Operator Theory, Analysis and Mathematical Physics OTAMP 2006

Book of abstracts

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Continuous Analogs of Orthogonal Polynomials with Respect to Non-Positive Measures and Singular Schrödinger and Dirac Operators

Vadym Adamyan
Odessa National I.I.Mechnikov University, Ukraine

In this talk we consider real functions of limited variation $\omega_\pm(\lambda)$ on the real axis and the system of functions $e(r,\lambda)$ which are defined and bounded for almost $r \in (0,\infty)$ and satisfying the conditions:

1. $e(r,\lambda)$ admits the representation
$$e(r,\lambda) := e^{i\lambda r}\left(1 - \int\limits_0^r \Gamma_r(s) e^{-i\lambda s} ds\right)$$
with continuous $\Gamma_r(s)$;

2. $$\int\limits_{-\infty}^{\infty} \overline{e(r,\lambda)} e(r',\lambda) d\left[\frac{\lambda}{2\pi} + \omega_+(\lambda) - \omega_-(\lambda)\right] = 0,\ r \neq r',\ 0 < r, r' < \infty.$$

We assume that
$$\sigma(\lambda) := \frac{\lambda}{2\pi} + \omega_+(\lambda)$$
is non-decreasing and $\omega_-(\lambda)$ is a non-decreasing step function.

Following M.G. Krein we call functions of the system $\{e(r,\lambda)\}$ continuous analogs of orthogonal trigonometric polinomials.

Put
$$\begin{aligned} &u(r) = \operatorname{Re}\Gamma_r(r), && v(r) = \operatorname{Im}\Gamma_r(r), \\ &\varphi(r,\lambda) := \operatorname{Re}\left(e^{-i\frac{1}{2}\lambda r} e(r,\lambda)\right), && \psi(r,\lambda) := \operatorname{Im}\left(e^{-i\frac{1}{2}\lambda r} e(r,\lambda)\right), \quad \operatorname{Im}\lambda = 0. \end{aligned}$$

Then $\varphi(r,\lambda)$, $\psi(r,\lambda)$ satisfy the Dirac type system
$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \psi' \\ \varphi' \end{pmatrix} = \frac{\lambda}{2}\begin{pmatrix} \psi \\ \varphi \end{pmatrix} + V(r)\begin{pmatrix} \psi \\ \varphi \end{pmatrix},\ V(r) = \begin{pmatrix} v(r) & u(r) \\ u(r) & -v(r) \end{pmatrix}. \tag{1}$$

If $\omega_\pm(\lambda)$ are odd then
$$-\varphi'' + \left[-u' + u^2\right]\varphi = \frac{\lambda^2}{4}\varphi.$$

Let $V_0(r)$ be the potential of the system (1) in the case: $\omega_-(\lambda) \equiv \text{const}$. Considering $V_0(r)$ and $\sigma(\lambda)$ as known we describe a class of singular potentials $V(r)$ and $-u' + u^2$ appearing as a result of taking some $\omega_-(\lambda)$ from $\sigma(\lambda)$.

Inverse Problems on Graphs–Trees

Sergei A. Avdonin
University of Alaska Fairbanks, USA

The spectral inverse problem for the Sturm-Liouville operator on a planar graph is considered. We suppose that the graph is a tree (i.e., it does not contain cycles), and on each edge the Schödinger equation (with a variable potential) is defined. The Weyl matrix function is introduced through all but one boundary vertices. We prove that, the Weyl matrix function is uniquely determines the graph (its topology and the lengths of the edges together with potentials on them). If the topology of the graph is known, the lengths of the edges and potentials on them are uniquely determined by the diagonal terms of either the Weyl matrix function, the response operator or by the back scattering coefficients.

This talk reports on a joint work with Pavel Kurasov.

Multilinear Estimates in Backscattering Theory

Ingrid Beltiţă
Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Romania

Consider the Schrödinger operator $H_v = -\Delta + v$ in $\mathbb{R}^n$, where the dimension $n \geq 3$ is odd, and $v \in L^q_{\mathrm{cpt}}(\mathbb{R}^n;\mathbb{R})$, where $q > n$. By definition, the backscattering transform $B(v)$ of v is, modulo a smooth term, the real part of the Fourier transform of the backscattering amplitude of H_v; then $v \to B(v)$ is an entire analytic function in $L^q_{\mathrm{cpt}}(\mathbb{R}^n;\mathbb{C})$. The N:th term $B_N(v)$ in the Taylor expansion of $B(v)$ at $v = 0$ is given by a N-linear singular integral operator B_N. The main interest lies in the recovering of v, or the singularities of v, from $B(v)$, and this motivates the study of continuity properties of B_N in various (weighted) Sobolev spaces contained in L^q. It has been shown that there is a sequence r_N tending to infinity with N such that $B_N(v) \in H_{(r_N)}$ for every N when $v \in L^q_{\mathrm{cpt}}$. This motivates a careful study of the regularity properties of the individual terms $B_N(v)$. The presentation will focus mainly on regularity results for B_2 and B_3, and on the structure of the operator B_N.

