LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
GEOMETRI
2004-1-16 - 2003-1-26

HEMTENTA. Lösningarna skall vara försedda med ordentliga motiveringar.

You may use any books and computer programs (e.g. Matlab and Maple), but it is not permitted to get help from other persons. Numerical examples may be useful, but they cannot provide sufficient proofs of any statements, except perhaps to give a counterexample.

Hand in solutions to 6 of problems below. For a passing grade (3), at least 3 problems have to be solved correctly. Credits can be given for partially solved problems. Write your solutions neatly and explain your calculations. Both the content and the format of your solutions, and also how difficult problems you choose, will affect your grade.

The exam has to be handed in to the Student's Expedition in the Mathematics Department at the latest January, 26 at 5.00 p.m. Write your name, section-year (or subject for Ph.D-students), id-number, and phone number or email address on the first page, and write your name on each of the following pages. The oral exams should take place in January or February (depending on your schedule).

Problems have some parameters $(a, c, d \ldots)$. The value of the parameters should be chosen according your personal number ascdeg-hklm. For example if your personal number is 650307-2384 then $a = 6, c = 0, l = 8, s = 5$ in your problems.

1. In which order should one take points $A = (0,0,0), B = (0,0,0), C = (1,a,c), D = (-1,k,l)$ to be control points of some Bézier curve $b(t)$ which does not go through origin and have positive first coordinate for $b'(0)$? Calculate $b'(t)$ without calculating $b(t)$. Calculate $b(t)$ and check that the derivative above was calculated correctly.

2. Let $y = x^3 + cx^2 + dx + e$ be a curve. Find a corresponding control points for a Bézier curve corresponding to the part of this curve with:
a) $0 \leq x \leq 1$;
b) $0 \leq x \leq 1/3$, - left part of a);
c) $1 \leq x \leq 3$, extrapolated part of a);
d) $-1 \leq x \leq 2$.

3. A polynomial $f(x)$ of grad 8 has a property that

$$f(x) = f(9-x),$$

$$f(1) = h, f(3) = a, f(5) = c, f(7) = d, f(9) = e.$$

Calculate $f(10)$.

4. Let a Bézier control net $P_{i,j}$ for a Bézier surface $b(u,v)$ be given by the following matrix:

$i\backslash j$	0	1
0	$(0,0,0)$	$(1,s,0)$
1	$(3,1,0)$	$(3,3,a)$

a) Calculate both $b(0.5,0.5)$ and $b'_v(0.5,0.5)$ without calculating $b(u,v)$.
b) Define the Bézier surface $b(u,v)$.
c) Find 9 points that produces the same surface.
d) Check the result in a) using $b(u,v)$.

5. Let $b_0 = (0,3), b_1 = (a,a), b_3 = (2,-a), b_4 = (0,x), b_5 = (-2,g), b_6 = (4,h)$ be 6 of 7 control points of some quadratic B-spline $s(t)$. Find $x, b_2, s(2)$ and $s(5)$ if the breakpoints divides the interval $[0,7]$ in three subintervals of lengths $\Delta_0, \Delta_1, \Delta_2$ and $\Delta_1 = 3\Delta_0$.

6. Experimental problem.
Try to draw the circle $x^2 + y^2 = 25$ using:
1) Lagrange interpolation;
2) Bézier curve (points should not be the same as above);
3) B-spline.
Use the same but small (not greater then 12) number of points. Use Maple or Matlab to draw the picture. Which method gives the best result?