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- Throw a dice five times. How many possible resulting sequences of outcomes are there?
- How many sequences where no number occurs more than once?
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Strings of letters

How many different words can you make from the letters i LEMURELL

- without restrictions?
- With no consequtive E's?
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The tennis vacation (from Grimaldi)

Susan has four-week vacation. She will play at least one set of tennis every day, but at most 40 sets in total. Show that there is a span of consecutive days during which she will play exactly 15 sets.

The flower shop

In a flower shop there are five display shelves and 10 different plants. How many arrangements of the flowers are there if we want to have at most three plants on each shelf?

Eulers φ function

Definition

$\varphi(n)$ is the number of integers between 1 and n that are relatively prime to n.

Theorem

If $n = p_1^{k_1} p_2^{k_2} p_3^{k_3} \cdots p_s^{k_s}$ *then*

$$\varphi(n) = n \prod_{j=1}^{s} (1 - 1/p_j)$$

Eulers φ function cont.

Proof (sketch): Use inclusion/exclusion. Let $S = \{1, 2, 3, \ldots, n\}$ and $c_j =$ p_i divides n. Assume $s = 3$.

$$S_1 = N(c_1) + N(c_2) + N(c_3) = \frac{n}{p_1} + \frac{n}{p_2} + \frac{n}{p_3} =$$

$$= n(\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3})$$

$$S_2 = N(c_1c_2) + N(c_1c_3) + N(c_2c_3) = \frac{n}{p_1p_2} + \frac{n}{p_1p_3} + \frac{n}{p_2p_3} =$$

$$= n(\frac{1}{p_1p_2} + \frac{1}{p_1p_3} + \frac{1}{p_2p_3})$$

$$S_3 = N(c_1c_2c_3) = \frac{n}{p_1p_2p_3}$$

Eulers φ function cont.

$$\varphi(n) = S_0 - S_1 + S_2 - S_3 =$$
$$= n(1 - (\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}) + (\frac{1}{p_1 p_2} + \frac{1}{p_1 p_3} + \frac{1}{p_2 p_3}) - \frac{1}{p_1 p_2 p_3})) =$$
$$= n(1 - \frac{1}{p_1})(1 - \frac{1}{p_2})(1 - \frac{1}{p_3})$$

More prime factors: same pattern (but more terms!)