Formelsamling

Formelsamligen utgör bara ett stöd för minnet. Beteckningar förklaras sålunda ej. Ej heller anges förutsättningar för formlernas giltighet.

Fysikaliska modeller

Kontinuitetsekvationen

$$\frac{\partial q}{\partial t} + \nabla\cdot \boldsymbol{j} = k.$$

Diffusion

$$\boldsymbol{j} = -D\,\nabla u,$$

$$\frac{\partial u}{\partial t} - D\,\Delta u = k. \qquad \text{(Allmännare } \frac{\partial u}{\partial t} - \nabla\cdot(D\,\nabla u) = k.\text{)}$$

Värmeledning

$$\boldsymbol{j} = -\lambda\,\nabla u, \quad \mathrm{d}q = \rho\,c\,\mathrm{d}u,$$

$$\frac{\partial u}{\partial t} - a\,\Delta u = \frac{a}{\lambda}k \quad \text{där } a = \frac{\lambda}{\rho\,c}. \qquad \text{(Allmännare } \rho\,c\,\frac{\partial u}{\partial t} - \nabla\cdot(\lambda\,\nabla u) = k.\text{)}$$

Elektrostatisk potential

$$\Delta u = -\frac{\rho}{\varepsilon\,\varepsilon_0}.$$

Svängande sträng och membran

$$\frac{\partial^2 u}{\partial t^2} - c^2\,\Delta u = \frac{f}{\rho} \quad \text{där } c^2 = \frac{S}{\rho}. \qquad \text{(Allmännare } \rho\,\frac{\partial^2 u}{\partial t^2} - \nabla\cdot(S\,\nabla u) = f.\text{)}$$

Longitudinella svängningar

$$\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = \frac{f}{\rho_l} \quad \text{där } c^2 = \frac{\alpha}{\rho_l}, \quad S = \alpha\,\frac{\partial u}{\partial x}.$$

Svängningar i gaser (ljud)

$$u = \frac{p - p_0}{p_0} \quad \text{(tryckstörning)},$$

$$\frac{\partial^2 u}{\partial t^2} - c^2\,\Delta u = 0 \quad \text{där } c^2 = \frac{\gamma\,p_0}{\rho_0}.$$

För svängningar i gaser (ljud) gäller efter linjärisering att

$$\begin{cases} \dfrac{1}{\gamma}\dfrac{\partial \tilde{p}}{\partial t} + v_0\dfrac{\partial \tilde{v}}{\partial x} = 0, \\[2ex] v_0\dfrac{\partial \tilde{v}}{\partial t} + \dfrac{p_0}{\rho_0}\dfrac{\partial \tilde{p}}{\partial x} = 0, \\[2ex] \tilde{p} = \gamma\,\tilde{\rho}. \end{cases}$$

där $\tilde{p} = \dfrac{p - p_0}{p_0}$ och $\tilde{v} = \dfrac{v}{v_0}$.

Vektoranalys

Gauss formel $\displaystyle\int_{\Omega} \nabla\cdot \boldsymbol{u}\,\mathrm{d}V = \oint_{\partial\Omega} \boldsymbol{u}\cdot\mathrm{d}\boldsymbol{S}.$

Stokes formel $\displaystyle\int_{S} \nabla\times \boldsymbol{u}\cdot\mathrm{d}\boldsymbol{S} = \oint_{\partial S} \boldsymbol{u}\cdot\mathrm{d}\boldsymbol{r}.$

Greens formel I $\displaystyle\int_{\Omega} \nabla u\cdot\nabla v\,\mathrm{d}V = \oint_{\partial\Omega} u\frac{\partial v}{\partial \boldsymbol{n}}\,\mathrm{d}S - \int_{\Omega} u\,\Delta v\,\mathrm{d}V.$

Greens formel II $\displaystyle\int_{\Omega} (u\,\Delta v - v\,\Delta u)\,\mathrm{d}V = \oint_{\partial\Omega}\Big(u\frac{\partial v}{\partial \boldsymbol{n}} - v\frac{\partial u}{\partial \boldsymbol{n}}\Big)\,\mathrm{d}S.$

