Assignment 3.

Latest due date: Sunday, February 10th

Note: please make sure to **include all requested figures and final results** in the **PDF report!**

1. Write a computer algorithm which computes and plots the natural cubic spline for the daily oil production data (in millions of barrels per day) in the world:

Year	1994	1995	1996	1997	1998	1999	2000	2001	2002
Barrels	67.052	67.908	69.503	71.924	73.340	72.063	74.569	74.507	74.065

   For full credit the report must include
   (a) the cubic spline polynomial for each section
   (b) the figure of your resulting cubic spline together with
   (c) the data points from the table above plotted on the same figure.

2. Obtain by hand the 3rd order Bézier curve for the following 4 control points:
   $(1,1), (2,3), (4,3)$ and $(6,4)$.
   Please provide in the report all details of your computations which are to be done by hand - not on the computer. The final polynomial must be simplified.

3. **(a)** Implement on the computer de Casteljau's algorithm and include in your report a plot of the control points provided below as well as the corresponding Bézier curves,

   (0,1) (0,1) (0,0) (0,0),
   (0,0) (0,1) (1,1) (1,0),
   (1,0) (1,1) (2,1) (2,0).

   (b) General question not related to the specific Bézier curves above:
   Are the second derivatives between Bézier curves equal with each other?
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4. Given data points $(x, y, z) = (0, 1, 3), (0, 1, 2), (1, 0, 3), (1, 0, 4), (1, 2, 6)$, find the equation of the plane $z = c_0 + c_1 x + c_2 y$ that best fits the data. For this problem you should start solving by hand and then use Matlab or a calculator to help you solve the resulting matrix system. Show all steps and all results in your report.

5. (computer) Let $x_1, \ldots, x_{11}$ be 11 equally spaced points in the interval $[2, 4]$ (including end points) and compute y from $y_i = 1 + x_i + x_i^2 + \cdots + x_i^d$. For each case below write-down the resulting polynomial in your report and answer the respective questions.
   (a) Given the data above use the normal equations to compute the best degree d polynomial, for $d = 5, 6$ and 8. Remark about the accuracy of the resulting polynomial coefficients. Is it influenced by d? Either way explain why yes or why not.
   (b) Now use QR factorization to compute the best degree d polynomial, for $d = 5, 6$ and 8. How many correct decimal places of the coefficients can be computed?