Modelling Non-linear and Non-stationary Time Series

Chapter 2: Conditional parametric model - An example

Henrik Madsen

Advanced Time Series Analysis

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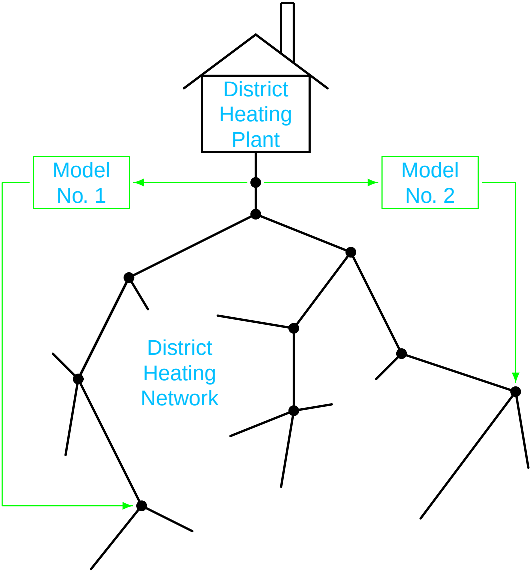
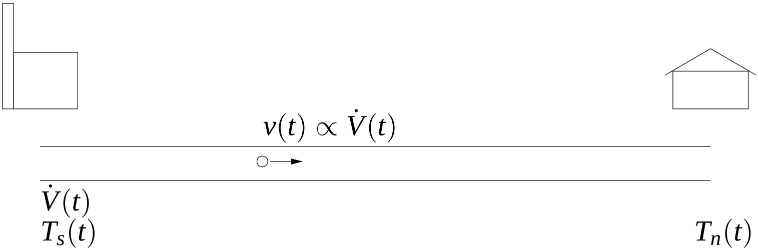
Contents

DH network principle



District Heating Network

- Simplification -



$$\text{Length of pipe} = \int_{t}^{t+\tau(t)} v(s)ds$$

Conditional parametric ARX-model

$$y_t = \sum_{i \in L_y} a_i(x_{t-m}) y_{t-i} + \sum_{i \in L_u} b_i(x_{t-m}) u_{t-i} + e_t,$$

- The **functions** $a_i(x_{t-m})$ and $b_i(x_{t-m})$ must be estimated
- The model may be written as $y_t = \boldsymbol{z}_t^T \boldsymbol{\theta}(\boldsymbol{x}_t) + e_t$

The Regression Case

$$y_i = \theta_0(\boldsymbol{x}_i) + \theta_1(\boldsymbol{x}_i) z_{1i} + \ldots + \theta_p(\boldsymbol{x}_i) z_{pi} + e_i$$

$$\begin{aligned} y_i &= \boldsymbol{z}_i^T \boldsymbol{\theta}(\boldsymbol{x}_i) + e_i \\ \boldsymbol{z}_i &= [1 \; z_{1i} \; \ldots \; z_{pi}]^T \\ \boldsymbol{\theta}(\boldsymbol{x}_i) &= [\theta_0(\boldsymbol{x}_i) \; \theta_1(\boldsymbol{x}_i) \; \ldots \; \theta_p(\boldsymbol{x}_i)]^T \end{aligned}$$

Observations: $(y_i, \boldsymbol{x}_i, \boldsymbol{z}_i);\; i = 1, \ldots, N$

Estimation

- Select points in the space spanned by $\boldsymbol{x}_i$
- For each point $\boldsymbol{x}$:
  - Weight on observations: $w_i(\boldsymbol{x}) = W\left(\frac{||\boldsymbol{x}_t - \boldsymbol{x}||}{h(\boldsymbol{x})}\right)$
  - Estimate $\boldsymbol{\theta}(\boldsymbol{x})$ (one value) by WLS
- Interpolate to obtain the whole of $\hat{\boldsymbol{\theta}}(\boldsymbol{x})$

Local Polynomial Approximation

- Example:

$$
\begin{aligned}
&y_i = \theta_0(x_i) + \theta_1(x_i) z_{1i} + e_i \\
&\theta_0(x) \approx \phi_{00}(x) + \phi_{01}(x)x \\
&\theta_1(x) \approx \phi_{10}(x) + \phi_{11}(x)x + \phi_{12}(x)x^2 \\
&\mathbf{z}_i = [1 \; x_i \quad z_{1i} \; z_{1i}x_i \; z_{1i}x_i^2]^T
\end{aligned}
$$