Laplaceoperatorn i cylindriska koordinater

$$\Delta = \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}$$

$$= \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}.$$

Laplaceoperatorn i sfäriska koordinater

$$\begin{aligned}
\Delta &= \frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{r^2}\Lambda = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} + \frac{1}{r^2}\Lambda \\
&= \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\Lambda, \\
\Lambda &= \frac{1}{\sin(\theta)}\frac{\partial}{\partial\theta}\sin(\theta)\frac{\partial}{\partial\theta} + \frac{1}{\sin^2(\theta)}\frac{\partial^2}{\partial\varphi^2}, \\
\Lambda &= \frac{\partial}{\partial s}(1-s^2)\frac{\partial}{\partial s} + \frac{1}{1-s^2}\frac{\partial^2}{\partial\varphi^2} \quad \text{om } s = \cos(\theta),
\end{aligned}$$

(θ polardistans, $0 < \theta < \pi$, φ längdgrad, $0 \leqslant \varphi < 2\pi$.

Ortogonalutvecklingar

$$(u\,|\,v) = \int_I \overline{u(x)}\,v(x)\,w(x)\,\mathrm{d}x, \qquad \|u\|^2 = (u\,|\,u).$$

Om $(\varphi_j\,|\,\varphi_k) = 0$, $j \neq k$, så $u = \sum c_k(u)\,\varphi_k$ med $c_k(u) = \dfrac{(\varphi_k\,|\,u)}{\rho_k}$, där $\rho_k = (\varphi_k\,|\,\varphi_k)$.

Parseval

$$(u\,|\,v) = \sum \frac{1}{\rho_k}\,\overline{(\varphi_k\,|\,u)}\,(\varphi_k\,|\,v) = \sum \rho_k\,\overline{c_k(u)}\,c_k(v).$$

Sturm-Liouville

$$\mathcal{A}u = \frac{1}{w}(-\nabla\cdot(p\,\nabla u) + q\,u).$$

Speciella funktioner

Gammafunktionen och Betafunktionen

$$\Gamma(z) = \int_0^\infty t^{z-1}\,\mathrm{e}^{-t}\,\mathrm{d}t, \qquad \Gamma(z+1) = z\,\Gamma(z), \qquad \Gamma(n+1) = n!, \qquad \Gamma(1/2) = \sqrt{\pi},$$

$$B(p,q) = \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)}.$$

Felfunktion/Error function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \mathrm{e}^{-y^2}\, \mathrm{d}y, \qquad \int_0^\infty \mathrm{e}^{-y^2}\, \mathrm{d}y = \frac{\sqrt{\pi}}{2}.$$

Besselfunktioner

$$\mathrm{e}^{\mathrm{i}r\sin(\theta)} = \sum_{-\infty}^{\infty} \mathrm{J}_n(r)\, \mathrm{e}^{\mathrm{i}n\theta},$$

$$\mathrm{J}_n(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathrm{e}^{\mathrm{i}(z\sin(\theta)-n\theta)}\, \mathrm{d}\theta, \quad n \text{ heltal},$$

$$\mathrm{J}_\nu(z) = \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{1}{k!\,\Gamma(k+\nu+1)} \left(-\frac{z^2}{4}\right)^k, \quad \nu \neq -1, -2, \ldots$$

Bessels differentialekvation

$$u'' + \frac{1}{r}u' + \left(\lambda - \frac{\nu^2}{r^2}\right)u = 0$$

har den allmänna lösningen

$$\begin{cases} a\,\mathrm{J}_\nu(\sqrt{\lambda}\,r) + b\,\mathrm{Y}_\nu(\sqrt{\lambda}\,r) & \text{om } \lambda > 0\,, \\ a\,r^\nu + b\,r^{-\nu} & \text{om } \lambda = 0,\ \nu \neq 0, \\ a + b\ln(r) & \text{om } \lambda = \nu = 0. \end{cases}$$

Normuttryck

$$\int_0^R \left|\mathrm{J}_\nu\!\left(\frac{r}{R}\alpha_{\nu k}\right)\right|^2 r\,\mathrm{d}r = \frac{R^2}{2}\,\mathrm{J}_{\nu+1}(\alpha_{\nu k})^2 = \frac{R^2}{2}\,\mathrm{J}'_\nu(\alpha_{\nu k})^2.$$

Nollställen till Besselfunktioner $\mathrm{J}_n(x)$, $\mathrm{J}_n(\alpha_{nk}) = 0$.

