Space-time downscaling
under error in computer model output

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joint work with Alan E. Gelfand, David M. Holland, Peter Guttorp and Peter Craigmile
Introduction

- In many environmental disciplines data come from two sources: monitoring networks and numerical models.

- **Numerical models** are deterministic mathematical models used to predict environmental spatio-temporal processes.

- Describe the underlying physical and chemical processes via partial differential equations.

- Equations solved via numerical methods by discretizing space and time.

- Predictions are given in terms of averages over grid cells.
Introduction

- Sparse locations
- Missing data
- Essentially, true value

- Large spatial domains
- No missingness
- Calibration concerns

Observed average temperature
Year 2007

Climate model output for average temperature
Year 2007

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Introduction

- Sparse locations
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- Essentially, true value

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Our goal

- Fuse the two sources of data
- Obtain improved predictions at point level by downscaling the computer model output
- Explicitly address the difference in spatial scale
  - Outputs from numerical models are given in terms of predictions over grid cells
  - Observations from monitors are collected at points
- Calibrate the numerical model
  - Correct outputs from numerical models
Previous approaches

Approaches for downscaling include:

- Atmospheric sciences approaches
  - Algorithmic

- Model-based approaches
  - Wikle and Berliner (2005)
  - Bayesian Melding of Fuentes and Raftery (2005)
Downscaler: main idea

Observed average temperature
Year 2007

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To each point $s$ in the domain $S$ with observation $Y(s)$ we associate the numerical model output at grid cell $B$, $X(B)$, where $s \in B$:

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Downscaler: main idea

To each point \( s \) in the domain \( S \) with observation \( Y(s) \) we associate the numerical model output at grid cell \( B \), \( X(B) \), where \( s \in B \):

\[
Y(s) \iff X(B)
\]
To each point $s$ in the domain $S$ with observation $Y(s)$ we associate the numerical model output at grid cell $B$, $X(B)$, where $B$ is such that $s \in B$:

$$Y(s) \iff X(B)$$
Downscaler

- Time $t$ is fixed. $Y(s)$ observation at point $s$, $X(B)$ numerical model output at grid cell $B$. For $s$ in $B$:

$$Y(s) = \tilde{\beta}_0(s) + \tilde{\beta}_1(s)X(B) + \epsilon(s) \quad \epsilon(s) \overset{\text{ind}}{\sim} N(0, \tau^2)$$

with $\tilde{\beta}_i(s) = \beta_i + \beta_i(s)$, $i=0,1$.

- $\beta_0(s)$ and $\beta_1(s)$ correlated mean-zero GP.
Downscaler

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- $\beta_0(s)$ and $\beta_1(s)$ correlated mean-zero GP.

- Extension to space-time: For $s$ in $B$ and for each $t$

$$Y(s, t) = \tilde{\beta}_0(s, t) + \tilde{\beta}_1(s, t)X(B, t) + \varepsilon(s, t) \quad \varepsilon(s, t) \sim N(0, \tau^2)$$

with $\tilde{\beta}_i(s, t) = \beta_{it} + \beta_i(s, t)$, $i=0,1$.

- Temporal dependence in $\beta_{it}$ and $\beta_i(s, t)$: dynamic or independent
Downscaler

- Very flexible and natural hierarchical Bayesian model specification
- Driven by true station data rather than uncalibrated model output
- Computationally feasible also for large spatial domains
- Allows local calibration of the numerical model output
- Endows the spatial process $Y(s)$ with a non-stationary covariance structure
- Straightforward downscaling prediction at an unmonitored sites
- Better predictive performance than other methods (geostatistical and model-based)
Extending the downscaler

- Propose a neighbor-based extension of the downscaler model
  - accounts for information in the numerical model output at neighboring grid cells
  - accounts for uncertainty in the association \( Y(s) \leftrightarrow X(B) \) with \( s \) in \( B \)
Extending the downscaler

- Propose a neighbor-based extension of the downscaler model
  - accounts for information in the numerical model output at neighboring grid cells
  - accounts for uncertainty in the association \( Y(s) \xleftarrow{} X(B) \) with \( s \) in \( B \)

- We call the new downscaler a **smoothed downscaler using spatially-varying random weights**
Static setting
Smoothed downscaler with spatially-varying weights

- Downscaler: \( Y(s) = \tilde{\beta}_0(s) + \tilde{\beta}_1(s)X(B) + \epsilon(s) \quad s \in B \)

We don't use the numerical model \( X(B) \) output directly anymore. For each \( s \), we introduce a new regressor \( \tilde{X}(s) \) and we assume that \( Y(s) = \tilde{\beta}_0(s) + \beta_1 \tilde{X}(s) + \epsilon(s) \)

where \( \tilde{X}(s) = \sum_{g_k=1}^{g} w_k(s)X(B_k) \), and \( w_k(s) \) random and spatially-varying.

