Lecture 10 Niels Chr. 2010-02-18

General homogeneous coord. on the line

0. We already know the projective plane $\mathbb{P}^2$ and the projective space $\mathbb{P}^3$. Let's consider the projective line $\mathbb{P}^1$:

$$\mathbb{P}^1 = \{\lambda(x,y) : (x,y) \neq (0,0),\ 0 \neq \lambda \in \mathbb{R}\}$$

(x,w) are thus homogeneous coordinates for a point in $\mathbb{P}^1$. If $y \neq 0$, then

$$(x,y) \sim \left(\frac{x}{y}, 1\right)$$

and $\hat{x} = \frac{x}{y}$ may be viewed as the usual affine coordinate of the point in question.

1. General homog. coord.; Choose $O\vec{e}_1\vec{e}_2$ such that $O \notin l$.

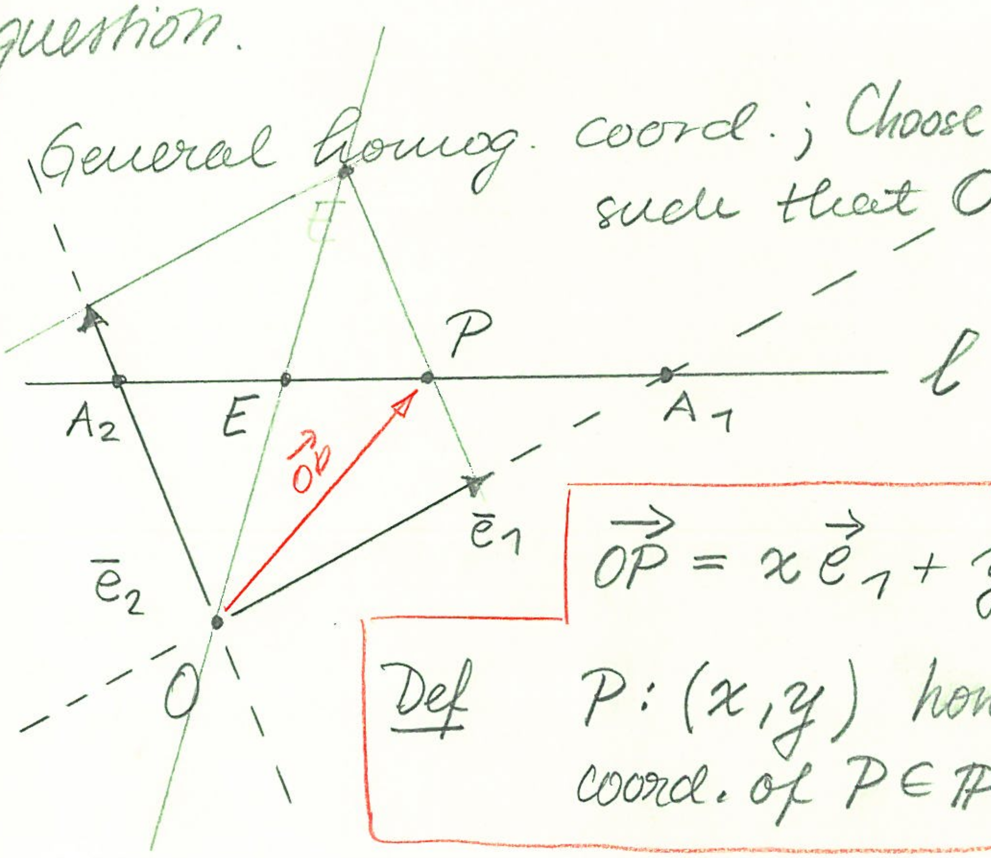
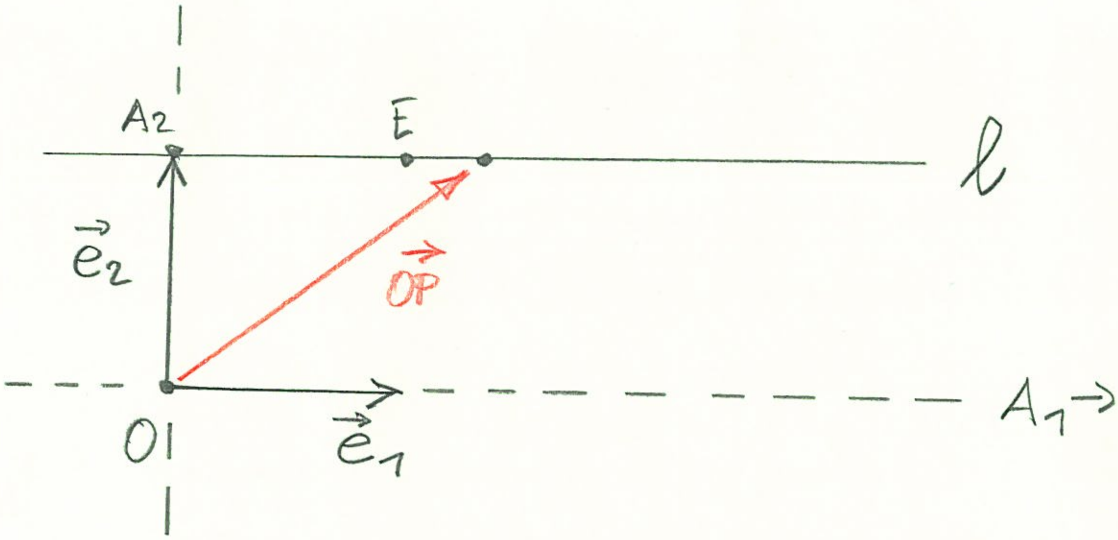


fig. 1

$$\boxed{\vec{OP} = x\vec{e}_1 + y\vec{e}_2}$$

Def $P : (x,y)$ homog. coord. of $P \in \mathbb{P}^1$.

T2.c Example (The "usual" homog. coord.)



3. <u>Def.</u> $(A_1 A_2 E)$ are called the <u>base points</u> for the homogeneous coordinates

<u>Remark</u> The base points are all we need to know in order to get homog. coord. of a point $P \in \mathbb{P}^1$. Jf. fig. 1

$$\frac{x}{y} = \frac{A_2E}{A_1E} \Big/ \frac{A_2P}{A_1P} = \underbrace{(A_1 A_2 E P)}_{\text{the cross ratio}}$$

(Sv. dubbeltförhållandet)