Time Series Analysis
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Andreas Jakobsson
Conditional expectations

\[
E\{Y|X = x\} = \int y f_{Y|X=x}(y) dy \\
= \int y \frac{f_{X,Y}(x, y)}{f_X(x)} dy \\
E\{g(X)Y\} = E\{g(X)E\{Y|X\}\} \\
C\{Y, Z|X\} = E\{(Y - E\{Y|X\})(Z - E\{Z|X\})^T X\} \\
C\{Y, Z\} = E\{C\{Y, Z|X\}\} + E\{E\{Y|X\}, E\{Z|X\}\} \\
\]

Linear projection

We defined the linear prediction of \( y \) on to \( x \) as

\[
E\{y|x\} = a + Bx \\
\]

Then, the prediction error is orthogonal with \( x \)

\[
C\{y - E\{y|x\}, x\} = 0 \\
\]

The optimal linear prediction is formed as

\[
E\{y|x\} = m_y - R_{y,x} R_x^{-1} (x - m_x) \\
\]

with error variance

\[
V\{e|x\} = R_{y,y} - R_{y,x} R_x^{-1} R_{y,x} \\
\]