This talk reports on a joint work with Anders Melin.

Scattering matrices and Weyl functions

Jussi Behrndt
TU Berlin, Germany

For a scattering system $\{A_\Theta, A_0\}$ consisting of selfadjoint extensions A_Θ and A_0 of a symmetric operator A with finite deficiency indices, the scattering matrix $\{S_\Theta(\lambda)\}$ and a spectral shift function ξ_Θ are calculated in terms of the Weyl function associated with a boundary triplet for A^*. The results are applied to singular Sturm-Liouville operators with scalar and matrix potentials, to Dirac operators and to Schrödinger operators with point interactions.

This is joint work with Mark M. Malamud and Hagen Neidhardt.

Uniqueness in Inverse Spectral Theory via Paley-Wiener Theorems

Christer Bennewitz
Lund University, Sweden

Extending the classical Borg-Marchenko uniqueness theorem for the inverse spectral theory of the one-dimensional Schrdinger equation $-u'' + qu = \lambda u$ to more general equations, of the form $-(pu')' + qu = \lambda wu$ or higher-order equations, seems difficult to do using the methods originally employed by Borg, Marchenko or other authors. I will show how such theorems may be proved by use of a novel idea involving Paley-Wiener type theorems for the generalized Fourier transforms associated with these equations.

On Asymptotics of Polynomial Eigenfunctions for Exactly-Solvable Differential Operators

Tanja Bergkvist
Department of Mathematics, Stockholm University

I will discuss asymptotic properties of zeros in families of polynomials satisfying certain linear differential equations. Namely, consider a linear differential operator $T = \sum_{j=1}^{k} Q_j D^j$ where $D = d/dz$ and the Q_j are complex polynomials in a single variable z. We are interested in the case when $\deg Q_j \leq j$ for all j, with equality for at least one j, and in particular $\deg Q_k < k$ for the leading term. Such operators are referred to as *degenerate exactly-solvable operators*. We show that for all such operators the root of the unique nth degree eigenpolynomial p_n with the largest absolute value tends to infinity when $n \to \infty$, as opposed to the non-degenerate case when $\deg Q_k = k$. Moreover we present an explicit conjecture and partial results on the growth of the largest root. Namely, let T be a degenerate exactly-solvable operator of order k, let j_0 be the largest j for which $\deg Q_j = j$ and denote by r_n be the largest modulus of all roots of the unique and monic nth degree eigenpolynomial p_n of T. Then

$$\lim_{n\to\infty} \frac{r_n}{n^d} = c_0,$$

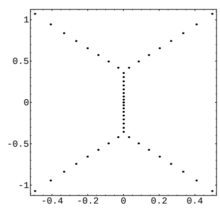
where $c_0 > 0$ is a positive constant and

$$d := \max_{j\in[j_0+1,k]} \left(\frac{j - j_0}{j - \deg Q_j} \right).$$

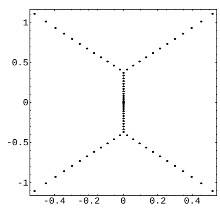
Based on this we introduce the scaled eigenpolynomial $q_n(z) = p_n(n^d z)$, where d is as above, which will have compactly supported zero distribution when $n \to \infty$. We deduce the algebraic equation satisfied by the asymptotic Cauchy transform of the appropriately scaled eigenpolynomials, and from this equation it is possible to obtain detailed information on the zero distribution.

Computer experiments indicate the existence of a limiting measure for the asymptotic zero distribution of the appropriately scaled eigenpolynomials. Conjecturally its support is the union of a finite number of analytic curves in the complex plane. Below a typical picture of the zero distribution of the scaled eigenpolynomial for a degenerate exactly-solvable operator.

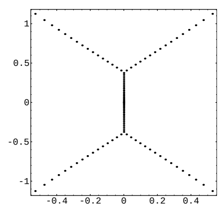
$T_3 = z^3 D3 + z^2 D4 + zD5$:



roots of
$q_{50}(z) = p_{50}(50^{1/2} z)$



roots of
$q_{75}(z) = p_{75}(75^{1/2} z)$



roots of
$q_{100}(z) = p_{100}(100^{1/2} z)$

A Lower Bound for Nodal Count on Discrete and Metric Graphs

Gregory Berkolaiko
Dept. of Mathematics, Texas A&M University, USA

According to a well-know theorem by Sturm, a vibrating string is divided into exactly n nodal intervals by zeros of its n-th eigenfunction. The Courant nodal line theorem carries over one half of Sturm's theorem for the strings to the theory of membranes: Courant proved that n-th eigenfunction cannot have more than n domains. A discrete analogue of Sturm's result (for discretizations of the interval) was discussed by Gantmacher and M.Krein.

Recently, it was discovered by Schapotschnikow that the nodal count for the Schrodinger operator on trees (where each edge is identified with an interval of the real line and some matching conditions are enforced on the vertices) is exact too: zeros of the n-th eigenfunction divide the tree into exactly n subtrees. We discuss two extensions of this result. One deals with the same continuous Schrodinger operator but on general graphs and another deals with discrete Schrodinger operator on combinatorial graphs.

