

# On a Modification to the Harris Corner Detector

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## Introduction

This year, it is 15 years ago C. Harris and M. Stephens [1] presented a corner detector which is widely used today. While the algorithm itself, (usually referred to as the Harris detector,) is well-known, there are some theoretical points which we feel have not been appreciated. This paper has two purposes. First we recall the theory behind the Harris detector. Then, as a result of the theoretical analysis, we are lead to suggest a modification to the Harris detector. This modification is achieved by replacing the response function used in [1, p.150](or Eq. (4) below) by a new response function which seems more natural from a mathematical point of view, and whose performance is (at least) just as good.

## Heuristics of Corner Detection

As a model of a gray scale image we use a non-negative function  $f = f(\mathbf{x})$ ,  $\mathbf{x} = (x, y) \in \mathbf{R}^2$ . In edge- and corner detection we are primarily interested in the gradient  $\nabla f = (f_x, f_y)^T$  of the image and its magnitude  $|\nabla f| = \sqrt{f_x^2 + f_y^2}$ . In order to reduce the sensibility to noise of the derivatives in the image, and to simplify the subsequent notation, we may assume that  $f$  has already been suitably mollified. Otherwise replace  $f$  by the convolution

$$w * f(\mathbf{x}) := \iint w(\mathbf{x} - \mathbf{y})f(\mathbf{y}) dxdy, \quad (1)$$

where  $w \geq 0$  is a smooth weight function with  $\iint w dxdy = 1$ . ( $w$  is usually taken to be the Gaussian filter  $G_\sigma(x, y) = (1/2\pi\sigma^2) \exp(-(x^2 + y^2)/2\sigma^2)$ , where  $\sigma > 0$  is the standard deviation.)

In detection of features in gray scale images the following principles play a major role:

- (i) If  $|\nabla f|$  is small in some region then the image is relatively 'flat' there, and the region contains no (useful) features.
- (ii) If  $|\nabla f|$  is large at some point, then we are either at an edge or at a corner.

But the magnitude of  $\nabla f$  alone is not enough to determine whether a given point is a corner or a part of an edge. To make such a distinction, we have to

consider the distribution of the directions of the gradients as well:

(iii) If the gradients  $\nabla f(\mathbf{x}')$  all point roughly in the same direction for all  $\mathbf{x}'$  in a neighbourhood of  $\mathbf{x}$ , then  $\mathbf{x}$  is a point on an edge.

(iv) If the gradients  $\nabla f(\mathbf{x}')$  varies significantly in direction as  $\mathbf{x}'$  runs through a neighbourhood of  $\mathbf{x}$ , then  $\mathbf{x}$  is (close to) a corner.

## The Gradient Density Matrices<sup>1</sup>

The problem, of course, is how to express the above principles analytically in order to get a working algorithm. Partly inspired by work of H. Moravec [2], Harris and Stephens came up with the following neat solution: They encoded the information in the gradient field  $\mathbf{x} \mapsto \nabla f(\mathbf{x})$  in a field of matrices  $\mathbf{x} \mapsto \mathbf{M}(\mathbf{x})$ , where  $\mathbf{M} = \mathbf{M}(\mathbf{x})$ ,  $\mathbf{x} \in \mathbf{R}^2$ , is the symmetric matrix defined by

$$\mathbf{M} = (\nabla f)(\nabla f)^T = \begin{bmatrix} f_x \\ f_y \end{bmatrix} \begin{bmatrix} f_x & f_y \end{bmatrix} = \begin{bmatrix} f_x^2 & f_x f_y \\ f_y f_x & f_y^2 \end{bmatrix}. \quad (2)$$

If  $|\nabla f| \neq 0$  then  $\mathbf{M}$  has the eigenvalues  $\lambda_1 = 0$  and  $\lambda_2 = |\nabla f|^2$ , and the corresponding eigenvectors are the non-zero multiples of  $(-f_y, f_x)^T$  and  $(f_x, f_y)^T$ , respectively. Thus  $\mathbf{M}$  is symmetric, has rank one, and is positive semi-definite ( $\mathbf{M} \geq 0$ ). Moreover,  $\nabla f$  can be recovered (up to signs) from  $\mathbf{M}$ .

Notice also that if the image  $f$  is replaced by a scalar multiple  $af$ ,  $a > 0$ , then  $\mathbf{M}$  is replaced by  $a^2\mathbf{M}$ .

