Assignment 4.

Latest due date: Sunday, February 17th

Note: please make sure to **include all figures and final results** in the **PDF report!**

1. Suppose you need to solve the equation $A^3x = b$, where A is nxn and invertible. Which of the following options is likely to take the least computer time?

   - Use QR factorization - also suggest how you would apply it.
   - Use LU factorization - also suggest how you would apply it.
   - Compute A^{-1} first, followed by $x = A^{-1}(A^{-1}(A^{-1}b))$.

   Substantiate your answer in your report for each case and provide an approximate operations count.

2. The age distribution in a population of female beetles can be modeled with the matrix A provided below which shows survival rates on a year by year basis. Let x_k denote the distribution of ages in year k, with $x_{k,1}$ standing for the number of beetles of age one in year k, and so on. Then the relation between populations in successive years can be expressed as $x_{k+1} = Ax_k$ where

$$A = \begin{pmatrix} 4 & 1 & 4 \\ 4 & 0 & 0 \\ 1 & 0 & 3 \end{pmatrix}.$$

   If after five years the beetle population has the distribution $(300, 60, 30)$, what was the original distribution? Outline the methodology you used in your report as well.

3. Suppose that the distance of a parachutist's descent s as a function of time t can be modeled by $s = at + bt^2e^{-0.1t}$. Provide the methodology of your solution in your report and find values of a and b that are reasonable in view of the data in the table below.

t	5	10	15	20	25	30
s	30	83	126	157	169	190

4. **(a)** Solve the sytem below graphically and provide the solution and the plot in your report.

$$\begin{aligned} 0.77x_1 + x_2 &= 14.25 \\ 1.2x_1 + 1.7x_2 &= 20 \end{aligned}$$

   (b) Approximate the eigenvalues of the system above and from those compute an approximate condition number. Write all answers in your report.