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Let \((\Omega, \Sigma, \lambda)\) be a measure space. For each “reasonable” measurable \(f : \Omega \to \mathbb{C}\), the non increasing rearrangement of \(f\) is a non increasing right continuous function \(f^*\) defined on the interval \((0, \infty)\), with the property that

\[
\left\{ t > 0 : f^*(t) > \alpha \right\} = \lambda \left( \left\{ x \in \Omega : |f(x)| > \alpha \right\} \right)
\]

for all \(\alpha > 0\).

Consequently, \(\int_0^\infty (f^*(t))^p \, dt = \int_\Omega |f(x)|^p \, d\mu(x)\) for all \(p > 0\).

Example: If \(f = 2 \chi_A + 4 \chi_B + 9 \chi_C\), where \(A\), \(B\) and \(C\) are disjoint subsets of \(\Omega\) of finite measure, then \(f^* = 9 \chi_I_C + 4 \chi_I_B + 2 \chi_I_A\), where \(I_C\), \(I_B\) and \(I_A\) are consecutive intervals, whose lengths equal \(\lambda(C)\), \(\lambda(B)\) and \(\lambda(A)\).

(Here is a picture: We suppose here that \(\lambda\) is two dimensional Lebesgue measure on some subset \(Q\) of \(\mathbb{R}^2\).)
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(Here is a picture:)

[Picture of a graph or diagram related to the rearrangement of a function.]}
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If \( f = 2\chi_A + 4\chi_B + 9\chi_C \),

where \( A \), \( B \) and \( C \) are disjoint subsets of \( Q \), then

\[
\begin{align*}
\hat{f} &= 9\chi_I C + 4\chi_I B + 2\chi_I A,
\end{align*}
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