Inga hjälpmedel. Använd institutionens papper, skriv på bara den ena sidan och högst en uppgift på varje papper. Skriv tydligt, ge klara och kortfattade motiveringar, rita gärna figur i förekommande fall och ge tydliga svar. Fyll i omslaget fullständigt och skriv initialer på varje papper.

No books, notes, computational devices etc. are allowed. Use paper supplied by the Department, write only on one side of each paper, and treat at most one exercise on each paper. Use clear handwriting and give clear careful motivations. Fill in the form completely and write your name on each sheet of paper.

Note: Partial solutions, solution-attempts and Ansatz-es for problems will be given attention in the evaluation and can amount to (partial) credit. Please, don’t hesitate to give such documentation.

1. Consider the polynomial 
   \[ p(z) = 1 + 3z + iz^5. \]
   Find the number of zeroes of \( p \) that are located in the open unit disc
   \[ \mathbb{D} = \{ z \in \mathbb{C} : |z| < 1 \}. \]

2. Calculate the integral
   \[ \int_0^\pi \frac{d\theta}{3 + \cos \theta}. \]

3. Calculate the integral
   \[ \int_0^\infty \frac{dx}{\sqrt{x}(1 + x)}, \]
   where \( \sqrt{x} \) for \( x > 0 \) denotes the usual positive square root.

4. Find an analytic function \( f \) mapping the open unit disc \( \mathbb{D} \) one-to-one onto the open right half-plane
   \[ \mathbb{C}_+ = \{ z \in \mathbb{C} : \text{Re}(z) > 0 \} \]
   such that \( f(0) = 1 \) and \( f'(0) = 2. \)

5. Let \( u : \Omega \to \mathbb{R} \) be a real-valued non-constant harmonic function in an open connected set \( \Omega \). Prove that the set of critical points
   \[ C = \{ z \in \Omega : u'_x(z) = u'_y(z) = 0 \} \]
   of \( u \) has no limit point in \( \Omega \), that is, prove that there does not exist a sequence of points \( \{ z_k \}_{k=1}^\infty \) from \( C \) such that \( z_k \to z_0 \) as \( k \to \infty \) for some point \( z_0 \in \Omega \).

Var god vänd!
Consider the function \( k \) in the unit disc \( \mathbb{D} \) given by the convergent power series expansion

\[
k(z) = \sum_{j=1}^{\infty} j z^j, \quad z \in \mathbb{D}.
\]

Prove that this function \( k \) maps \( \mathbb{D} \) one-to-one onto the slit plane \( \mathbb{C} \setminus (-\infty, -1/4] \), or show that this is not the case. Here

\[
(-\infty, -1/4] = \left\{ x \in \mathbb{R} : -\infty < x \leq -1/4 \right\}
\]

considered as a subset of the complex plane \( \mathbb{C} \).