The result that we derive applies to both types of graphs: the number of nodal domains of the n-th eigenfunction is bounded below by n-l, where l is the number of links that distinguish the graph from a tree (defined as the dimension of the cycle space or the rank of the fundamental group of the graph).

On the Classification of Hyperbolicity and Stability Preservers

Julius Borcea
Stockholm University, Sweden

A linear operator T on $\mathbb{C}[z]$ is called hyperbolicity-preserving or an HPO for short if $T(P)$ is hyperbolic whenever $P \in \mathbb{C}[z]$ is hyperbolic, i.e., it has all real zeros. One of the main challenges in the theory of univariate complex polynomials is to describe the monoid of all HPOs. This reputably difficult problem goes back to Pólya-Schur's characterization of multiplier sequences of the first kind, that is, HPOs which are diagonal in the standard monomial basis of $\mathbb{C}[z]$. Pólya-Schur's celebrated result (Crelle, 1914) generated a vast literature on this subject and related topics at the interface between analysis, operator theory and algebra but so far only partial results under rather restrictive conditions have been obtained.

In this talk I will report on the progress towards solutions to both this problem and its analog for stable polynomials as well as their multivariate extensions made in an ongoing series of papers jointly with Petter Brändén and Boris Shapiro.

Spectrum of a Periodic Operator Perturbed by a Small Nonself-Adjoint Operator

Denis I. Borisov
Nuclear Physics Institute, Czech Republic

The work is devoted to the study of the perturbation of the operator

$$\mathcal{H}_0 := -\frac{d}{dx}p(x)\frac{d}{dx} + q(x)$$

in $L_2(\mathbb{R})$ whose domain is $W_2^2(\mathbb{R})$. Here $p = p(x)$ is 1-periodic piecewise continuously differentiable real-valued function, $q = q(x)$ is 1-periodic piecewise continuous real-valued function, and

$$p(x) \geqslant p_0 > 0, \quad x \in \mathbb{R}.$$

Let Q be a some fixed interval in the real axis, $L_2(\mathbb{R}; Q)$ be the subset of the functions from $L_2(\mathbb{R})$ whose support lies in Q, $0 < \varepsilon \ll 1$ be a small parameter. By $\mathcal{L}\varepsilon$ we denote the family of linear operators acting from $W_{2,loc}^2(\mathbb{R})$ into $L_2(\mathbb{R}; Q)$ and meeting the uniform on ε estimate:

$$\|\mathcal{L}\varepsilon u\|_{L_2(Q)} \leqslant C\|u\|_{W_2^2(Q)}. \tag{2}$$

The operator in question is $\mathcal{H}\varepsilon := (\mathcal{H}_0 - \varepsilon\mathcal{L}\varepsilon)$ considered as an operator in $L_2(\mathbb{R})$ with the domain $W_2^2(\mathbb{R})$. We show that the operator $\mathcal{H}\varepsilon$ is closed.

The main aim of the work is to study the behaviour of the spectrum of the operator $\mathcal{H}\varepsilon$ as $\varepsilon \to 0$. We prove that the continuous spectrum of the operator $\mathcal{H}\varepsilon$ is independent on $\mathcal{L}\varepsilon$ and is the same as one of $\mathcal{H}_0$. As a result, we conclude that the continuous spectrum of the operator $\mathcal{H}\varepsilon$ has a band structure. At the same time, the operator $\mathcal{H}\varepsilon$ can have eigenvalues. We show that that the set of the eigenvalues of the operator $\mathcal{H}\varepsilon$ is at most countable. As $\varepsilon \to 0$, each eigenvalue of the operator $\mathcal{H}\varepsilon$ converges to one of the edges of the gaps in the continuous spectrum. We construct the asymptotic expansions for these eigenvalues of the operator $\mathcal{H}\varepsilon$, as well as the asymptotic expansions for the associated eigenfunctions.

Under some additional restrictions we also show that the continuous spectrum has no embedded eigenvalues. At the same time, without these restrictions the operator $\mathcal{H}\varepsilon$ can have eigenvalues embedded into the continuous spectrum. If exist, such eigenvalues should tend to infinity as $\varepsilon \to 0$. We provide the example of the operator $\mathcal{L}\varepsilon$ so that the operator $\mathcal{H}\varepsilon$ has an embedded eigenvalue which tends to infinity as $\varepsilon \to +\infty$.

This is joint work with R. Gadyl'shin.

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Singular Spectrum for Schrödinger Operators on Certain Sparse Trees

Jonathan Breuer
The Hebrew University of Jerusalem, Israel

We present examples of rooted tree graphs for which the Laplacian has singular continuous spectrum. For some of these models, we further establish fractional Hausdorff dimensions of all spectral measures. Moreover, we show for these models that the singular continuous components occur with arbitrarily high multiplicities. An interesting property of these trees is that they interpolate between the Bethe lattice and the line, in the sense that examples may be constructed with finite 'dimensions'. For the Anderson model on such trees we obtain complete (spectral) localization for all energies and all couplings for all finite dimensions.