$k\backslash n$	0	1	2	3	4	5	6	7	8	9	10
1	2,405	3,832	5,136	6,380	7,588	8,771	9,936	11,086	12,225	13,354	14,475
2	5,520	7,016	8,417	9,761	11,065	12,339	13,589	14,821	16,038	17,241	18,433
3	8,654	10,173	11,620	13,015	14,372	15,700	17,004	18,288	19,554	20,807	22,047
4	11,791	13,324	14,796	16,223	17,616	18,980	20,321	21,641	22,945	24,234	25,509
5	14,931	16,471	17,960	19,409	20,827	22,218	23,586	24,935	26,267	27,584	28,887
6	18,071	19,616	21,117	22,583	24,019	25,430	26,820	28,191	29,546	30,885	32,212
7	21,212	22,760	24,270	25,748	27,199	28,627	30,034	31,423	32,796	34,154	35,500
8	24,352	25,904	27,421	28,908	30,371	31,812	33,233	34,637	36,026	37,400	38,762
9	27,493	29,047	30,569	32,065	33,537	34,989	36,422	37,839	39,240	40,628	42,004
10	30,635	32,190	33,716	35,219	36,699	38,160	39,603	41,031	42,444	43,844	45,232

Nollställen till $\mathrm{J}'_n(x)$, $\mathrm{J}'_n(\alpha'_{nk}) = 0$.

$k\backslash n$	0	1	2	3	4	5	6	7	8	9	10
1	0,000	1,841	3,054	4,201	5,317	6,416	7,501	8,578	9,647	10,711	11,771
2	3,832	5,331	6,706	8,015	9,282	10,520	11,735	12,932	14,115	15,287	16,448
3	7,016	8,536	9,969	11,346	12,682	13,987	15,268	16,529	17,774	19,005	20,223
4	10,173	11,706	13,170	14,586	15,964	17,313	18,637	19,942	21,229	22,501	23,761
5	13,324	14,864	16,347	17,789	19,196	20,575	21,932	23,268	24,587	25,891	27,182
6	16,471	18,015	19,513	20,972	22,401	23,804	25,184	26,545	27,889	29,219	30,534
7	19,616	21,164	22,672	24,145	25,590	27,010	28,410	29,791	31,155	32,505	33,842
8	22,760	24,311	25,826	27,310	28,768	30,203	31,618	33,015	34,397	35,764	37,118
9	25,904	27,457	28,978	30,470	31,938	33,385	34,813	36,224	37,620	39,002	40,371

Sfäriska Besselfunktioner

Differentialekvationen

$$u'' + \frac{2}{z}u' + \Big(\lambda - \frac{\ell(\ell+1)}{z^2}\Big)u = 0$$

har den allmänna lösningen

$$\begin{cases} a\,\mathrm{j}_\ell(\sqrt{\lambda}\,z) + b\,\mathrm{y}_\ell(\sqrt{\lambda}\,z) & \text{om } \lambda > 0, \\ a\,z^\ell + b\,z^{-\ell-1} & \text{om } \lambda = 0,\ \ell \neq -1/2, \\ \dfrac{a + b\ln(z)}{\sqrt{z}} & \text{om } \lambda = 0,\ \ell = -1/2, \end{cases}$$

där

$$\mathrm{j}_\ell(z) = \sqrt{\frac{\pi}{2z}}\,\mathrm{J}_{\ell+1/2}(z), \quad \mathrm{y}_\ell(z) = \sqrt{\frac{\pi}{2z}}\,\mathrm{Y}_{\ell+1/2}(z)\,.$$