Simulations

- Model -

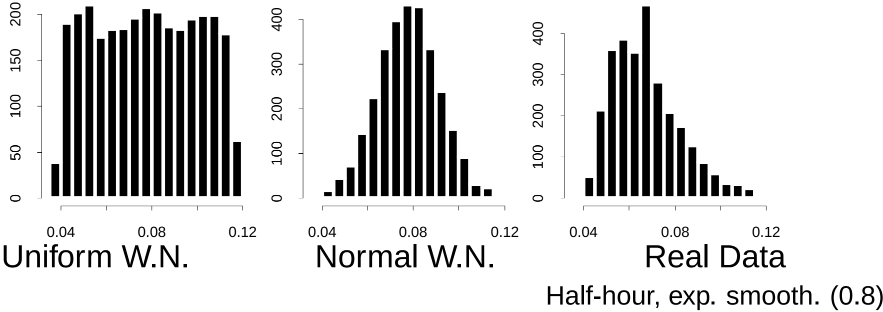
$$y_t = a_1(x_{t-1})y_{t-1} + \sum_{i \in \{2,4\}} b_i(x_{t-1})u_{t-i} + e_t$$

y_t	$\sim$	Network temperature
u_t	$\sim$	Supply temperature
x_t	$\sim$	Flow

Simulations - Estimation

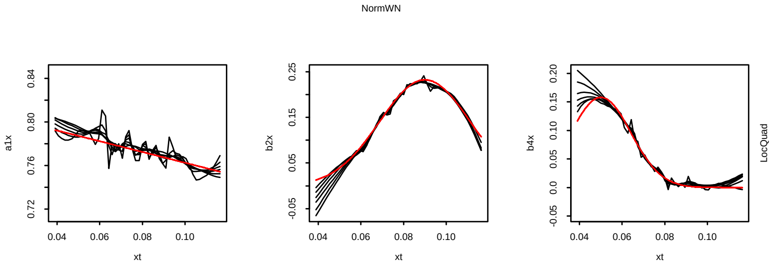
- Nearest neighbour: $10\%, 20\%, \ldots, 60\%$
- Local constant approximations
- Local quadratic approximations
- System noise $N(0, 0.27^2)$

Number of 1/2 hours versus flow (m^3/s)
3000 observations

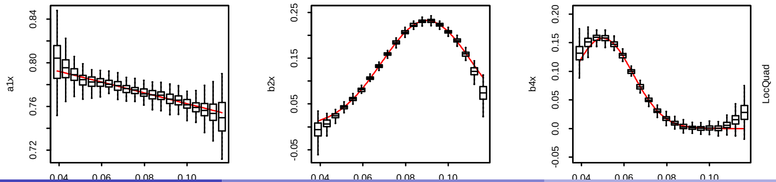


Simulations

Input: Normal White Noise



Box plots: 100 realizations of *system* noise (30%)



Simulations - Conclusions

- Local constants inferior to local quadratic
- Local quadratic in general quite reliable
- Noise, and maybe bias, in border regions

Application

- One consumer consisting of 84 households
- Measurements:
  - Flow at plant
  - Temperature at plant
  - Temperature at consumer

