Warrant the full potential of the deflation techniques

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It is well known that the convergence of Krylov subspace iterative solution methods is severely hampered by the presence of small eigenvalues. Techniques how to avoid or diminish this effect are also well-known, under the names *bordering*, *augmenting* or *deflation*. All those techniques are based on the assumption that we know the smallest eigenvalues and their corresponding eigenvectors, or, at least some good enough approximations of those. At the same time, we are aware that, in general, to compute or estimate a number of eigenvalues and eigenvectors is not an easy task, in particular, for large size matrices. Note that computing eigenpair might be practically infeasible when we are aiming to determine a number of interior eigenvalues or all eigenvalues in an interval.

The idea to *deflate* the small eigenvalues has been incorporated in particular implementations of some of the most used Krylov subspace iteration methods, such as the Conjugate Gradient method, (deflated CG), BiCG, GMRES. Deflation techniques are advocated also in the context of communication-avoiding techniques and pipelined versions of various iterative solvers, highly relevant for High Performance computations.

In this talk we show that for certain classes of problems we are able to compute (all) the eigenvalues exactly or within machine precision, without even constructing the matrices explicitly. The target matrices arise from discretizations of partial differential models, discretized using local methods, such as Finite Elements (FEM), Finite Differences (FD), Finite Volumes and Iso-geometric Analysis (IgA). For some problems it is possible to construct also explicitly the eigenvectors.

The mean to do this is based on the so-called 'Generalized Locally Toeplitz' (GLT) sequences. For certain classes of structured matrices, much richer than the classical Toeplitz matrices, GLT offers the possibility to associate an analytical function to (a sequence of) matrices, referred to as the symbol of the matrices. Sampling the symbol gives an information about the spectrum of the corresponding matrix, namely, a curve (for s.p.d. matrices) on which all eigenvalues are located, except for possibly a finite number of outliers. Until recently, it was not known however, where exactly on that curve the exact eigenvalues are located.

A significant improvement recently has been achieved, providing a methodology to compute exactly (or up to machine accuracy) *all* eigenvalues of a matrix of the considered class, only based on the symbol, in a cheap and easy to implement way.

Knowing the exact eigenvalues opens the door to reviving various methods, based on eigenvalue information. Computing the eigenvectors needs, however, more intention.

At this stage of development of the techniques the discretization meshes are regular and tensor-based. The latter, although seen as a disadvantage for some problems, is also attractive

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for HPC applications. We show the idea on how to compute the eigenvalues based on the matrix symbol and illustrate the effect on solving the linear systems with deflated methods on problems, discretized by FD, FEM and IgA.