Spectral Results for the p-Laplacian on the Half-Line

Malcolm B. Brown
Computer Science, Cardiff University, UK

A powerful method for obtaining information on eigenvalues of the Strum-Liouville problem on a compact or singular interval is provided by the Prufer transformation. In this talk I will show that these ideas can be translated to the p-Laplaian with potential function on the half-line in order to obtain information about the eigenvalues of that problem.

Spectral Properties and Semi-Classical Asymptotics for Shrödinger Equations on Quantum Graphs

Vsevolod L. Chernyshev
Moscow State Technical University n.a. N.E. Bauman, Russia

There are a number of works devoted to differential operators on geometrical graphs (see e.g. [1] and references there).

This report addresses two topics, eigenvalue problem for Shrödinger operator in semi-classical limit and description of Laplace kernels on quantum graph. One of the main results is an algorithm for constructing quantization rules analogues to the Bohr-Sommerfeld quantization rule. We consider second order differential operator on a geometrical graph and look for solution of the asymptotical eigenvalue problem. First of all we modify our graph by adding new vertices (turning points) and removing some edges, which may result in splitting of the net into several connected components. Then, using Maslov canonical operator (see [2]), we construct egienfunctions. The algorithm is illustrated by several

examples. The quantization rules may have standard form as described in [3] (in other words, look like an equality between integral and integer number plus index) or may be represented as a transcendental equation (general case). It is also possible that the spectrum covers the whole complex plane. An asymptotic eigenvalue that one can find using the algorithm, approximates the real eigenvalue of the original operator in the selfadjoint case.

The relationship between kernel of Laplace operator acting on k-forms and topological characteristics of a differentiable manifold without border is well known (see [4] and references there). It turns out that analogous properties are valid for nets. Namely, the following statement is correct for a compact geometrical graph without pendant vertices. Dimension of the kernel of Laplace operator acting on 0-forms defined on the geometrical graph is equal to the number of connected components of the net. In the selfadjoint case dimension of the kernel of Laplace operator acting on 1-forms defined on the geometrical graph is equal to the first Betti number. In the former case the proof simply arises from results presented in [1]. In the latter case it is based on topological properties of the graph. Note that close fact was described in a recent article devoted to the inverse spectral problem on quantum graph [5].

[1] Pokornyi Yu. V., Penkin O. M., Pryadiev V. L., Borovskih A. V., Lazarev K. P., Shabrov S. A. Differntial equations on geometrical graphs.//(in russian) – Moscow: Fizmatlit, 2004.

[2] Maslov V. P., Fedorjuk M. V. Semi-classical aymptotics for in quantum mechanis. – Reidel: Dordrecht 1981.

[3] (Collected works)/Editors: Bolsinov A. V., Fomenko A. T., Shafarevich A. I. Toplological methods in hamiltonian systems theory. //(in russian) Moscow.: Factrorial, 1998.

[4] Cycon, H. L., Froese, R. G., Kirsch, W., Simon, B., Schrodinger Operators with Application to Quantum Mechanics and Global Geometry, Springer, Berlin, 1987.

[5] Kurasov P., Nowaczyk M., Inverse spectral problem for quantum graphs.// J. Phys. A: Math. Gen. 38 p. 4901-4915, 2005.

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On the Wave Operator for the Sturm-Liouville Operators with Spectral Singularities

Evgeney V. Cheremnikh
Lviv Polytechnic National University, Ukraine

There are many works concerning the notion of absolutely continuous subspaces, separation of spectral composants, scattering theory for non-selfadjoints operators. Recall for example the works of Pavlov B. S., Naboko S. N. within well-known functional model.

We consider the wave operators $W_{\pm}(A, A_1)$ where the operators A, A_1 are generated in the space $L^2(0,\infty)$ by the expression $ly = -y''$ and non-local boundary conditions respectively $y(0) + (y,\eta)_{L^2(0,\infty)} = 0$ and $y(0) + (y,\eta_1)_{L^2(0,\infty)} = 0$ (cf. the condition in [1]). We suppose that the operators A and A_1 have the same set of spectral singularities (inclusively their multiplicities).

We start from the study of the exponential functions $\exp(itA)$ and $\exp(itA_1)$.

We are looking for the subspaces corresponding respectively to the operators A and A_1 which play role of the subspace of definition and the subspace of values of the wave operators $W_{\pm}(A, A_1)$. We want that this wave operator will be bounded (with his inverse operator).

As result we obtain that such subspace for the operator A depends on the operator A_1, so, it is natural to define the analogue of absolutely continuous subspace simultaneously for two operators A and A_1, but not for one isolated operator.

[1] Pavlov B. S., Spectral analysis of the differential operators with "spreaded" boundary condition, Probl. math. phys., 1973, n.6, 101–119.