Application - Models

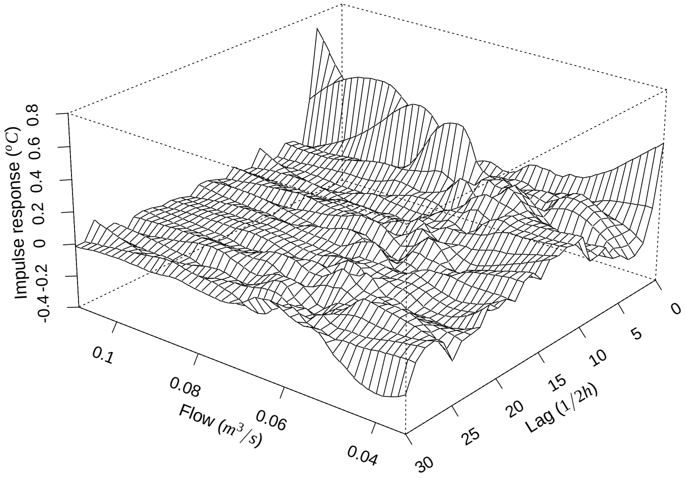
- FIR-model

$$y_t = \sum_{i=0}^{30} b_i(x_t)u_{t-i} + e_t$$

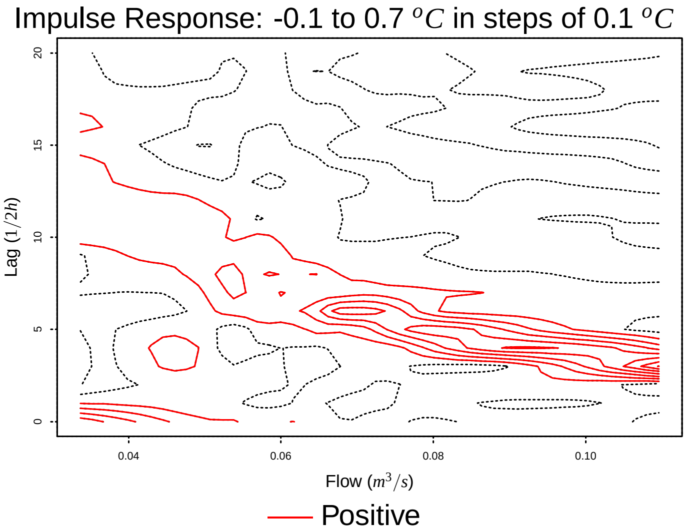
- ARX-model

$$y_t = a(x_t)y_{t-1} + \sum_{i=3}^{15} b_i(x_t)u_{t-i} + e_t$$

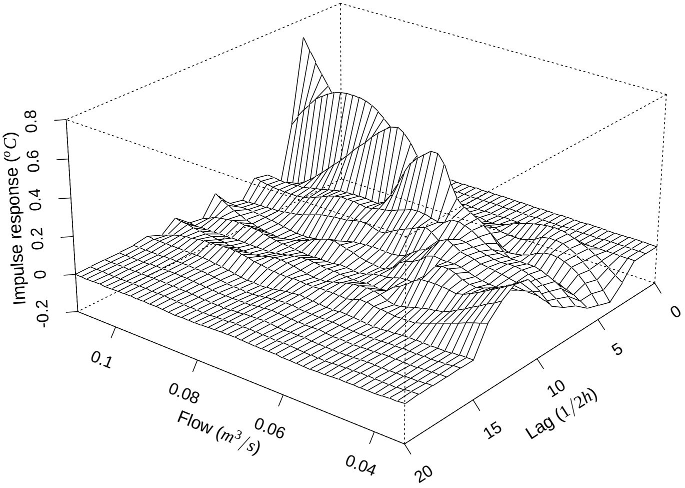
Impulse Response of FIR-model (40%)

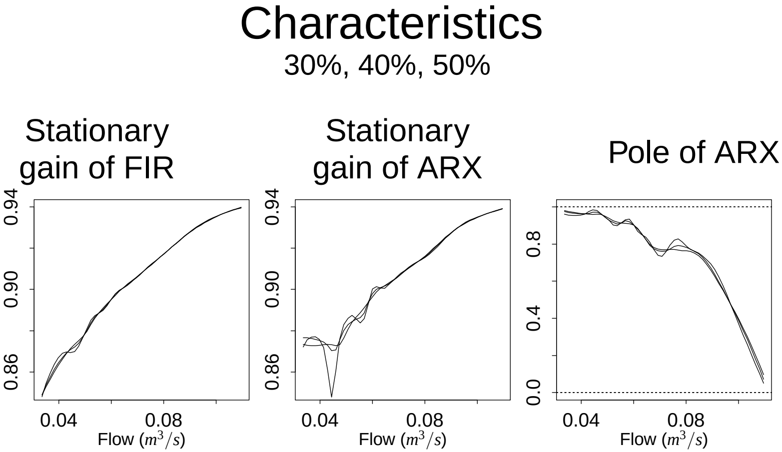


Impulse Response - FIR model



Impulse Response of ARX-model (40%)




Characteristics
30%, 40%, 50%
Stationary gain of FIR
Stationary gain of ARX
Pole of ARX
0.86
0.90
0.94
0.0
0.4
0.8
0.04
0.08
Flow (m^3/s)

Conclusions (Application)

- Time delay decreasing with increasing flow
- 6-15% temperature loss depending on flow
- Possible inaccuracy of the model at low flows (input design?)