time of the averages of the individual claims which are smaller than .04 MSEK, together with standard errors, and of the averages against storm severity as measured by the total number of claims. A weighted regression analysis, with weights proportional to the number of values in the individual averages, gives the regression line \( 0.007350 + 0.000058 \times \text{storm number} \) with the coefficient for the storm number significant at the 1% level, and hence the sizes of the small claims are increasing approximately \( 0.000058 \times 10 \times 46/12 \approx 0.0022 \) MSEK per decade faster than inflation. Also for the small claims there seems to be no relation between storm severity and average claim size.

### 6. USE OF METEOROLOGICAL INFORMATION

This section contains an investigation of the feasibility of using meteorological information to improve the precision for the risk estimates for the windstorm losses. To obtain as homogeneous data as possible, we in this section only consider claims from farm insurance. In addition we restrict attention to the province Skåne in the south of Sweden. This is an important farming area, and contains much open terrain, and in fact 43% of the total claims from farm insurance in the windstorm loss data came from Skåne.

The Swedish Meteorological and Hydrological Institute (SMHI) provided a database containing the 78 most severe windstorms in the south of Sweden during the timeperiod 1982–1993. For each storm the database contained measured 10 minute wind speed averages at 6 stations in or close to Skåne. In the analysis below, we have only used the squared maximum windspeed at each individual location in the storms and the duration of the storms. The selection of the storms was
Fig. 11. Plot of the mean of the individual claims which are smaller than .04 MSEK in a storm, against a) the time of the storm and b) the number of claims under MSEK 0.04. The dashed lines show \(1.96 \times \text{the standard error.}\)

performed in two steps. In the first step the 5 strongest storms at each station were selected. In the second step, only those storms were selected which belonged to the first selection for at least two of a larger set of 15–20 stations south of the 58-th latitude. In addition the measured values for 12 storms that didn't meet this criterion, but which had caused total losses in Sweden above .9 MSEK were
included. The reason for using the square of the windspeed is that this is proportional to pressure, which ought to be closely related to the damage caused. As an additional background we have had access to a report Geographic Analysis Project (1995) prepared by David Simmons at Greig Fester.

Fig. 12 contains plots of the logarithm of the losses against the square of the windspeed for the measuring stations, and Fig. 13 contains the corresponding plots with the squared windspeed replaced by the product of the squared windspeed and the duration of the storm. Storms with high windspeeds at all stations are not necessarily accompanied by any substantial damage, and storms with rather low windspeeds at all stations may still cause rather large losses. For an example of this see Table 5.

Fig. 12. Plot of the logarithm of the losses against the square of the maximal windspeed in the storm. The stations Falsterbo, Kullen and Sandhammaren are close to the coast.

Table 5. Table of storm loss in MSEK and squared maximal windspeed in \((m/s)^2\) at 6 stations in Skåne

<table>
<thead>
<tr>
<th>Storm</th>
<th>Loss</th>
<th>BAR</th>
<th>FAL</th>
<th>KUL</th>
<th>LJU</th>
<th>SAN</th>
<th>STU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aug 83</td>
<td>5.8</td>
<td>100</td>
<td>144</td>
<td>256</td>
<td>100</td>
<td>169</td>
<td>144</td>
</tr>
<tr>
<td>Jan 89</td>
<td>0.0005</td>
<td>121</td>
<td>196</td>
<td>529</td>
<td>144</td>
<td>225</td>
<td>169</td>
</tr>
</tbody>
</table>
Fig. 13. Plot of the logarithm of the losses against the product of square of the maximal windspeed and the duration of the storm. The stations Falsterbo, Kullen and Sandhammaren are close to the coast.

As expected, the windspeeds are considerably higher for the coastal location Kullen, and since the windspeeds from this location correlates poorly with loss, it is not used further.

We then made a multiple regression analysis of the logarithm of the losses against the squared windspeeds. Standard significance tests shows that the stations Barkåkra and Sandhammaren can also be dropped from the regression line. For remaining stations the estimated coefficients and their standard deviation are:

|            | Value | Std. Error | t value | Pr(>|t|) |
|------------|-------|------------|---------|---------|
| (Intercept)| 8.298 | 0.4960     | 16.5927 | 0.0000  |
| fal^2      | 0.0021| 0.0021     | 0.9940  | 0.3237  |
| lju^2      | 0.0116| 0.0027     | 4.3089  | 0.0001  |
| stu^2      | 0.0048| 0.0021     | 2.2653  | 0.0267  |

Fig. 14 illustrates the result. The residual standard deviation is 1.54. A corresponding analysis with the product of squared windspeeds and duration of storms gave slightly larger residuals.
7. DISCUSSION

In this section, we summarise and comment on the analysis of the windstorm loss data. We then argue that a computed "worst possible loss" should always have a risk level attached, and that in windstorm insurance it is useful (and feasible) to compute the entire distribution of the reclaimed amount in the reinsurance layers, and not just, say, the mean and standard deviation. Finally, we list some needs for further research.