Let \( r_k, k = 1, \ldots, g \) be the centroids of numerical model grid cells \( B_k, k = 1, \ldots, g \), then

\[
\tilde{X}(s) = \frac{K(s - r_k; \psi) \cdot \exp(Q(r_k))}{\sum_{g_l=1}^{g} K(s - r_l; \psi) \cdot \exp(Q(r_l))}
\]

- \( Q(\cdot) \) mean-zero GP with variance \( \sigma^2_Q \), exponential covariance function and decay parameter \( \phi_Q \).
- \( K(s - r_k; \psi) = \exp(-\psi \| s - r_k \|) \)
Smoothed downscaler with spatially-varying weights

- **Downscaler:** \( Y(s) = \tilde{\beta}_0(s) + \tilde{\beta}_1(s)X(B) + \varepsilon(s) \quad s \in B \)

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where \( \tilde{X}(s) = \sum_{k=1}^{g} w_k(s)X(B_k) \), and \( w_k(s) \) random and spatially-varying.
Smoothed downscaler with spatially-varying weights

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w_k(s) := \frac{\mathcal{K}(s - r_k; \psi) \cdot \exp(Q(r_k))}{\sum_{l=1}^{g} \mathcal{K}(s - r_l; \psi) \cdot \exp(Q(r_l))}
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- \( Q(\cdot) \) mean-zero GP with variance \( \sigma_Q^2 \), exponential covariance function and decay parameter \( \phi_Q \).
- \( \mathcal{K}(s - r_k; \psi) = \exp(-\psi\|s - r_k\|) \).
Smoothed downscaler with spatially-varying random weights
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To each point $s$ in the domain $S$ with observation $Y(s)$ we associate $\tilde{X}(s)$, where $\tilde{X}(s) = \sum_{k=1}^{g} w_k(s) x(B_k)$.

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Smoothed downscaler with spatially-varying random weights

Observed average temperature
Year 2007

Borlänge
Stockholm

Climate model output for average temperature
Year 2007

To each point \( s \) in the domain \( S \) with observation \( Y(s) \) we associate \( \tilde{X}(s) \), where

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Observed average temperature
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Climate model output for average temperature
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To each point $s$ in the domain $S$ with observation $Y(s)$ we associate $\tilde{X}(s)$, where $\tilde{X}(s) = \sum_{k=1}^{g} w_k(s) x(B_k) Y(s) \rightarrow \tilde{X}(s)$.
Smoothed downscaler with spatially-varying random weights

To each point $s$ in the domain $S$ with observation $Y(s)$ we associate $\tilde{X}(s)$, where $\tilde{X}(s) = \sum_{k=1}^{g} w_k(s) x(B_k)$. 

$Y(s) \rightarrow \tilde{X}(s)$
Spatio-temporal setting
Smoothed downscaler with spatially-varying weights

- $Y(s, t)$ observations at $s$ and time $t$, $X(B, t)$ numerical model output at grid cell $B$ and time $t$:

$$Y(s, t) = \tilde{\beta}_0(s, t) + \beta_1, t \tilde{X}(s, t) + \varepsilon(s, t) \quad \varepsilon(s, t) \overset{\text{ind}}{\sim} N(0, \tau^2)$$

with $\tilde{\beta}_0(s, t) = \beta_0 + \beta_0(s, t)$, $\tilde{X}(s, t) = \sum_{k=1}^{g} w_k(s, t) X(B_k, t)$, and $w_k(s, t)$ random and varying both in space and time.
Smoothed downscaler with spatially-varying weights

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$$Y(s, t) = \tilde{\beta}_0(s, t) + \beta_1, t \tilde{X}(s, t) + \varepsilon(s, t) \quad \varepsilon(s, t) \overset{\text{ind}}{\sim} \mathcal{N}(0, \tau^2)$$

