Some recent results, challenges and ideas for optimizing data-driven interpolatory model reduction methods

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Model order reduction (MOR) aims at replacing complex, large-scale models of time-varying processes with simpler, smaller models that could be efficiently used as surrogates for tasks such as large-scale simulations, analysis or control. High dimensional models are often obtained after spatial discretization of partial differential equations.
We are interested in interpolation based data-driven MOR methods. In the usual framework, reduced models are computed directly from measured or computed data by means of matching certain input-output mappings of the original system.
The primary method of this study is the Loewner framework (see [1, 2]). It constructs a reduced model directly from transfer function measurements of the underlying system. Additionally, by compressing the usually large data set, it extracts the dominant features and eliminates the inherent redundancies. While the classical Loewner framework is fairly developed, there are still a number of issues to be better understood and improved. For example, the procedure in [1, 2] relies on a full SVD (singular value decomposition) in order to compress the raw data model. This might prove costly to perform in terms of processing time for large data sets (thousands of measurements). For even larger problems, i.e., dimension of tens of thousands, using the standard SVD becomes impractical (storing the full matrices might also be prohibitive).
In order to surpass this shortcoming, we consider a more efficient computational approach which replaces the SVD by a CUR factorization (with the main algorithm based on the one in [4]). We propose a modified Loewner framework that chooses a subset of special interpolation points from within the set of original given points. The selection of these points depends on the maximum volume low-rank matrix that approximates the original Loewner matrix. Another possibility is to replace the classical SVD with a randomized algorithm that computes partial matrix decomposition (known as randomized SVD in [5]). This method uses random sampling to identify a subspace that captures most of the action of the full Loewner matrix. Finally, we also analyze the algorithm in [6] that avoids explicitly forming the Loewner matrix. Instead, it exploits the structure and rapid decay of singular values of the matrix to compute only the dominant singular values and singular vectors.
Although a direct method, the classical Loewner framework in [1, 2] provides good approximation results without relying on any optimization tools. We analyze extensions of this method that try to minimize the deviation from the original data in certain norms. For example, one approach is the AAA algorithm from [3]. It is an adaptive and iterative version of the Loewner framework and aims to optimize the approximation error by means of a least squares approach.
The test cases include physical problems such as semi-discretized Euler-Bernoulli clamped beam and heat diffusion models as well as artificial examples which commonly appear in approximation theory, e.g., Bessel and hyperbolic sine functions.

References

[1] A. J. Mayo and A. C. Antoulas, A framework for the generalized realization problem, Linear Algebra and Its Applications, Vol. 426, pp. 634–662, 2007.

[2] A C. Antoulas and S. Lefteriu and A C. Ionita, A tutorial introduction to the Loewner framework for model reduction, Model Reduction and Approximation for Complex Systems, Series: Computational Science & Engineering, pp. 335–376, SIAM, 2017.

[3] Y. Nakatsukasa, O. Séte, L.N. Trefethen, The AAA algorithm for rational approximation, SIAM Journal on Scientific Computing, Vol. 40, No. 3, pp. A1494–A1522, 2018.

[4] D.C. Sorensen, M. Embree, A DEIM induced CUR factorization, SIAM Journal on Scientific Computing, Vol. 38, No. 3, pp. 1454–1482, 2016.

[5] N. Halko, P. G. Martinsson, and J. A. Tropp, Finding structure with randomness: probabilistic algorithms for constructing approximate matrix decompositions, SIAM Review, Vol. 53, No. 2, pp. 217–288, 2011.

[6] A. Hochman, Fast singular-value decomposition of Loewner matrices for state-space macromodeling, 2015 IEEE 24th Conference on Electrical Performance of Electronic Packaging and Systems (EPEPS), San Jose, US, 25–28 October 2015.