Orthogonal Polynomials and Doubly Infinite Jacobi Matrices

Jacob S. Christiansen
Caltech, USA

We exploit the fact that orthogonal polynomials within the q-analogue of the Askey scheme not only satisfy a three- term recurrence relation but also are eigenfunctions of a second order q-difference operator. In particular, we develop the spectral theory for this operator on certain weighted L2-spaces corresponding to solutions to the associated moment problem.

The talk is based on joint work with E. Koelink, TU Delft.

Semiclassical Rresults in the Linear Response Theory

Monique Combescure
CNRS, France

We consider a quantum system of non-interacting fermions at temperature T, in the framework of linear response theory. We show that semiclassical theory is an appropriate tool to describe some of their thermodynamic properties, in particular through asymptotic expansions in $\hbar$ (Planck constant) of the dynamical susceptibilities. We show how the closed orbits of the one-particle classical motion in phase space manifest themselves in these expansions, in the regime where T is of the order of $\hbar$.

This is joint work with D. Robert

Mixed Spectra and Local Perturbations of Sturm-Liouville Operators

Rafael R. del Rio
IIMAS-UNAM, Mexico

We give conditions which imply equivalence of the Lebesgue measure with respect to a measure μ generated as an average of spectral measures corresponding to Sturm–Liouville operators in the half axis. We apply this to prove that some spectral properties of these operators hold for large sets of boundary conditions if and only if they hold for large sets of positive local perturbations.

This is joint work with O. Tchebotareva.

The Pseudo-Spectra of Systems of Semiclassical Operators

Nils Dencker
Lund University, Sweden

The pseudo-spectra (spectral instability) of operators is a topic of current interest in applied mathematics. In fact, for non-selfadjoint operators the resolvent could be very large far away from the spectrum, making the numerical computation of eigenvalues impossible. This has importance, for example, for the stability of flows and the onset of turbulence for solutions to the Navier-Stokes equations. The occurence of pseudo-spectra for semiclassical operators is due to the existence of quasi-modes, i.e., approximate local solutions to the eigenvalue problem. For scalar operators, the quasi-modes appear since the Nirenberg-Treves condition (Ψ) is not satisfied for topological reasons, see the paper of Dencker, Sjöstrand and Zworski in CPAM 2004. In this talk we shall explain how these result can be generalized to systems of semiclassical operators, for which new phenomena appear.

Quadratic Hamiltonians and Their Renormalization

Jan Dereziński
Warsaw University, Poland

I consider Hamiltonians acting on a bosonic Fock space given by a quadratic polynomial in the fields. If the number of degrees of freedom is infinite, the theory of such Hamiltonians is surprisingly rich. In particular, it may involve an infinite renormalization.

Jacobi Matrices: Eigenvalues and Spectral Gaps

Joanne M. Dombrowski
Wright State University, Dayton, Ohio USA

This talk will investigate the existence of spectral gaps, and eigenvalues within these spectral gaps, for Jacobi matrices obtained by specific types of oscillating perturbations of unbounded Jacobi matrices with smooth weights. Some results on absolute continuity for such operators will also be presented.

A Hardy-Inequality in Twisted Waveguides

Tomas Ekholm
University of Lisbon, Grupo de Fisica Matematica, Portugal

We show that twisting of an infinite straight three-dimensional waveguide with non-circular cross-section gives rise to a Hardy-type inequality for the associated Dirichlet Laplacian. As an application we prove certain stability of the spectrum of the Dirichlet Laplacian in locally and mildly bent tubes. Namely, it is known that any local bending, no matter how small, generates eigenvalues below the essential spectrum of the Laplacian in the tubes with arbitrary cross-sections rotated along a reference curve in an appropriate way. In the present paper we show that for any other rotation some critical strength of the bending is needed in order to induce a non-empty discrete spectrum.

A Uniqueness Result for Steady Symmetric Water Waves with Afinne Vorticity

Mats Ehrnström
Lund University, Sweden

Under reasonable hypotheses, the theory of steady water waves with affine vorticity is related to the study of eigenfunctions of the Laplacian in bounded domains in the plane. Combining that theory with sharp maximum principles for second-order elliptic operators, we prove uniqueness for symmetric water waves of this type. The result holds with or without surface tension and gravity.

L^p Boundedness of the Wave Operators for Schrödinger Operators with Threshold Singularities, Even Dimensional Case

Domenico Finco
Institut fur Angewandte Mathematik, Bonn, Germany

We consider the wave operators $W_\pm$ for a Schrödinger operator H in $\mathbf{R}^n$ with $n \geq 4$ even and we discuss the L^p boundedness of $W_\pm$ assuming a suitable decay at infinity of the potential V. The analysis depends on the singularities of the resolvent for small energy, that is if 0-energy eigenstates exist. If such eigenstates do not exist $W_\pm : L^p \to L^p$ are bounded for $1 \leq p \leq \infty$ otherwise this is true for $\frac{n}{n-2} < p < \frac{n}{2}$. Several applications of this result are presented.

Applications of Fractional Calculus to Nonlinear Evolution Equations.

