35. Determine all possible Laurent expansions, in powers of $z$, of the function $\frac{1}{1 + z + z^2}$.

36. Suppose $n$ is a natural number and $p \neq 1$ a positive number. Calculate $\int_0^{2\pi} \frac{\cos n \theta}{1 - 2p \cos \theta + p^2} d\theta$.

37. Calculate $\int_{-\infty}^{\infty} (x^{2n} + 1)^{-1} dx$ where $n$ is a positive integer.

38. Calculate $\int_0^{\infty} \frac{\sqrt{x}}{1 + x^n} dx$ for $n = 2, 3, 4, \ldots$

39. Evaluate $\int_{-\infty}^{\infty} (1 + x)^{-1} x^a \, dx$ for $-1 < a < 1$.

40. Evaluate $\int_0^{\infty} \frac{\sqrt{x} \ln x}{(1 + x)^2} \, dx$.

41. Show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ by integrating the function $z^{-2} \cot(\pi z)$ around the circle $|z| = k + \frac{1}{2}$ and then letting $k \to \infty$ through the integers.

42. Find the number of zeros of the polynomial $z^4 - z^3 + 2z^2 + z - 3$ which have positive real part.

43. Find the number of zeros in the first quadrant of the polynomial $z^5 + z^2 + 1$.

44. Show that if $a > e$, then the equation $e^z = az^n$ has $n$ roots in the unit disk.

45. Prove the fundamental theorem of algebra using Rouché’s theorem.

46. A function analytic in a domain $D$ is called schlicht (a German word) if it is one-to-one. Show that if $f$ is schlicht, then $f'$ does not vanish in $D$, and then conclude that $f^{-1}$ is schlicht on $f(D)$. 