Speciellt är

$$\mathrm{j}_0(z) = \frac{\sin(z)}{z}\,, \qquad \mathrm{j}_1(z) = \frac{\sin(z) - z\cos(z)}{z^2}\,,$$
$$\mathrm{y}_0(z) = -\frac{\cos(z)}{z}\,, \quad \mathrm{y}_1(z) = -\frac{\cos(z) + z\sin(z)}{z^2}\,.$$

Legendrefunktioner

Legendrepolynomen $(\mathrm{P}_\ell)_0^\infty$ är ortogonala i $\mathrm{L}_2(I)$, $I = (-1, 1)$.
Legendres differentialekvation

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big((1-x^2)\frac{\mathrm{d}u}{\mathrm{d}x}\Big) + \ell(\ell+1)\,u = 0, \quad \ell = 0, 1, 2, \ldots$$

har allmänna lösningen

$$a\,\mathrm{P}_\ell(x) + b\,\mathrm{Q}_\ell(x)$$

där Q_ℓ ej är begränsad i $(-1, 1)$ och

$$\mathrm{P}_\ell(x) = \frac{1}{2^\ell \ell!}\,\mathrm{D}^\ell (x^2-1)^\ell.$$

Rekursionsformel för Legendrepolynom:

$$\mathrm{P}_0(x) = 1\,, \quad \mathrm{P}_1(x) = x\,, \quad \mathrm{P}_{\ell+1}(x) = \frac{2\ell+1}{\ell+1}\,x\,\mathrm{P}_\ell(x) - \frac{\ell}{\ell+1}\,\mathrm{P}_{\ell-1}(x).$$

Associerade Legrendreekvationen

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big((1-x^2)\frac{\mathrm{d}u}{\mathrm{d}x}\Big) - \frac{m^2}{1-x^2}\,u + \ell(\ell+1)\,u = 0$$

har allmänna lösningen

$$a\,\mathrm{P}_\ell^m(x) + b\,\mathrm{Q}_\ell^m(x)$$

där Q_ℓ^m ej är begränsad och

$$\mathrm{P}_\ell^m = (1-x^2)^{m/2}\,\mathrm{D}^m\,\mathrm{P}_\ell(x).$$

Greenfunktioner

Fundamentallösningar till Laplaceoperatorn ($-\Delta \mathrm{K} = \delta$)

$$\mathrm{K}(\boldsymbol{x}) = -\frac{1}{2\pi}\ln|\boldsymbol{x}| \quad \text{i } \mathbb{R}^2,$$

$$\mathrm{K}(\boldsymbol{x}) = \frac{1}{4\pi|\boldsymbol{x}|} \quad \text{i } \mathbb{R}^3.$$

Poissonkärnor

$$\mathrm{P}(r,\theta) = \frac{1}{2\pi}\frac{1-r^2}{1+r^2-2r\cos(\theta)} \qquad \text{(enhetscirkeln)},$$

$$\mathrm{P}(x,y) = \frac{1}{\pi}\frac{y}{x^2+y^2} \qquad \text{(halvplanet } y>0\text{)}.$$

Greenfunktion för Dirichlets problem

$$\begin{cases} -\Delta_{\mathbf{x}}\mathrm{G}(\boldsymbol{x},\boldsymbol{\alpha}) = \delta_{\boldsymbol{\alpha}}(\boldsymbol{x}), & \boldsymbol{x}\in\Omega, \\ \mathrm{G}(\boldsymbol{x},\boldsymbol{\alpha}) = 0, & \boldsymbol{x}\in\partial\Omega. \end{cases}$$

Om $-\Delta u = f$ i Ω, $u = g$ på $\partial\Omega$ så

$$u(\boldsymbol{x}) = \int_\Omega \mathrm{G}(\boldsymbol{x},\boldsymbol{\alpha}) f(\boldsymbol{\alpha})\,\mathrm{d}V_{\boldsymbol{\alpha}} - \oint_{\partial\Omega} \frac{\partial \mathrm{G}}{\partial \boldsymbol{n}_{\boldsymbol{\alpha}}}(\boldsymbol{x},\boldsymbol{\alpha})\, g(\boldsymbol{\alpha})\,\mathrm{d}S_{\boldsymbol{\alpha}}.$$

