Solutions may of course be presented in Swedish! The sheets of the department should be used. Write your initials and at most one solution on each sheet handed in. Give careful motivations to your solutions!

1. Let \( \sigma = (1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8) \in S_8. \)
   (i) Express \( \sigma \) as a product of disjoint cycles. (2p)
   (ii) Is \( \sigma \) an even or an odd permutation? (1p)
   (iii) Find the order of \( \sigma \). (2p)

2. Let \( d, n \) be positive integers with \( d \mid n \). Show that \( \mathbb{Z}_n/([d]) \simeq \mathbb{Z}_d \) (ring isomorphism).

3. Give an example of a noncommutative ring \( R \) and a proper ideal \( I \) in \( R \) such that \( R/I \) is commutative.

4. Let \( H \) be a subgroup of a group \( G \). Show that
   \[
   N = \bigcap_{g \in G} gHg^{-1}
   \]
   is a normal subgroup of \( G \).

5. Give an example of a (6,3) code where the Hamming weight of each nonzero code word is at least 3.

6. For an element \( x \) in a group \( G \), the conjugacy class of \( x \) is defined as
   \[ccl(x) = \{gxg^{-1} \mid g \in G\} \]
   The centralizer \( C_G(x) \) is the subgroup
   \[C_G(x) = \{h \in G \mid hx = xh\} \]
   Assume that \( G \) is a finite group. Show that
   \[|ccl(x)| = (G : C_G(x)).\]