Assignment 4.
Latest due date: Sunday, February 17th

Note: please make sure to include all figures and final results in the PDF report!

1. Suppose you need to solve the equation $A^3x = b$, where $A$ is nxn and invertible.
Which of the following options is likely to take the least computer time?

- Use QR factorization - also suggest how you would apply it.
- Use LU factorization - also suggest how you would apply it.
- Compute $A^{-1}$ first, followed by $x = A^{-1}(A^{-1}b))$.

Substantiate your answer in your report for each case and provide an approximate operations count.

2. The age distribution in a population of female beetles can be modeled with the matrix $A$ provided below which shows survival rates on a year by year basis. Let $x_k$ denote the distribution of ages in year $k$, with $x_{k,1}$ standing for the number of beetles of age one in year $k$, and so on. Then the relation between populations in successive years can be expressed as $x_{k+1} = Ax_k$ where

$$A = \begin{pmatrix} 4 & 1 & 4 \\ 4 & 0 & 0 \\ 1 & 0 & 3 \end{pmatrix}.$$ 

If after five years the beetle population has the distribution $(300, 60, 30)$, what was the original distribution? Outline the methodology you used in your report as well.

3. Suppose that the distance of a parachutist’s descent $s$ as a function of time $t$ can be modeled by $s = at + bt^2e^{-0.1t}$. Provide the methodology of your solution in your report and find values of $a$ and $b$ that are reasonable in view of the data in the table below.

<table>
<thead>
<tr>
<th>$t$</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td>30</td>
<td>83</td>
<td>126</td>
<td>157</td>
<td>169</td>
<td>190</td>
</tr>
</tbody>
</table>

4. (a) Solve the sytem below graphically and provide the solution and the plot in your report.

$$0.77x_1 + x_2 = 14.25$$
$$1.2x_1 + 1.7x_2 = 20$$

(b) Approximate the eigenvalues of the system above and from those compute an approximate condition number. Write all answers in your report.