Formelsamling


**Fysikaliska modeller**

Kontinuitetsekvationen

\[
\frac{\partial q}{\partial t} + \nabla \cdot \mathbf{j} = k.
\]

**Diffusion**

\[
\mathbf{j} = -D \nabla u,
\]

\[
\frac{\partial u}{\partial t} - D \Delta u = k.
\]  
(Allmännare \( \frac{\partial u}{\partial t} - \nabla \cdot (D \nabla u) = k \).)

**Värmeledning**

\[
\mathbf{j} = -\lambda \nabla u,
\]

\[
\frac{\partial u}{\partial t} - a \Delta u = \frac{a}{\lambda} k \quad \text{där} \quad a = \frac{\lambda}{\rho c^2}.
\]  
(Allmännare \( \rho c \frac{\partial u}{\partial t} - \nabla \cdot (\lambda \nabla u) = k \).)

**Elektrostatisk potential**

\[
\Delta u = -\frac{\rho}{\varepsilon \varepsilon_0}.
\]

**Svängande sträng och membran**

\[
\frac{\partial^2 u}{\partial t^2} - c^2 \Delta u = \frac{f}{\rho} \quad \text{där} \quad c^2 = \frac{S}{\rho}.
\]  
(Allmännare \( \rho \frac{\partial^2 u}{\partial t^2} - \nabla \cdot (S \nabla u) = f \).)

**Longitudinella svängningar**

\[
\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = \frac{f}{\rho_i} \quad \text{där} \quad c^2 = \frac{\alpha}{\rho_i}, \quad S = \alpha \frac{\partial u}{\partial x}.
\]

**Svängningar i gaser (ljud)**

\[
u = \frac{p - p_0}{p_0} \quad \text{(tryckstörning)},
\]

\[
\frac{\partial^2 u}{\partial t^2} - c^2 \Delta u = 0 \quad \text{där} \quad c^2 = \frac{\gamma p_0}{\rho_0}.
\]

För svängningar i gaser (ljud) gäller efter linjärisering att

\[
\begin{cases}
\frac{1}{\gamma} \frac{\partial \tilde{p}}{\partial t} + v_0 \frac{\partial \tilde{v}}{\partial x} = 0, \\
v_0 \frac{\partial \tilde{v}}{\partial t} + \frac{p_0}{\rho_0} \frac{\partial \tilde{p}}{\partial x} = 0,
\end{cases}
\]

\[
\tilde{p} = \gamma \tilde{\rho}.
\]

där \( \tilde{p} = \frac{p - p_0}{p_0} \) och \( \tilde{v} = \frac{v}{v_0} \).
Vektoranalys

Gauss formel \[ \int_{\Omega} \nabla \cdot u \, dV = \oint_{\partial \Omega} u \cdot dS. \]

Stokes formel \[ \int_{S} \nabla \times u \cdot dS = \oint_{\partial S} u \cdot d\mathbf{r}. \]

Greens formel I \[ \int_{\Omega} \nabla \cdot \nabla u \, dV = \oint_{\partial \Omega} u \, \partial n \, dS - \int_{\Omega} u \nabla \cdot \nabla \Delta V \, dV. \]

Greens formel II \[ \int_{\Omega} (u \nabla \Delta v - v \nabla \Delta u) \, dV = \oint_{\partial \Omega} (u \partial v \partial n - v \partial u \partial n) \, dS. \]

Laplaceoperatorn i cylindriska koordinater \[ \Delta = \frac{1}{r} \nabla \cdot \nabla = \nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}. \]

Laplaceoperatorn i sfäriska koordinater \[ \Delta = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial \varphi^2} \frac{1}{\sin^2(\theta)}, \]
\[ \Lambda = \frac{\partial}{\partial \rho} (1 - \rho^2) \frac{\partial}{\partial \rho} + \frac{1}{1 - \rho^2} \frac{\partial^2}{\partial \varphi^2} \text{ om } \rho = \cos(\theta), \]
\( \theta \text{ polardistans, } 0 < \theta < \pi, \varphi \text{ längdgrad, } 0 \leq \varphi < 2\pi. \)

