Assignment 2.

Latest due date: Sunday, February 3rd

1. **(a)** On paper (no computer), solve the system of equations $A\mathbf{x} = b$ below using LU factorization,
$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 1 & 3 \\ 6 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$
Make sure to show what is L what is U and the solution x.
Exactly how many **operations** did it take to produce the solution $\mathbf{x}$?
Does that number agree with theory?

   (b) Now perform, **by hand**, 2 iterations of the Jacobi iterative scheme and provide an approximation of the solution $\mathbf{x}$ starting from an initial value of $\mathbf{x}_0 = [0, 1, 1]^T$.
Note that $\mathbf{x}_0^T$ denotes the transpose of the vector $\mathbf{x}_o$. Thus $\mathbf{x}_0^T$ is really a column vector.

2. **(a)** On paper (no computer), use the Lagrange interpolation method to find a polynomial in the interval $[0, 3]$ that passes through the points (0,1), (2,3) and (3,0). Simplify as much as possible the resulting polynomial.

   (b) Calculate by hand the roots for the Chebyshev interpolating polynomial of the same degree as the Lagrange polynomial above in the (same) interval $[0, 3]$.
Note: do not compute the Chebyshev polynomial! Just compute its roots in that interval.

3. **(a)** Write an algorithm which computes and plots both the Lagrange (evenly spaced interpolation) and Chebyshev interpolation polynomials for $f(x) = e^{-x^2}$ in the interval $[-1, 1]$ for $n = 10$ and also for $n = 20$ (in total there will be 4 polynomials).

   (b) We now compute the actual error of the interpolating polynomials above.
Use a step size of 0.01 to plot the absolute error between the function $f(x)$ and each of the interpolating polynomials in the interval $[-1, 1]$.
Can Runge's phenomenon be observed in this problem?

4. Decide using analysis (no programming) and what you learned in class whether the equations below form a cubic spline. Explain fully!

   (a)
$$S(x) = \begin{cases} x^3 + x - 1 & x \in [0, 1] \\ -(x-1)^3 + 3(x-1)^2 + 3(x-1) + 1 & x \in [1, 2] \end{cases}$$

   (b)
$$S(x) = \begin{cases} 2x^3 + x^2 + 4x + 5 & x \in [0, 1] \\ (x-1)^3 + 7(x-1)^2 + 12(x-1) + 12 & x \in [1, 2] \end{cases}$$