The matrices  $\mathbf{M}$  can be used to express (iii) and (iv) analytically. Let  $W \geq 0$  be a weight function with  $\iint W dxdy = 1$  whose mass is concentrated near the origin. That is, either the support of  $W$  is a neighbourhood of  $\mathbf{x} = 0$ , or  $W(\mathbf{x})$  falls off rapidly as  $|\mathbf{x}| \rightarrow \infty$ . (Here the Gaussian filter  $G_\sigma$ , with  $\sigma > 0$  small, will do.) Then we form the averaged field of matrices:

$$\overline{\mathbf{M}} = W * \mathbf{M}. \quad (3)$$

For want of a better name we shall refer to  $\overline{\mathbf{M}}$  as *the gradient density matrices*, or the *structure tensor* for  $f$ . If one was to take the average  $W * \nabla f$  of the gradients, then variations in magnitudes and directions would simply add out, (in fact  $W * \nabla f = \nabla(W * f)$ ),

<sup>1</sup>Also known as the *structure tensor* of the image.

and no new information is obtained. Now each  $\mathbf{M}$  is positive semi-definite and  $W \geq 0$ , so

$$\overline{\mathbf{M}} \geq 0.$$

Therefore the information about  $\nabla f$  contained in the  $\mathbf{M}$ 's from the support of  $W$  does not cancel, but is accumulated in  $\overline{\mathbf{M}}$ .

In terms of the gradient density matrices the principles (iii) and (iv) become:

(iii') At an ideal edge all gradients point in the same direction, so  $\overline{\mathbf{M}}$  has one eigenvalue equal to zero and the other one positive and large. At a real edge we expect  $\overline{\mathbf{M}}$  to have one small and one large eigenvalue. (That is,  $\overline{\mathbf{M}}$  is ill-conditioned.)

(iv') At an ideal corner the gradients point in orthogonal directions, so  $\overline{\mathbf{M}}$  has two equally large positive eigenvalues. At a real corner the eigenvalues are expected to be large and approximately equal. ( $\overline{\mathbf{M}}$  is well conditioned.)

## The Harris Detector

Let  $\lambda_1 = \lambda_1(\mathbf{x})$  and  $\lambda_2 = \lambda_2(\mathbf{x})$  denote the eigenvalues of  $\overline{\mathbf{M}} = \overline{\mathbf{M}}(\mathbf{x})$ . Recall that  $\lambda_1, \lambda_2 \geq 0$ , since  $\overline{\mathbf{M}} \geq 0$ , and that

$$\det \overline{\mathbf{M}} = \lambda_1 \lambda_2, \quad \text{tr } \overline{\mathbf{M}} = \lambda_1 + \lambda_2.$$

It is suggested in [1] that one considers the response function

$$R_{\text{harris}} = \det \overline{\mathbf{M}} - \kappa (\text{tr } \overline{\mathbf{M}})^2, \quad (4)$$

where  $\kappa > 0$  is a parameter. One usually takes  $\kappa \approx 0.04$ , cf. [3, p.33]. It follows from (iii') and (iv') that  $\det \overline{\mathbf{M}}$  is small at edges and large at corners, while  $\text{tr } \overline{\mathbf{M}}$  remains of the same order of magnitude. (The change in  $\text{tr } \overline{\mathbf{M}}$  from an edge to a corner is approximately a factor two.) So, due to the small value of the constant  $\kappa$ , we have that

$$\text{Corners} = \text{Local maxima of } R_{\text{harris}}. \quad (5)$$

This is *the Harris corner detector*. Notice that if we search for minima of  $R_{\text{harris}}$  then we obtain edges, as indicated by the title of [1], but we shall not say more about this here.

## The Modified Harris Detector

Harris and Stephens admit that (4) is an "...inspired formulation for the corner response" [1, p.150]. The reason why it is still around is probably that it works well in practice, at least when  $\kappa$  is tuned to the right value. It would nevertheless be nice if (4) could be replaced by a response function based on some (simple) mathematical principle. This we do below in three steps, in order to explain the thoughts behind the construction. We end up with a modified response function (7). While this response function may not

perform significantly better than (4), it does have some additional properties which makes it attractive.

The starting point is (iii') and (iv'). At a corner the eigenvalues of  $\overline{\mathbf{M}}$  satisfy  $\lambda_1 \approx \lambda_2$ , so the scalar

$$R_{**} = \frac{(\lambda_1 - \lambda_2)^2}{(\lambda_1 + \lambda_2)^2}$$

is close to zero at corners. Since  $\lambda_1, \lambda_2 \geq 0$  it is easy to see that  $0 \leq R_{**} \leq 1$ , and that  $R_{**} = 1$  only if one eigenvalue is = 0, that is, we are at an edge. Observe that the numerator of  $R_{**}$  is nothing but the discriminant of the characteristic polynomial  $\det(\lambda - \overline{\mathbf{M}})$ , and that

$$(\lambda_1 - \lambda_2)^2 = (\lambda_1 + \lambda_2)^2 - 4\lambda_1\lambda_2 = (\text{tr } \overline{\mathbf{M}})^2 - 4 \det \overline{\mathbf{M}}. \quad (6)$$

If we replace  $R_{**}$  by  $R_* = 1 - R_{**}$ , then  $0 \leq R_* \leq 1$  still holds, and we get a response close to 1 at corners and close to 0 at edges. It follows from (6) that

$$R_* = \frac{4 \det \overline{\mathbf{M}}}{(\text{tr } \overline{\mathbf{M}})^2}.$$

This expression still has some drawbacks.  $R_*$  can distinguish between equal and unequal eigenvalues, but not between equal eigenvalues which are large and equal eigenvalues which are small. In other words,  $R_*$  does not incorporate the heuristic principles (i) and (ii). One way to repair this is to add a regularizing constant to the denominator of  $R_*$ :

$$R = \frac{4 \det \overline{\mathbf{M}}}{\delta^4 + (\text{tr } \overline{\mathbf{M}})^2}. \quad (7)$$

Here  $\delta > 0$  is a parameter. The reason for the exponent of four will become clear shortly. This modified response function has the following nice properties.