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$$w_k(s, t) := \frac{\mathcal{K}(s - r_k; \psi) \cdot \exp(Q(r_k, t))}{\sum_{l=1}^{g} \mathcal{K}(s - r_l; \psi) \cdot \exp(Q(r_l, t))}$$
Smoothed downscaler with spatially-varying weights

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\[
Y(s, t) = \tilde{\beta}_0(s, t) + \beta_{1,t} \tilde{X}(s, t) + \varepsilon(s, t) \quad \varepsilon(s, t) \overset{\text{ind}}{\sim} N(0, \tau^2)
\]

with \( \tilde{\beta}_0(s, t) = \beta_{0t} + \beta_0(s, t) \), \( \tilde{X}(s, t) = \sum_{k=1}^g w_k(s, t) X(B_k, t) \), and \( w_k(s, t) \) random and varying both in space and time.

\[
w_k(s, t) := \frac{\mathcal{K}(s - r_k; \psi) \cdot \exp(Q(r_k, t))}{\sum_{l=1}^g \mathcal{K}(s - r_l; \psi) \cdot \exp(Q(r_l, t))}
\]

- Temporal dependence in \( \beta_{it} \), \( i = 0, 1 \), \( \beta_0(s, t) \) and \( Q(r_k, t) \): either (i) independent in time, or (ii) dynamic.
Application

- Climate model output for southern Sweden: the Rossby Centre Regional Climate model RCA3 at 12-km grid resolution ($g=2,640$).
- 17 monitoring sites used for fitting the model and 2 sites used for validation: Borlange and Goteborg
- Missing data: 28.5%
- Time-varying parameters: independent in time
Scatterplot of observed yearly average temperature at the 17 training sites versus the climate model output at the corresponding grid cells.

Correlation coefficient: $R = 0.80$
Observations, climate model output and predictions

- We will look at observations and predictions at the 17 training sites by the climate model output and the downscaler model with spatially-varying random weights.
Observations, climate model output and predictions

- Observed yearly average temperature at the 17 training sites (black line), climate model output (blue line), downscaler predictions (red line)
**Observations, climate model output and predictions**

- Observed yearly average temperature at the 17 training sites (black line), climate model output (blue line), downscaler predictions (red line)
Mean absolute error at the 17 training sites: climate model output (blue line), downscaler (red line)
Results

- Predictive performance at the 17 training sites, averaged over space and time.
- Results are in degree Celsius.

<table>
<thead>
<tr>
<th>Method</th>
<th>MSPE</th>
<th>MAPE</th>
<th>Coverage 95% PI</th>
<th>Avg. length 95% PI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regional climate model</td>
<td>1.90</td>
<td>0.64</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Smoothed downscaler with spatially-varying weights</td>
<td>0.47</td>
<td>0.33</td>
<td>92.5%</td>
<td>7.02</td>
</tr>
</tbody>
</table>
Observations, climate model output and predictions

- Observed yearly average temperature at **Goteborg** (black line), climate model output (blue line), downscaler predictions (red line). The shaded grey area indicates the 95% prediction interval.
• Observed yearly average temperature at **Borlange** (black line), climate model output (blue line), downscaler predictions (red line). The shaded grey area indicates the 95% prediction interval.
Results

- Results are in degree Celsius.

<table>
<thead>
<tr>
<th>City</th>
<th>Method</th>
<th>MSPE</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Goteborg</td>
<td>Regional climate model</td>
<td>2.17</td>
<td>1.34</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Smoothed downscaler with spatially-varying weights</td>
<td>2.17</td>
<td>1.32</td>
<td>96.7%</td>
<td>5.87</td>
</tr>
<tr>
<td>Borlange</td>
<td>Regional climate model</td>
<td>3.56</td>
<td>1.05</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Smoothed downscaler with spatially-varying weights</td>
<td>1.56</td>
<td>0.92</td>
<td>93.8%</td>
<td>5.49</td>
</tr>
</tbody>
</table>
Predictions: year 1994

Climate model output
Year 1994

Downscaler
Year 1994

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Predictions: year 1994

Downscaler–Climate model output
Year 1994

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Predictions: year 2007

Climate model output
Year 2007

Downscaler
Year 2007

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Conclusions

• Presented a method to downscale the output from numerical models that accounts for uncertainty in the association of a site to its putatively associated grid cells.

• In our application, predictions are less biased than the predictions from a regional climate model.

• Currently, we are comparing the merits of this approach versus those of an upscaling approach.