1. Show that any Möbius transformation which leaves the real axis invariant may be written with real coefficients.

2. Find the Möbius transformation that carries 0, i, −i in order into 1, −1, 0.

3. Find the Möbius transform which carry the circle |z| = 2 into |z + 1| = 1, the point −2 into the origin, and the origin into i.

4. Find all Möbius transforms that leave the circle |z| = R invariant.

5. Find all Möbius transforms that map the unit disk onto itself.

6. Show that the map \( z \mapsto \frac{\bar{z} - 1}{\bar{z} + 1} \) maps the right half-plane (i.e., the set \( \Re z > 0 \)) onto the interior of the unit circle.

7. Suppose a Möbius transform maps a pair of concentric circles onto a pair of concentric circles. Prove that the ratio of the radii is invariant under the map.

8. Find all circles that are orthogonal to |z| = 1 and |z − 1| = 4.

9. Find all linear transformations that are rotations of the Riemann sphere.

10. Exercise 1.51 in the lecture notes.