Assignment 6.

Latest due date: Sunday, March 3rd.

Note: please make sure to **include all figures and final results** in the **PDF report!**

Task 1. **(a)** Write an algorithm which implements the trigonometric interpolating function $P_n(t)$ based on the formula provided in class.

(b) Find the order 10 trigonometric interpolating function for $f(t) = e^t$ for points $(j/10, f(j/10))$ for $j = 0, \ldots, 9$. Include in your report
1. the trigonometric polynomial $P_{10}(t)$,
2. a plot of the data points and $P_{10}(t)$ on the same figure.

Task 2. Program two different quadrature rules to compute the integral $\int_a^b f(x)\mathrm{dx}$ of a function f, over a given interval $[a, b]$ and a given number of steps n. Use:
a) the following three point composite Simpson rule,

$$\int_a^b f(x)\mathrm{dx} \approx \frac{h}{6}\left[f(a) + 4f(a + h/2) + \sum_{j=1}^{n-1} 2f(x_j) + 4f(x_j + h/2) + f(b)\right]$$

with error $\frac{(b-a)}{2880}h^4 f^{(4)}(\xi)$, where $h = (b - a)/n$ and $x_j = a + jh$ and
b) the following 3-point composite Gauss rule,

$$\int_a^b f(x)\mathrm{dx} \approx \frac{h}{18}\sum_{j=0}^{n-1}\left[5f\left(x_j + \frac{5-\sqrt{15}}{10}h\right) + 8f(x_j + \frac{5}{10}h) + 5f\left(x_j + \frac{5+\sqrt{15}}{10}h\right)\right].$$

where $h = h = (b - a)/n$ and $x_j = a + jh$.
Advice: Test on non-polynomial functions for which you know the exact integral.
1. What is the observed **order of convergence** of each of these quadrature rules?
2. Include in your report the error of approximation from Task 1 (b): $|f(t) - P_{10}(t)|$ using the formula from above which has the better order of convergence.

Task 3. Derive on paper (no computer) a Gaussian quadrature formula which has degree of precision (ADA) 5. Show that your formula has indeed ADA 5.