Ortogonalutvecklingar \[ (u \mid v) = \int_{I} u(x) v(x) w(x) \, dx, \quad \|u\|^2 = (u \mid u). \]

Om \( (\varphi_j \mid \varphi_k) = 0 \), \( j \neq k \), så \( u = \sum c_k(u) \varphi_k \) med \( c_k(u) = \frac{(\varphi_k \mid u)}{\rho_k} \), där \( \rho_k = (\varphi_k \mid \varphi_k) \).

Parseval \[ (u \mid v) = \sum \frac{1}{\rho_k} (\varphi_k \mid u) (\varphi_k \mid v) = \sum \rho_k c_k(u) c_k(v). \]

Sturm-Liouville \[ Au = \frac{1}{w}(- \nabla \cdot (p \nabla u) + q u). \]

Speciella funktioner

Gammafunktionen och Betafunktionen \[ \Gamma(x) = \int_{0}^{\infty} t^{x-1} e^{-t} \, dt, \quad \Gamma(x + 1) = x \Gamma(x), \quad \Gamma(n + 1) = n!, \quad \Gamma(1/2) = \sqrt{\pi}, \]

\[ B(p, q) = \frac{\Gamma(p) \Gamma(q)}{\Gamma(p + q)}. \]
**Felfunktion/Error function**

\[
\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt, \quad \int_0^\infty e^{-t^2} \, dt = \frac{\sqrt{\pi}}{2}.
\]

**Beselfunktioner**

\[
e^{ir \sin(\theta)} = \sum_{n=-\infty}^{\infty} I_n(r) e^{i n \theta},
\]

\[
J_n(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i(nz - n\theta)} \, d\theta, \quad n \text{ heltal},
\]

\[
J_v(z) = \left(\frac{z}{2}\right)^v \sum_{k=0}^{\infty} \frac{1}{k! \Gamma(k+v+1)} \left( -\frac{z^2}{4} \right)^k, \quad v \neq -1, -2, \ldots
\]

**Bessels differentialekvation**

\[u'' + \frac{1}{r} u' + \left( \frac{\lambda - \nu^2}{r^2} \right) u = 0\]

har den allmänna lösningen

\[
\begin{cases}
  a J_v(\sqrt{\lambda} r) + b Y_v(\sqrt{\lambda} r) & \text{om } \lambda > 0, \\
  a r^\nu + b r^{-\nu} & \text{om } \lambda = 0, \nu \neq 0, \\
  a + b \ln(r) & \text{om } \lambda = \nu = 0.
\end{cases}
\]

**Normuttryck**

\[
\int_0^R \left| J_v \left( \frac{r}{R} \alpha_{nk} \right) \right|^2 r \, dr = \frac{R^2}{2} J_{v+1}(\alpha_{nk})^2 = \frac{R^2}{2} J_v'(\alpha_{nk})^2.
\]

**Nollställen till Beselfunktioner** \( J_n(x) \), \( J_n(\alpha_{nk}) = 0 \).

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<tr>
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**Nollställen till \( J_n'(x) \), \( J_n'(\alpha_{nk}) = 0 \).**

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<th>( n )</th>
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Sfäriska Beselfunktioner

Differentialekvationen

\[ u'' + \frac{2}{z} u' + \left( \lambda - \frac{\ell(\ell + 1)}{z^2} \right) u = 0 \]

har den allmänna lösningen

\[
\begin{cases}
    a j_{\ell}(\sqrt{\lambda} z) + b y_{\ell}(\sqrt{\lambda} z) & \text{om } \lambda > 0, \\
    a z^\ell + b z^{-\ell-1} & \text{om } \lambda = 0, \ell \neq -1/2, \\
    a + b \ln(z) & \text{om } \lambda = 0, \ell = -1/2,
\end{cases}
\]

där

\[ j_{\ell}(z) = \sqrt{\frac{\pi}{2z}} J_{\ell + 1/2}(z), \quad y_{\ell}(z) = \sqrt{\frac{\pi}{2z}} Y_{\ell + 1/2}(z). \]