Magnus Fontes
Lund University, Sweden

We will discuss a method to establish new types of apriori estimates to treat questions of existence and uniqueness of solutions to certain types of nonlinear evolution equations. In particular we will look at Burgers and Navier-Stokes equations with irregular forcing terms.

On Lieb-Thirring-Hardy Inequalities

Rupert Frank
KTH Stockholm, Sweden

We show that the Lieb-Thirring inequality on moments of negative eigenvalues remains true, with a possibly different constant, when the critical Hardy-weight is subtracted from the Laplace operator. A similar statement is true for fractional powers of the Laplacian, in particular relativistic Schrödinger operators.

The talk is based on joint works with T. Ekholm and with E. Lieb and R. Seiringer.

Trapped Modes in Elastic Media

Clemens Förster
IADM, University Stuttgart, Germany

We consider a linear elastic medium of the shape $R2 \times [0, \pi]$ with free boundary conditions in the limit case of vanishing Poisson coefficient. We show that under a local change of the material properties infinitely many eigenvalues arise in the essential spectrum which accumulate to a non-zero threshold. We give an estimate on the accumulation rate.

The Asymptotics of Orthogonal Polynomials with Unbounded Recurrence Coefficients

Jeffrey S. Geronimo
School of Mathematics, Georgia Tech, USA

We will use the theory of external fields and epsilon difference equations to obtain the asymptotics for orthogonal polynomials with unbounded recurrence coefficients. Time permitting we will also present a method for the construction of the orthogonality measure.

Some Applications of (Modified) Fredholm Determinants

Fritz Gesztesy
University of Missouri, USA

We intend to recall that several familiar objects in applied and mathematical physics, such as, Jost functions, Floquet discriminants, and Evans functions, are actually (modified) Fredholm determinants associated with appropriate Birman-Schwinger-type integral operators.

We will describe an abstract perturbation approach (essentially, due to Kato) and some elements of the Birman-Schwinger principle in connection with non-self-adjoint operators in Hilbert space, and then discuss some recent applications to spectral and scattering theory in connection with two- and three-dimensional Schrödinger operators. In particular, we will discuss multi-dimensional analogs of (ratios of) Jost functions in terms of modified Fredholm determinants.

Higher Derivatives of Spectral Functions Associated with One Dimensional Schrödinger Operators

Daphne J. Gilbert
Dublin Institute of Technology, Ireland

We investigate the existence and behaviour of higher derivatives of the spectral function, $\rho(\lambda)$, for one dimensional Schrödinger operators on the half-line with decaying potentials. In particular, we establish minimal sufficient conditions for the nth derivative of $\rho(\lambda)$ to exist and be continuously differentiable on $(0, \infty)$, and demonstrate that even if the minimal conditions fail to hold, an nth spectral derivative may still exist and be continuously differentiable for sufficiently large λ, and eventually approach the nth derivative of the spectral function for the free Hamiltonian as $\lambda \to \infty$. In suitable cases, the methodology of the proof also enables numerical upper bounds on turning points of lower derivatives of $\rho(\lambda)$ to be estimated.

This is joint work with B.J. Harris and S.M. Riehl.

The Dirichlet Problem for Quantum Graphs

Daniel Grieser
Carl von Ossietzky Universität Oldenburg, Germany

We consider a graph embedded with straight edges in Euclidean space. We look at the eigenvalue problem for the Laplace operator on the set given by the ϵ-neighborhood of the graph (a more general version involving manifold cross sections will also be considered). The question is how these eigenvalues behave as ϵ tends to zero. For Neumann boundary conditions, or if the cross sectional manifolds have no boundary, this problem was solved by various authors (in this generality by Exner and Post). We consider the Dirichlet problem, where the eigenvalues diverge. The exact rate of divergence can be determined. The Dirichlet problem behaves essentially different from the Neumann problem in some respects, in particular is the leading asymptotic behavior not determined by the underlying metric graph alone. We analyze this using a suitable rescaling and scattering theory on manifolds with cylindrical ends.

General Invariant Random Matrix Ensembles and Supersymmetry

Thomas Guhr
Matematisk Fysik (LTH), Lunds Universitet, Sweden

Random Matrix Theory (RMT) is a powerful tool to model statistical properties in a wide class of complex systems, including chaotic and mesoscopic systems, molecules, atoms, nuclei and even quantum chromodynamics. The correlation functions of classical RMT can be obtained by the orthogonal polynomial method. Many advanced issues in RMT are tackled with Supersymmetry. It has often been argued that Supersymmetry is restricted to Gaussian probability densities for the elements of the random matrices. We show that this is not so. We considerably extent the present supersymmetric formulations by extending the Hubbard-Stratonovich transformation. This also embeds the above mentioned orthogonal polynomial representations into much more general structures.

