Krylov methods for Hermitian nonlinear eigenvalue problems

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We consider the Hermitian nonlinear eigenvalue problem (NEP) of the type: given $M : \mathbb{C} \to \mathbb{C}^{n\times n}$ a holomorphic function such that $M(\lambda)^H = M(\bar{\lambda})$, determine $(\lambda, v) \in \mathbb{C} \times (\mathbb{C}^n \setminus \{0\})$ such that

$$M(\lambda)v = 0. \tag{1}$$

The vector v is called eigenvector and the scalar λ eigenvalue. Hermitian standard eigenvalues problems are very well studied in both theoretical and algorithmic aspects and they can be efficiently solved with variations of the Lanczos algorithm. Certain nonlinear eigenvalue problems can be reduced to standard eigenvalue problems (or larger size) with linearizations techniques. More specifically, Hermitian polynomial and rational eigenvalue problems can reduced to Hermitian standard eigenvalue problem by means for structured linearizations [1, 2].

Analogous to this, we reduce the Hermitian nonlinear eigenvalue problem in the more general form (1) to an infinite dimensional linear eigenvalue problem that preserves the Hermitian structure. We derive and analyse a variation of the Lanczos method that can be applied to this infinite dimensional linear eigenvalue problem. The resulting method is an iterative procedure that in each iteration dynamically expand an Hermitian linearization and compute a basis of the Krylov space with a short-term recurrence. We illustrate how the performances benefit by exploiting the Hermitian structure of the problem.

References

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