Speciellt är

\[ j_0(z) = \frac{\sin(z)}{z}, \quad j_1(z) = \frac{\sin(z) - z \cos(z)}{z^2}, \]
\[ y_0(z) = -\frac{\cos(z)}{z}, \quad y_1(z) = -\frac{\cos(z) + z \sin(z)}{z^2}. \]

Legendrefunktioner

Legendrepolynomen \((P_{\ell})_0^\infty\) är ortogonala i \(L^2(I)\), \(I = (-1, 1)\).

Legendres differentialekvation

\[ \frac{d}{dx} \left( (1-x^2) \frac{du}{dx} \right) + \ell(\ell + 1) u = 0, \quad \ell = 0, 1, 2, \ldots \]

har allmänna lösningen

\[ a P_{\ell}(x) + b Q_{\ell}(x) \]

där \(Q_{\ell}\) ej är begränsad i \((-1, 1)\) och

\[ P_{\ell}(x) = \frac{1}{2\ell!} D^\ell (x^2 - 1)^\ell. \]

Rekursionsformel för Legendrepolynom:

\[ P_0(x) = 1, \quad P_1(x) = x, \quad P_{\ell+1}(x) = \frac{2\ell + 1}{\ell + 1} x P_\ell(x) - \frac{\ell}{\ell + 1} P_{\ell-1}(x). \]

Associerade Legendreekvationen

\[ \frac{d}{dx} \left( (1-x^2) \frac{du}{dx} \right) - \frac{m^2}{1-x^2} u + \ell(\ell + 1) u = 0 \]

har allmänna lösningen

\[ a P_{\ell}^m(x) + b Q_{\ell}^m(x) \]

där \(Q_{\ell}^m\) ej är begränsad och

\[ P_{\ell}^m = (1-x^2)^{m/2} D^m P_\ell(x). \]
Greenfunktioner
Fundamentallösningar till Laplaceoperatorn \((-\Delta K = \delta)\)

\[
K(x) = \frac{1}{2\pi} \ln|x| \quad \text{i } \mathbb{R}^2,
\]

\[
K(x) = \frac{1}{4\pi|x|} \quad \text{i } \mathbb{R}^3.
\]

Poissonkärnor

\[
P(r, \theta) = \frac{1}{2\pi} \frac{1 - r^2}{1 + r^2 - 2r \cos(\theta)} \quad \text{(enhetscirkeln)},
\]

\[
P(x, y) = \frac{1}{\pi} \frac{y}{x^2 + y^2} \quad \text{(halvplanet } y > 0).\]

Greenfunktion för Dirichlets problem

\[
\begin{cases}
-\Delta_x G(x, \alpha) = \delta_\alpha(x), & x \in \Omega, \\
G(x, \alpha) = 0, & x \in \partial \Omega.
\end{cases}
\]

Om \(-\Delta u = f \text{ i } \Omega, u = g \text{ på } \partial \Omega\) så

\[
u(x) = \int_\Omega G(x, \alpha)f(\alpha) \, dV_\alpha - \int_{\partial \Omega} \frac{\partial}{\partial n_\alpha}(x, \alpha)g(\alpha) \, dS_\alpha.
\]

Konjugerade punkter med avseende på cirkeln (sfären) \(|x| = \rho\)

\[
|\alpha| |\tilde{\alpha}| = \rho^2,
\]

\[
|x - \alpha| = \frac{|\alpha| |x - \tilde{\alpha}|}{\rho} \quad \text{då } |x| = \rho.
\]

