# Assignment 1.
## Latest due date: January Sunday 27th

1. Consider the equation $x^4 = x^3 + 10$.

    (a) Find an interval $[a, b]$ of length one inside which the equation has a solution.

    (b) Starting with $[a, b]$, how many steps of the Bisection Method are required to calculate the solution within $10^{-10}$?

2. Write an algorithm for the bisection method and use it to calculate the solution of $\cos x = \sin x$ in the interval $[0, 1]$ within 6 correct decimal digits. Now also use `fzero` in matlab to solve the problem and compare with your solution.

3. Find three different ways to express each equation below as a fixed-point problem $x = g(x)$.

    (a) $x^3 - x + e^x = 0$

    (b) $3x^{-2} + 9x^3 = x^2$

4. Write an M-file for the Fixed-Point Iteration and use it to calculate the solution of each equation to 8 correct decimal digits.

    (a) $x^3 = 2x + 2$

    (b) $e^x + x = 7$

    (c) $e^x + \sin x = 4$

5. **a)** Apply Theorem 1.6 (see below) to show that $g(x) = \cos x$ will converge if we use the Fixed-Point Iteration. Is the same true for $g(x) = \cos^2 x$?
   **b)** Numerically approximate the fixed point to 6 correct decimal digits, and report the number of iterations needed. Discuss the convergence (based on your results and the expectations Theorem 1.6).

    **Theorem. 1.6.** *Assume that $g$ is continuously differentiable, that $g(r) = r$, and that $S = |g'(r)| < 1$. Then Fixed-Point Iteration converges linearly with rate $S$ to the fixed point $r$ for initial guesses sufficiently close to $r$.*

6. Write an algorithm which implements the Newton-Raphson method. Find a function $f$ of your own (not a function given in the class or in this assignment) and initial guess $x_0$ for which Newton-Raphson's method diverges.

7. Consider the function $f(x) = e^{\sin^3 x} + x^6 - 2x^4 - x^3 - 1$ on the interval $[-2, 2]$. Plot the function and find all three roots to 6 correct decimal digits using the Newton-Raphson method. Determine which roots converge quadratically and find the multiplicity of the roots that converge linearly. Use `fzero` and `fsolve` and compare.

8. Write an algorithm which implements the secant method. Apply it to each of the following equations with initial guesses $x_0 = 1$ and $x_1 = 2$. Discuss briefly the convergence (linear, quadratic, etc) of your results.

    (a) $x^3 = 2x + 2$

    (b) $e^x + x = 7$

    (c) $e^x + \sin x = 4$