Konjugerade punkter med avseende på cirkeln (sfären) $|\boldsymbol{x}| = \rho$

$$|\boldsymbol{\alpha}||\tilde{\boldsymbol{\alpha}}| = \rho^2,$$
$$|\boldsymbol{x}-\boldsymbol{\alpha}| = \frac{|\boldsymbol{\alpha}|}{\rho}|\boldsymbol{x}-\tilde{\boldsymbol{\alpha}}| \quad \text{då } |\boldsymbol{x}| = \rho.$$

Värmeledning

$$\begin{cases} \mathrm{G}(x,t) = \dfrac{1}{\sqrt{4\pi a t}}\,\mathrm{e}^{-x^2/4at}, & x\in\mathbb{R},\ t>0, \\ \dfrac{\partial \mathrm{G}}{\partial t} - a\dfrac{\partial^2 \mathrm{G}}{\partial x^2} = 0, & x\in\mathbb{R},\ t>0, \\ \mathrm{G}(x,0) = \delta(x), & x\in\mathbb{R}. \end{cases}$$

Vågutbredning

d'Alembert

$$\begin{cases} u(x,t) = \dfrac{1}{2}(g(x-ct)+g(x+ct)) + \dfrac{1}{2c}\displaystyle\int_{x-ct}^{x+ct} h(y)\,\mathrm{d}y, \\ g(x) = u(x,0),\ h(x) = u_t(x,0). \end{cases}$$

Karakteristikor

$$\begin{cases} a_{11}u''_{xx} + 2a_{12}u''_{xy} + a_{22}u''_{yy} + F(x,y,u,u_x,u_y) = 0, \\ a_{11}\,\mathrm{d}y^2 - 2a_{12}\,\mathrm{d}x\,\mathrm{d}y + a_{22}\,\mathrm{d}x^2 = 0. \end{cases}$$

Kvasilinjära

$$\begin{cases} \alpha\dfrac{\partial u}{\partial x} + \beta\dfrac{\partial u}{\partial y} = f, \\ u(x_0,y_0) = u_0(x_0,y_0), \quad \text{för } g(x_0,y_0) = 0, \end{cases} \qquad \begin{cases} \dot{x} = \alpha, & x(0) = x_0, \\ \dot{y} = \beta, & y(0) = y_0, \\ \dot{z} = f, & z(0) = u_0(x_0,y_0). \end{cases}$$

Fouriertransformer

$$\mathcal{F}f(\omega) = \hat{f}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} f(t)\, dt,$$

$$(\mathcal{F}^{-1}\hat{f})(t) = f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t} \hat{f}(\omega)\, d\omega.$$

Parsevals formel

$$\int_{-\infty}^{\infty} \overline{f(t)} g(t)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{\hat{f}(\omega)} \hat{g}(\omega)\, d\omega.$$

		$\xrightarrow{\mathcal{F}}$
(1)	$\lambda f(t) + \mu g(t)$	$\lambda \hat{f}(\omega) + \mu \hat{g}(\omega)$
(2)	$f(at)$	$\frac{1}{\lvert a\rvert}\hat{f}\left(\frac{\omega}{a}\right)$
(3)	$f(t-t_0)$	$e^{-it_0\omega}\, \hat{f}(\omega)$
(4)	$e^{i\omega_0 t}\, f(t)$	$\hat{f}(\omega-\omega_0)$
(5)	$f'(t)$	$i\omega \hat{f}(\omega)$
(6)	$t f(t)$	$i\frac{d}{d\,\omega}\hat{f}(\omega)$
(7)	$f * g(t)$	$\hat{f}(\omega)\, \hat{g}(\omega)$
(8)	$\delta(t)$	1
(9)	1	$2\pi\, \delta(\omega)$
(10)	$e^{-t}\, \theta(t)$	$\frac{1}{1+i\omega}$
(11)	$e^{-\lvert t\rvert}$	$\frac{2}{1+\omega^2}$
(12)	$\frac{1}{1+t^2}$	$\pi\, e^{-\lvert\omega\rvert}$
(13)	e^{-t^2}	$\sqrt{\pi}\, e^{-\omega^2/4}$
(14)	$\theta(t+1)-\theta(t-1)$	$2\frac{\sin(\omega)}{\omega}$
(15)	$\theta(t)$	$\frac{1}{i}\mathcal{P}\left(\frac{1}{\omega}\right) + \pi\, \delta(\omega)$

