Dugga 6/10-05 fms121

① a) $E(X) = \sum_k k p_X(k) = 0 \cdot 0.3 + 1 \cdot 0.5 + 2 \cdot 0.2 = 0.9$

$E(X^2) = \sum_k k^2 p_X(k) = 0^2 \cdot 0.3 + 1^2 \cdot 0.5 + 2^2 \cdot 0.2 = 1.3$

$V(X) = E(X^2) - E(X)^2 = 1.3 - 0.9^2 = 0.49$

b) Låt Y = Totala antalet större reparationer under 1 månad

$Y = \sum_{i=1}^{121} X_i$ där X_i har förd. enl. a)

$E(Y) = E\left(\sum_{i=1}^{121} X_i\right) = \sum_{i=1}^{121} E(X_i) = 121 \cdot 0.9 = 108.9$

$V(Y) = V\left(\sum_{i=1}^{121} X_i\right) =$ [Antag olika X_i ober av varandra]

$= \sum_{i=1}^{121} V(X_i) = 121 \cdot 0.49 = 59.29$

Dogaja FMS121 - 05

2

$$f_X(x) = \begin{cases} \frac{3}{x^4} & , x \geq 1 \\ 0 & , x < 1 \end{cases}$$

$3t^{-4} \to \frac{3t^{-3}}{-3}$

$F_X(x)$ ser ut att bli användbar, så börja med den.

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(t)\,dt \overset{\text{om } x \geq 1}{=}$$

$$= \int_1^x \frac{3}{t^4}\,dt = \left[\frac{3}{-3t^3}\right]_1^x = 1 - \frac{1}{x^3} \quad , x \geq 1$$

a)

$$P(X \geq 3 \mid X \geq 2) = \frac{P(X \geq 3 \cap X \geq 2)}{P(X \geq 2)} = \frac{P(X \geq 3)}{P(X \geq 2)} =$$

$$= \frac{1 - P(X < 3)}{1 - P(X < 2)} = \frac{1 - F_X(3)}{1 - F_X(2)} = \frac{\frac{1}{3^3}}{\frac{1}{2^3}} = \frac{8}{27}$$

b) $Y = \ln X$

$$F_Y(y) = P(Y \leq y) = P(\ln X \leq y) = P(X \leq e^y)$$

$$= F_X(e^y)$$

Alt 1

$$= 1 - \frac{1}{(e^y)^3} = 1 - e^{-3y}, \quad e^y \geq 1 \text{ dvs } y \geq 0$$

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy}(1 - e^{-3y}) = 3e^{-3y} \quad , y \geq 0$$

Alt 2

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} F_X(e^y) = f_X(e^y) \cdot e^y$$

$$= \frac{3}{(e^y)^4} e^y = 3e^{-3y} \quad , e^y \geq 1 \text{ dvs } y \geq 0$$

dugga FMS121 - 05

3 Låt A = Propeller tillv. av maskin A

B = ——— " ——— B

S = En propeller är skev.

Givet $P(B) = 0.45 \Rightarrow P(A) = 0.55$

$P(S) = 0.05$ $\quad (B = A^*)$

$P(A|S) = 0.5 \Rightarrow P(B|S) = 0.5$

a) $$P(S|A) = \frac{P(A \cap S)}{P(A)} = \frac{P(A|S) \cdot P(S)}{P(A)} =$$

$$= \frac{0.5 \cdot 0.05}{0.55} = 0.0455$$

b) $$P(A|S^*) = \frac{P(A \cap S^*)}{P(S^*)} = \frac{P(S^*|A) \cdot P(A)}{P(S^*)}$$

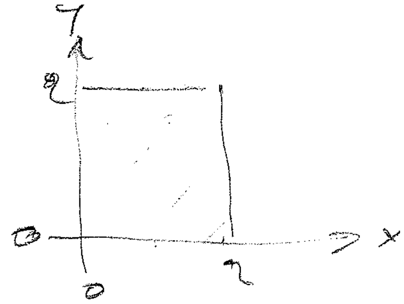
$$= \frac{(1 - P(S|A)) \cdot P(A)}{1 - P(S)} = \frac{(1 - 0.0455) \cdot 0.55}{1 - 0.05}$$

$$= 0.5526$$

Rogga FMS121 - 05

(4)

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{8}(x+y) & , \ 0 \le x,y \le 2 \\ 0 & , \ \text{f ö} \end{cases}$$



a) X o Y ober

$\iff$

$$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y) \quad \text{för alla } (x,y)$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy = \int_0^2 \frac{1}{8}(x+y)\,dy =$$

$$= \frac{1}{8}\left[xy + \frac{y^2}{2}\right]_0^2 = \frac{1}{8}\left(2x + \frac{4}{2}\right) = \frac{x}{4} + \frac{1}{4} =$$

$$= \frac{1}{4}(x+1) \quad , \ 0 \le x \le 2$$

$$f_Y(y) = [\text{Symmetri}] = \frac{1}{4}(y+1) \quad , \ 0 \le y \le 2$$

$$f_X(x) \cdot f_Y(y) = \frac{1}{8}(x+1)(y+1) \neq f_{X,Y}(x,y)$$

Nej, inte ober

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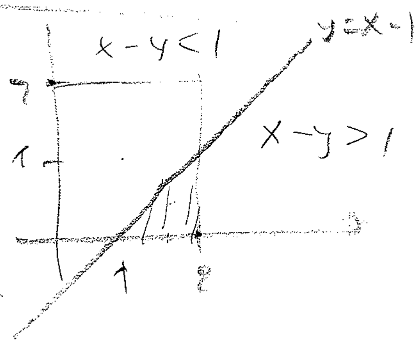
b) $$P(X - Y > 1) = \iint_{x-y>1} f_{X,Y}(x,y)\,dx\,dy$$



$$= \int_{x=1}^{2} \int_{y=0}^{x-1} \frac{1}{8}(x+y)\,dy\,dx = \frac{1}{8}\int_1^2 \left[xy + \frac{y^2}{2}\right]_0^{x-1} dx$$

$$= \frac{1}{8}\int_1^2 \left(x(x-1) + \frac{(x-1)^2}{2}\right) dx = \frac{1}{8}\int_1^2 \left(x^2 - x + \frac{x^2}{2} - x + \frac{1}{2}\right) dx$$

$$= \frac{1}{8}\int_1^2 \left(\frac{3x^2}{2} - 2x + \frac{1}{2}\right) dx = \frac{1}{8}\left[\frac{x^3}{2} - x^2 + \frac{x}{2}\right]_1^2 = \frac{1}{8}$$