Värmeledning

\[
\begin{cases}
G(x, t) = \frac{1}{\sqrt{4\pi at}} e^{-x^2/4at}, & x \in \mathbb{R}, \ t > 0, \\
\frac{\partial}{\partial t} G - \frac{\partial^2}{\partial x^2} G = 0, & x \in \mathbb{R}, \ t > 0, \\
G(x, 0) = \delta(x), & x \in \mathbb{R}.
\end{cases}
\]

Vågutbredning

d’Alembert

\[
\begin{cases}
u(x, t) = \frac{1}{2} \left(g(x + ct) + g(x - ct)\right) + \frac{1}{2c} \int_{x-ct}^{x+ct} h(y) \, dy, \\
g(x) = u(x, 0), \ h(x) = u_t(x, 0).
\end{cases}
\]

Karakteristiker

\[
\begin{cases}
a_{11} u_{xx}'' + 2a_{12} u_{xy}'' + a_{22} u_{yy}'' + F(x, y, u, u_x, u_y) = 0, \\
a_{11} dy^2 - 2a_{12} dx \, dy + a_{22} dx^2 = 0.
\end{cases}
\]

Kvasilinjära

\[
\begin{cases}
x \frac{\partial u}{\partial x} + \beta \frac{\partial u}{\partial y} = f, \\
u(x_0, y_0) = u_0(x_0, y_0), \text{ för } g(x_0, y_0) = 0,
\end{cases}
\]

\[
\begin{cases}
\dot{x} = x, \ x(0) = x_0, \\
\dot{y} = y, \ y(0) = y_0,
\end{cases}
\]

\[
\begin{cases}
\dot{z} = f, \ z(0) = u_0(x_0, y_0).
\end{cases}
\]
Fouriertransformer

\[ \mathcal{F} f(\xi) = \hat{f}(\xi) = \int_{-\infty}^{\infty} e^{-i\xi x} f(x) \, dx, \]

\[ (\mathcal{F}^{-1} \hat{f})(x) = f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\xi x} \hat{f}(\xi) \, d\xi. \]

Parsevals formel

\[ \int_{-\infty}^{\infty} f(x) g(x) \, dx = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\xi) \hat{g}(\xi) \, d\xi. \]

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<th>(\lambda \hat{f}(\xi) + \mu \hat{g}(\xi))</th>
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</tr>
<tr>
<td>(13)</td>
<td>(\Theta(x))</td>
<td>(\frac{1}{i} \mathcal{P}\left(\frac{1}{\xi}\right) + \pi \delta)</td>
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</tbody>
</table>

\[ \Theta(x) = \begin{cases} 
1, & x > 0, \\
0, & x \leq 0,
\end{cases} \quad \Theta' = \delta, \quad \text{sgn}(x) = \begin{cases} 
1, & x > 0, \\
-1, & x < 0,
\end{cases} \quad f(x) \delta = f(0) \delta, \quad f(x) \delta' = f(0) \delta' - f'(0) \delta. \]
Laplace transform

\[ \mathcal{L}f(s) = \mathcal{L}_{II}f(s) = \int_{-\infty}^{\infty} e^{-st} f(t) \, dt, \quad \alpha < \Re s < \beta, \quad s = \sigma + i\omega, \]

\[ f(t) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} e^{st} F(s) \, ds, \quad \alpha < \sigma < \beta, \]

\[ \mathcal{F}f(\omega) = \mathcal{L}_{II}f(i\omega), \]

\[ \mathcal{L}_{I}f = \mathcal{L}_{II}(\theta f). \]