(a) The inequalities  $0 \leq R \leq 1$  still hold. (In fact  $0 \leq R < 1$ .) This is in contrast to Harris' response function  $R_{\text{harris}}$  whose numerical range is not known. This means, for instance, that we are able to make a crude localization of corners by using thresholding.

(b)  $R$  is asymptotically independent of the overall brightness of the image. As remarked earlier, if the image  $f$  is replaced by a scalar multiple  $af, a > 0$ , then  $\mathbf{M}$  is replaced by  $a^2\mathbf{M}$ , hence  $\overline{\mathbf{M}}$  by  $a^2\overline{\mathbf{M}}$ . It follows that  $R$  should be replaced by

$$\frac{a^4 4 \det \overline{\mathbf{M}}}{\delta^4 + a^4 (\text{tr } \overline{\mathbf{M}})^2} \rightarrow R_*,$$

as  $a \rightarrow \infty$  (in practice,  $a \gg \delta$  suffices).

(c) Furthermore, the expression above shows that the parameter  $\delta$  may be interpreted as a threshold beyond which  $|\nabla f|$  is regarded as large. Thus the principles (i) and (ii) are built into (7).

Since  $R \approx 1$  at corners, we define:

$$\text{Corners} = \text{Local maxima of } R. \quad (8)$$

This is *the modified Harris detector*.

## Implementation and Results

A simple implementation of the modified Harris detector (8) was programmed in MATLAB. The weight functions  $w$  and  $W$  were chosen to be 21-by-21 masks sampled from Gaussian filters  $G_\sigma$  with  $\sigma = 0.5$  pixels and  $\sigma = 1.5$  pixels, respectively. The regularizing parameter  $\delta$  was set to be the average of  $|\nabla f|$  over the whole image. The local maxima of  $R$  were found in the following way: A pixel in the image of  $R$  is a local maximum if its gray scale value is  $\geq$  the values of its eight neighbouring pixels.

The experiments were performed on an image containing classical computer science artifacts, fig.1. The result is shown in fig.2. The detector usually finds a lot of corners, so fig.2 appears quite messy, we have therefore included a detail from the latter in fig.3, where it is easier to see the results of the experiment.

Finally some concluding remarks. While the theory has been presented in detail, the implementation and testing is still in a preliminary phase, so it is still too early to say exactly how well the modified Harris detector will perform. But, as stated earlier, we have indications which show that it is just as good as the standard Harris detector. We hope to return to the subject in a later paper.



Figure 1: The original image.

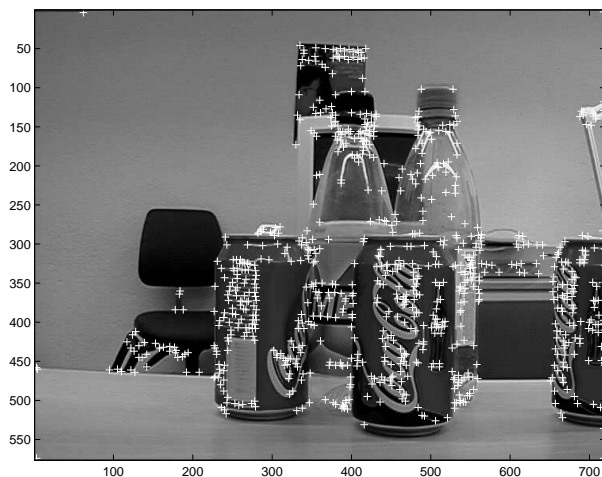


Figure 2: The modified Harris detector, with the parameters specified above, found 827 corners, marked with white '+' in the figure.

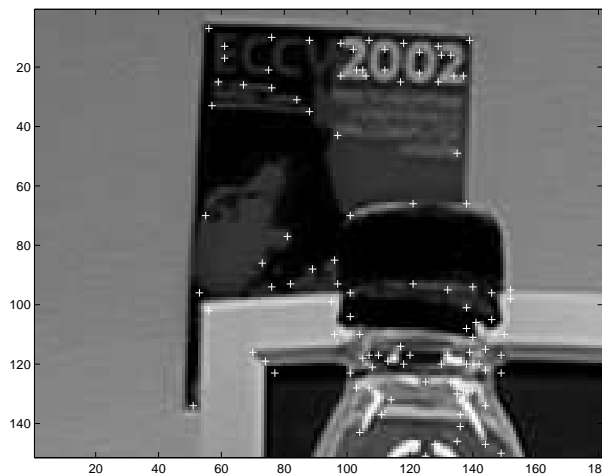


Figure 3: A detail from fig.2 which demonstrates the performance of the modified Harris detector.

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## References

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