Optimal Lieb-Thirring Constants for an Exactly Solvable Magnetic Schrödinger Operator

Anders Hansson
KTH, Sweden

We explicitly compute the spectrum and eigenfuntions of a class of two-dimensional magnetic Schrödinger operators, $(i\nabla + \vec{A})^2 + V$, which describe a quantum particle interacting with a constant magnetic field and one of the Aharonov-Bohm type. There is a set of allowed degrees of homogeneity which V must possess if exact solutions are to exist. The spectral information thus obtained is used to determine sharp constants in the Cwikel-Lieb-Rozenblyum inequality. The constants turn out to be non-classical and depend on the fractional part of the Aharonov-Bohm flux. In the case of a pure Aharonov-Bohm field and quadratic potential, we prove that the Lieb-Thirring inequality holds for all exponents $\gamma \geq 1$ with the classical constant, just as one expects from the non-magnetic system, the harmonic oscillator. A numerical study of the interval $0 < \gamma < 1$ completes these results.

Potential Theory for the p-Parabolic Operator

Per-Anders Ivert
Lund Univeristy, Sweden

We derive a fundamental existence theorem for the equation

$$u_t'(x,t) - \operatorname{div}(|\nabla_x u|^{p-2}\nabla_x u) = 0$$

with continuous initial and boundary data in a cylinder and discuss the notion of p-superparbolic functions, thereby indicating the way for a potential theoretic treatment of the nonlinear parabolic operator under consideration.

KAM Method and Limit-Periodic Potentials

Yulia Karpeshina
UAB, USA

We consider the application of KAM (Kolmogorov-Arnold-Moser) method for spectral investigation of the Schrödinger operator $H = -\Delta + V(x)$ with a limit-periodic potential $V(x)$ in dimension two. We prove that the spectrum of H contains a semiaxis and there is a family of generalized eigenfunctions at every point of this semiaxis with the following properties. First, the eigenfunctions are close to plane waves $e^{i\langle\vec{k},\vec{x}\rangle}$ at the high energy region. Second, the isoenergetic curves in the space of momenta $\vec{k}$ corresponding to these eigenfunctions have a form of slightly distorted circles with holes (Cantor type structure).

Absence of Absolute Continuous Spectrum for One Dimensional Delone Hamiltonians

Steffen Klassert
Technische Universitaet Chemnitz, Germany

We discuss the almost sure absence of absolutely continuous spectrum for certain families of one dimensional Schrödinger operators on aperiodic Delone dynamical systems. The proof uses geometrical reasoning and Kotani theory which relates aperiodicity, the vanishing set of the Lyapunov exponent and the absolutely continuous spectrum.

Position Dependent Non-Linear Schrödinger Hierarchies: Involutivity, Commutation Relations, Renormalisation and Classical Invariants

Torbjörn Kolsrud
Royal Institute of Technology, Sweden

We consider a family of explicitly position dependent hierarchies $(I_n)_0^\infty$, containing the NLS (non-linear Schrödinger) hierarchy. All $(I_n)_0^\infty$ are involutive and fulfill $\mathsf{D}I_n = nI_{n-1}$, where $\mathsf{D} = D^{-1}V_0$, V_0 being the Hamiltonian vector field $v\frac{\delta}{\delta v} - u\frac{\delta}{\delta u}$ afforded by the common ground state $I_0 = uv$. The construction requires renormalisation of certain function parameters.

It is shown that the 'quantum space' $\mathbb{C}[I_0, I_1, ...]$ projects down to its classical counterpart $\mathbb{C}[p]$, with $p = I_1/I_0$, the momentum density. The quotient is the kernel of D. It is identified with classical semi-invariants for forms in two variables.

Scattering by Locally Deformed Quantum Wires

Sylwia M. Kondej
Institute of Physics, University of Zielona Gora, Poland

We consider the Hamiltonian in two dimensional system with singular potential supported by an infinite curve. The existence of wave operators is proven for the case of locally deformed curve. We also derive S-matrix for the above mentioned quantum system.

The results are common work with P. Exner.

Schrödinger Operators on Zigzag Periodic Graphs

Evgeny Korotyaev
Institut für Mathematik, Humboldt Universitat zu Berlin, Germany

Firstly, we consider the Schrodinger operator on the so-called zigzag periodic metric graph (a continuous version of zigzag nanotubes) with a periodic potential. The spectrum of this operator consists of an absolutely continuous part (intervals separated by gaps) plus an infinite number of eigenvalues with infinite multiplicity. We describe all compactly supported (localization) eigenfunctions with the same eigenvalue. We define a Lyapunov function, which is analytic on some Riemann surface. On each sheet, the Lyapunov function has the same properties as in the scalar case, but it has branch points, which we call resonances. We prove that all resonances are real. We determine the asymptotics of the periodic and anti-periodic spectrum and of the resonances at high energy. We show that there exist two types of gaps: 1) stable gaps, where the endpoints are periodic and anti-periodic eigenvalues, 2) unstable (resonance) gaps, where the endpoints are resonances (i.e., real branch points of the Lyapunov function). We obtain the following results from the inverse spectral theory: 1) we describe all finite gap potentials, 2) the mapping: potential – all eigenvalues is a real analytic isomorphism for some class of potentials.