$$\theta(t) = \begin{cases} 1, & t > 0, \\ 0, & t < 0, \end{cases} \qquad \theta'(t) = \delta(t), \qquad \operatorname{sgn}(t) = \begin{cases} 1, & t > 0, \\ -1, & t < 0, \end{cases}$$

$$f(t)\, \delta(t) = f(0)\, \delta(t), \qquad f(t)\, \delta'(t) = f(0)\, \delta'(t) - f'(0)\, \delta(t).$$

Laplacetransformer

$$\mathcal{L}f(s) = \mathcal{L}_{II}f(s) = \int_{-\infty}^{\infty} \mathrm{e}^{-st} f(t)\,\mathrm{d}t, \qquad \alpha < \operatorname{Re} s < \beta, \quad s = \sigma + i\omega,$$

$$f(t) = \frac{1}{2\pi \mathrm{i}} \int_{\sigma - \mathrm{i}\infty}^{\sigma + \mathrm{i}\infty} \mathrm{e}^{st} F(s)\,\mathrm{d}s, \qquad \alpha < \sigma < \beta,$$

$$\mathcal{F}f(\omega) = \mathcal{L}_{II}f(\mathrm{i}\omega),$$

$$\mathcal{L}_I f = \mathcal{L}_{II}(\theta f).$$

$$\xrightarrow{\mathcal{L}_{II}}$$

(16)	$\lambda f(t) + \mu g(t)$	$\lambda F(s) + \mu G(s)$
(17)	$f(at)$	$\frac{1}{\lvert a \rvert} F\left(\frac{s}{a}\right)$
(18)	$f(t - t_0)$	$\mathrm{e}^{-t_0 s} F(s)$
(19)	$\mathrm{e}^{at} f(t)$	$F(s - a)$
(20)	$f'(t)$	$s\,F(s)$
(21)	$t\,f(t)$	$-\frac{d}{ds} F(s)$
(22)	$f * g(t)$	$F(s)\,G(s)$
(23)	$\theta(t) f'(t)$	$s\,\mathcal{L}_{II}(\theta f)(s) - f(0)$
(24)	$\delta(t)$	1
(25)	$\theta(t)$	$\frac{1}{s}, \quad \sigma > 0$
(26)	$\theta(t) - 1$	$\frac{1}{s}, \quad \sigma < 0$
(27)	$t^k\,\mathrm{e}^{at}\,\theta(t)$	$\frac{k!}{(s-a)^{k+1}}, \quad \sigma > \operatorname{Re}(a)$
(28)	$\sin(bt)\,\theta(t)$	$\frac{b}{s^2 + b^2}, \quad \sigma > 0$
(29)	$\cos(bt)\,\theta(t)$	$\frac{s}{s^2 + b^2}, \quad \sigma > 0$
(30)	e^{-t^2}	$\sqrt{\pi}\,\mathrm{e}^{s^2/4}$
(31)	$t^{\alpha - 1}\,\theta(t)$	$\frac{\Gamma(\alpha)}{s^{\alpha}}, \quad \operatorname{Re}(\alpha) > 0,\ \operatorname{Re}(s) > 0$
(32)	$\frac{\lvert a \rvert}{\sqrt{4\pi}} \frac{\mathrm{e}^{-a^2/4t}}{t^{3/2}}\,\theta(t)$	$\mathrm{e}^{-\lvert a \rvert \sqrt{s}}$
(33)	$\frac{1}{\sqrt{\pi t}}\,\mathrm{e}^{-a^2/4t}\,\theta(t)$	$\frac{\mathrm{e}^{-\lvert a \rvert \sqrt{s}}}{\sqrt{s}}$

