

The mathematical manuscripts of Karl Marx

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Outline

This talk will shortly consider:

- Publishing details of the manuscripts



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- A brief history of differential calculus



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Publishing details

- About 1000 manuscript pages
- Partial edition in german/russian from 1933.
- Extended (sovietic) edition from 1968 commented by mathematicians.
- Translation to some european languages in the 70's
- Present english version (1983): Translation of 1968 edition.
- Lately: Marxist writers comment the text.

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Differential calculus

- Newton & Leibnitz introduce infinitesimal variations.
- D'Alembert introduces the finite difference
- Lagrange treats "analytic" functions
- Cauchy attempts the first formalization of the concept of limit
- Bolzano, Weierstrass: "modern" (current) version of the concept of limit and continuity.



Differential calculus II

- Uniform convergence of function series
- Fourier
- Lacroix, Cauchy, Moigno
- Abel
- Seidel

Robinson and non-standard analysis.



Differential calculus III

The problem:

1600-1900: The scientific conception of the world struggles to impose its views to that of preexistent alternative conceptions.

D'Alembert (1743): ..Up to the present...more concern has been given to enlarging the building than to illuminate the entrance, to raising it higher than to giving proper strength to the foundations...

Hegel (1812-6): ...For the infinitesimal calculus permits and requires modes of procedure which mathematics must wholly reject when operating with finite quantities...

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The manuscripts

Index

- On the concept of the derived function
- On the differential (including three drafts and a “final version” of this manuscript).
- On the history of differential calculus (including loose pages).

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On the concept of the derived function

- Marx works with examples.

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- A similar reasoning can be done for other elementary functions (exponentials, roots, Marx mentions also *log* and *trig*).
- Without the idea of limit, a *recipe* is needed to produce $g(x_0)$ starting from $p(x)$ without falling in "0/0".

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On the differential

Considerations on the relationship among:

$$\Delta f, \quad df, \quad \Delta x, \quad dx$$

and their quotes.

Includes reasoning around the idea of derivative of a product.

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On the history of differential calculus

- Newton & Leibnitz: Mystical calculus. $du dv$ is set equal to zero.
- D'Alembert: Rational calculus. The increment quote $\Delta f / \Delta x$.
- Lagrange: Pure algebraic caaculus. "...The whole problem is then resolved into finding (algebraic) methods of developing all kinds of functions of $x + h$ in integral ascending powers of $h...$ "
- Some notes on Taylor and McLaurin's formulæand their use in the theory of functions by Lagrange.

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Other mathematics in Marx

- In some of the drafts the expressions “limit”, “limit value” are found.
- Moigno is named on a bibliographic list.
- The concept of operational rule.
- The use of the equal sign.

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- Appearance of different equivalent axiom systems (relativism).
- Mathematics as a calculational tool (Marx, post modernism).



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