Instructions. It is not allowed to get help from other human beings, but it is allowed to use pen and paper, books, computers, et cetera. Please hand in your solutions not later than January 24. The assignment is followed by a short oral exam. When handing in your assignment, send an email to tomasp@maths.lth.se to arrange a time for the oral exam. Failure to comply with this instruction may lead to a failed assignment.

1. Consider the dynamical system $f: \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$f(x, y) = \left( x - \frac{1}{4}y, y + x(1 - x^2) \right).$$

Find all fixed points and describe their stable and unstable manifolds locally. Use a computer to describe them globally.

2. Let $f: [0, 1) \to [0, 1)$ be defined by $f(x) = 5x \mod 1$. Let $E$ be the set of numbers $x \in [0, 1)$ such that

$$f^n(x) \not\in \left[ \frac{1}{5}, \frac{2}{5} \right], \quad f^n(x) \not\in \left[ \frac{3}{5}, \frac{4}{5} \right],$$

for all $n$.

What is the box counting dimension of $E$?

3. Let $k = \frac{\sqrt{5} + 1}{2}$ and define $f: [-1, 1] \to [-1, 1]$ by

$$f(x) = \begin{cases} 
kx + 1 & \text{if } x < 0 \\
kx - 1 & \text{if } x \geq 0
\end{cases}$$

Find a Markov partition for $f$. How many periodic points are there of period 5? Is there a point with orbit which is dense in $[-1, 1]$? Are the periodic points dense in $[-1, 1]$?

4. Consider the Hénon map

$$f(x, y) = \left( 1 - ax^2 + y, bx \right),$$

where $a = 1.4$ and $b = 0.3$. Do a numerical estimate of the box counting dimension of the attractor and the Lyapunov exponents of some appropriately chosen point. Compare the two results.