Fourierserier

$$f(t) = \sum_{k=-\infty}^{\infty} c_k \, \mathrm{e}^{\mathrm{i}k\omega t} = c_0 + \sum_{k=1}^{\infty} a_k \cos k\omega t + b_k \sin k\omega t, \qquad \omega T = 2\pi,$$

$$c_k = \frac{1}{T} \int_{\text{period}} \mathrm{e}^{-\mathrm{i}k\omega t} \, f(t) \, \mathrm{d}t, \qquad \begin{cases} a_k = \dfrac{2}{T} \displaystyle\int_{\text{period}} \cos(k\omega t) f(t) \, \mathrm{d}t, \\ b_k = \dfrac{2}{T} \displaystyle\int_{\text{period}} \sin(k\omega t) f(t) \, \mathrm{d}t, \end{cases}$$

$$\begin{cases} a_k = c_k + c_{-k}, \\ b_k = \mathrm{i}(c_k - c_{-k}), \end{cases} \qquad \begin{cases} c_k = \dfrac{1}{2}(a_k - \mathrm{i}b_k), \\ c_{-k} = \dfrac{1}{2}(a_k + \mathrm{i}b_k). \end{cases}$$

Parsevals formel

$$\frac{1}{T} \int_{\text{period}} \overline{f(t)} g(t) \, \mathrm{d}t = \sum_{k=-\infty}^{\infty} \overline{c_k(f)} c_k(g),$$

$$\frac{1}{T} \int_{\text{period}} |f(t)|^2 \, \mathrm{d}t = \sum_{k=-\infty}^{\infty} |c_k|^2, \qquad \frac{1}{T} \int_{\text{period}} |f(t)|^2 \, \mathrm{d}t = |c_0|^2 + \frac{1}{2} \sum_{k=1}^{\infty} (|a_k|^2 + |b_k|^2).$$

Halvperiodutvecklingar

Cosinusserie

$$f(x) = c_0 + \sum_{k=1}^{\infty} \alpha_k \cos\Big(\frac{k\pi}{L} x\Big),$$

$$\alpha_k = \frac{2}{L} \int_0^L f(x) \cos\Big(\frac{k\pi}{L} x\Big) \, \mathrm{d}x,$$

$$c_0 = \frac{1}{L} \int_0^L f(x) \, \mathrm{d}x.$$

Sinusserie

$$f(x) = \sum_{k=1}^{\infty} \beta_k \sin\Big(\frac{k\pi}{L} x\Big),$$

$$\beta_k = \frac{2}{L} \int_0^L f(x) \sin\Big(\frac{k\pi}{L} x\Big) \, \mathrm{d}x,$$

Några trigonometriska formler

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta),$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta),$$

$$\cos(\alpha) + \cos(\beta) = 2\cos\Big(\frac{\alpha+\beta}{2}\Big)\cos\Big(\frac{\alpha-\beta}{2}\Big),$$

$$\cos(\alpha) - \cos(\beta) = -2\sin\Big(\frac{\alpha+\beta}{2}\Big)\sin\Big(\frac{\alpha-\beta}{2}\Big),$$

$$\sin(\alpha) + \sin(\beta) = 2\sin\Big(\frac{\alpha+\beta}{2}\Big)\cos\Big(\frac{\alpha-\beta}{2}\Big),$$

$$\sin(\alpha) - \sin(\beta) = 2\cos\Big(\frac{\alpha+\beta}{2}\Big)\sin\Big(\frac{\alpha-\beta}{2}\Big),$$

$$a\cos(\alpha) + b\sin(\alpha) = c\cos(\alpha - \gamma), \qquad c = \sqrt{a^2 + b^2}, \quad \tan(\gamma) = b/a.$$