<table>
<thead>
<tr>
<th>( \mathcal{L}_{II} \rightarrow )</th>
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<tbody>
<tr>
<td>( \lambda f(t) + \mu g(t) )</td>
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<tr>
<td>( f(at) )</td>
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<tr>
<td>( f(t - t_0) )</td>
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<tr>
<td>( e^{st} f(t) )</td>
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<td>( f'(t) )</td>
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<td>( tf(t) )</td>
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<td>( (f * g)(t) )</td>
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<td>( \theta(t)f'(t) )</td>
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<td>( \delta )</td>
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<td>( \theta(t) )</td>
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<td>( \theta(t) - 1 )</td>
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<tr>
<td>( t^k e^{st} \theta(t) )</td>
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<tr>
<td>( \sin(bt) \theta(t) )</td>
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<td>( \cos(bt) \theta(t) )</td>
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<tr>
<td>( e^{-t^2} )</td>
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<td>( t^{\alpha-1} \theta(t) )</td>
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<td>( \frac{</td>
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<tr>
<td>( \frac{1}{\sqrt{\pi t}} e^{-a^2/4t} \theta(t) )</td>
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</tbody>
</table>
Fourierserier

\[ f(t) = \sum_{k=\pm \infty} c_k e^{i k \omega t} = c_0 + \sum_{k=1}^\infty a_k \cos k \omega t + b_k \sin k \omega t, \quad \omega T = 2\pi, \]

\[ c_k = \frac{1}{T} \int_{\text{period}} e^{-i k \omega t} f(t) \, dt, \]

\[ a_k = \frac{2}{T} \int_{\text{period}} \cos(k \omega t) f(t) \, dt, \quad b_k = \frac{2}{T} \int_{\text{period}} \sin(k \omega t) f(t) \, dt, \]

\[ \begin{cases} a_k = c_k + c_{-k} , \\ b_k = i(c_k - c_{-k}) \end{cases} \]

\[ \begin{cases} c_k = \frac{1}{2}(a_k - ib_k) , \\ c_{-k} = \frac{1}{2}(a_k + ib_k) \end{cases} \]

Parsevals formel

\[ \frac{1}{T} \int_{\text{period}} f(t) g(t) \, dt = \sum_{k=\pm \infty} c_k(f) c_k(g), \]

\[ \frac{1}{T} \int_{\text{period}} |f(t)|^2 \, dt = \sum_{k=\pm \infty} |c_k|^2 , \quad \frac{1}{T} \int_{\text{period}} |f(t)|^2 \, dt = |c_0|^2 + \frac{1}{2} \sum_{k=1}^\infty (|a_k|^2 + |b_k|^2). \]

Halvperiodutvecklingar

\[ f(x) = c_0 + \sum_{k=1}^\infty \alpha_k \cos \left( \frac{k\pi}{L} x \right), \quad \beta_k \sin \left( \frac{k\pi}{L} x \right), \]

\[ \alpha_k = \frac{2}{L} \int_0^L f(x) \cos \left( \frac{k\pi}{L} x \right) \, dx, \quad \beta_k = \frac{2}{L} \int_0^L f(x) \sin \left( \frac{k\pi}{L} x \right) \, dx, \]

\[ c_0 = \frac{1}{L} \int_0^L f(x) \, dx. \]

Nägra trigonometriska formler

\[ \cos(x + \beta) = \cos(x) \cos(\beta) - \sin(x) \sin(\beta), \]

\[ \sin(x + \beta) = \sin(x) \cos(\beta) + \cos(x) \sin(\beta), \]

\[ \cos(x) + \cos(\beta) = 2 \cos \left( \frac{x + \beta}{2} \right) \cos \left( \frac{x - \beta}{2} \right), \]

\[ \cos(x) - \cos(\beta) = -2 \sin \left( \frac{x + \beta}{2} \right) \sin \left( \frac{x - \beta}{2} \right), \]

\[ \sin(x) + \sin(\beta) = 2 \sin \left( \frac{x + \beta}{2} \right) \cos \left( \frac{x - \beta}{2} \right), \]

\[ \sin(x) - \sin(\beta) = 2 \cos \left( \frac{x + \beta}{2} \right) \sin \left( \frac{x - \beta}{2} \right), \]

\[ a \cos(x) + b \sin(x) = c \cos(x - \gamma), \quad c = \sqrt{a^2 + b^2}, \quad \tan(\gamma) = b/a. \]