The QQ-plot and the median excess plot (Figs 3 and 5), and the dispersion test for the Poisson process of excess times indicated that the peaks over thresholds model could be used for the windstorm loss data. The model produced estimates of quantile PML-s (Table 1), of the risk of exceeding a given reinsurance level, and of the conditional distribution of the excess loss (Fig. 4) which in our opinion are reasonable and agree with experiences from other portfolios. The windstorm loss data contained 46 storm events. If one has substantially less data, the estimation errors become very large, and one has to resort to credibility (or Bayesian) methods (see e.g. Hesselager, 1993 for a Bayesian approach to the Pareto model).

We found no trends in the aggregate losses, in the number of claims per storm, or in the number of storms per year (Section 5). We then looked at the individual claims within storm events, treating large claims ($\geq 0.04$ MSEK) and small claims separately. Both for large and small claims, the claim distribution seemed to be the
same for storms with many claims and storms with few claims. We didn’t discern any trend in the individual large claims. For the claims which were smaller than 0.04 MSEK there was an increase in the average claim amount of 0.0022 MSEK/decade in the inflation corrected loss amounts. However, the time period for the loss data is rather short and hence small, or even moderate, trends may be hidden by the large random variation in the data.

The multiple regression line of the logarithm of the aggregate claims in Skåne on the wind data from 6 meteorological stations fitted rather well. Still the residual standard deviation was 1.54 in the logarithm of losses. This corresponds to a factor 0.2 down and 4.7 up in losses in MSEK (Section 6). Further, there were some storms with rather high windspeeds at all stations which caused little damage, and storms with low windspeeds at all stations which lead to considerable losses. Hence, we believe that windspeed data cannot completely explain the sizes of losses. However, meteorological data may perhaps still be used to improve parameter estimates. The extent of the possible improvement remains to be determined.

Next, a basic property of our model for windstorm losses is that it predicts that very large losses may occur, albeit with low probability, and that the size of the loss contains a large unpredictable chance component. As argued above, this seems to agree with experience. In our opinion, it also agrees with the way large accumulated losses occur—e.g. moderate changes in the path of a storm, whether it rains or not, or whether a windstorm causes a large electrical shortage may have a dramatic influence on the loss. It should be remarked that even the largest loss considered in this paper (2497 MSEK in Table 1) only constitutes less than 0.3% of the total sum insured in the windstorm portfolio. An argument is sometimes put forward that for meteorological reasons windstorms in Sweden cannot be much worse than what has already been observed. However, meteorology is not able to set this down into hard numbers, and it doesn’t agree with loss experiences in Sweden and elsewhere. In conclusion we believe that Table 1 and Fig. 4 give a qualitatively correct picture of the situation. There are three consequences of this.

(i) Windstorm insurance contains an element of gambling—either a very large accumulated claim occurs, or it doesn’t occur. The way to avoid this gambling seems to be to put a contractual limit to the accumulated loss caused by a single storm event.

(ii) All of the very different loss amounts in Table 1 give relevant information about the possibility of large losses. Any method to predict “the worst conceivable loss” which doesn’t include a risk estimate corresponds to choosing a number from Table 1 without explaining which one, and seems to be of little use. In particular this applies to the “engineering” method discussed in the introduction. Nevertheless, this method of course still could be very useful as a way of finding a “storm severity index” for subsequent use in a statistical analysis. It also has clear “pedagogical” advantages.

(iii) For small claims insurance, all relevant information on claim sizes is summarised by the mean, or the mean and standard deviation of the distribution of the
claims. Clearly this isn’t the case for the distributions in Table 1 and Fig. 4. The situation may be similar for the reclaimed amounts for the higher reinsurance layers. We believe that one hence should estimate the entire distribution of the reclaimed amounts, including reinstatements and other contractual rules. Once the peaks over thresholds model has been fitted to the available data, it is completely straightforward to do this by simulation. Of course, in the end this information has to be distilled down to a price of reinsurance—our contention is just that the entire claims distribution is useful in judging this price. Ideally, this analysis should be complemented by a sensitivity study, where the same computations are made for a set of different values of the parameters.

Finally, a list of areas where we think there is a significant need for further research.

- bias reduction and small sample confidence intervals for the estimators in the peaks over thresholds method
- use of information on individual claim sizes and on the distribution of the number of claims in the estimation of accumulated loss sizes
- use of meteorological information to improve parameter estimates
- Bayes and credibility methods for the peaks over thresholds model
- how to include information on estimation uncertainty, e.g. confidence intervals, in the risk analysis.

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Address for correspondence:
Holger Rootzén
Department of Mathematics
Chalmers University of Technology
S-412 96 Gothenburg
Sweden
rootzen@math.chalmers.se

Nader Tajvidi
Department of Mathematics
Chalmers University of Technology
S-412 96 Gothenburg
Sweden
nader@math.chalmers.se