Secondly, we turn on the constant magnetic field and study the corresponding spectral problems. We determine how eigenvalues, the spectral bands and gaps depend on the magnetic field, potentials and the graph.

This is the joint result with Igor Lobanov.

Discrete Spectrum of Schrödinger Operators on Regular Metric Trees

Hynek Kovařík
Stuttgart University, Germany

We study the connection between the discrete spectrum of a Schrödinger operator $-\Delta+\lambda V$ with a central potential V on regular metric trees and the geometry of the tree. In particular, we investigate the dependence of the bound states on the coupling constant λ when λ is small. We show how the corresponding asymptotic behavior of the bound states depends on the geometric parameters of the tree. We also discuss the asymptotic distribution of negative eigenvalues in the limit $\lambda \to \infty$.

Inverse Scattering, Asymptotic Properties of Reproducing Kernels and a Canonical System

Stanislav Kupin
University of Provence, France

We are interested in asymptotic properties of reproducing kernels coming from a special functional model. The model gives rise to a class of canonical systems containing Schrödinger (Sturm-Liouville) operators. The results are then applied to the inverse scattering for these systems.

This is a joint work with F. Peherstorfer, A. Volberg and P. Yuditskii.

Some Problems of Spectral Stability

Yoram Last
The Hebrew University of Jerusalem, Israel

The talk will discuss the stability of spectral properties of Schroedinger operators under decaying perturbation potentials. The primary focus will be on one-dimensional operators and preservation of absolutely continuous spectrum. The talk will review some known results, present new results, and discuss some open problems and conjectures.

Recent Progress in the Filter Diagonalization Technique

Tatiana Levitina
Institut Computational Mathematics, TU Braunschweig, Germany

The eigenfunctions of the Finite Fourier Transform (FFT) are employed as filtering functions in the Filter Diagonalization Method [M. R. Wall and D. Neuhauser, J. Chem. Phys. 102, 8011 (1995)], which serves for spectrum estimation of a Hamiltonian within a selected energy range. As a consequence and advantage the contribution of eigenstates outside the prescribe interval is completely suppressed. Also filtering with FFT eigenfunctions allows one to exactly determine the number of eigenstates located within the above interval. Further sampling with FFT egenfunctions simplifies the auxiliary calculations.

This is joint work with E. J. Brändas.

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Singular Differential Operators: Titchmarsh-Weyl m-Function and an Operator Model

Annemarie Luger
Berlin University of Technology, Germany

We explore the connection between a (generalized) Titchmarsh-Weyl-coefficient for a singular Sturm-Liouville operator and a certain singular perturbation of this operator.

This talk is based on joint work with Pavel Kurasov.

Boundary Controllability of the Wave Equation on Graphs

Victor S. Mikhaylov
University of Alaska, Fairbanks, USA

We study the boundary control problem for the wave equation on a planar graph. We suppose that the graph is a tree (i.e. it does not contain cycles) and on each edge a string equation with a variable density is given. The control is acting through the Dirichlet or Neumann boundary conditions applied to the exterior vertices. The exact controllability in L_2 classes of controls is proved and a sharp estimate on the time of controllability (which equal to the optical diameter of the graph) is obtained.

Let $\Omega \subset L_2$ be a finite connected planar graph with edges $\{e\} = E$ (intervals), vertices $\{v_1, \ldots, v_m\} = V$, boundary vertices $\{\gamma_1, \ldots, \gamma_n\} = \Gamma \subset V$. The graph is equipped with a density $\rho > 0$ and ρ is C_2-smooth function on each edge.

We associate the dynamical system with the graph

$$\begin{array}{l} \rho u_{tt} - u_{xx} = 0 \quad \text{in } \{\Omega \backslash V\} \times (0, T), \\ \sum_{e \sim v} u_e \mid_{x=v} = 0 \quad \text{for all } v \in V \backslash \Gamma, \\ u \mid_{t=0} = 0, \quad u_t \mid_{t=0} \quad \text{in } \Omega, \\ u = f \quad \text{on } \Gamma \times [0, T]. \end{array} \tag{3}$$

Here $(\cdot)_e$ is differentiation along the edge; the sum is taken over all edges incident to the vertex v; $(\cdot)_e \mid_v$ is the derivative along edge e in the direction from vertex v taken at v; $f = f(\gamma, t)$ is the boundary control. We denote $u^f(x, t)$ the solution of above boundary value problem.

The optical diameter of graph Ω is defined by the formula:

$$d(\Omega) = \max_{a,b \in \Gamma} \int_{\pi[a,b]} \sqrt{\rho} |dx|.$$

Here $|dx|$ is the length element on Ω, induced by $\mathbb{R}^2$ metric.

The main result is formulated as follows:

Theorem. *If $T \geq d(\Omega)$, then for any $y \in L_2(\Omega)$, $z \in H^{-1}(\Omega)$ there exist a control $f \in L_2(\Gamma \times [0,T])$ such that*

$$u^f(\cdot, T) = y, \quad u_t^f(\cdot, T) = z.$$

In the case of Neumann boundary control

$$u_e = f \quad \text{on } \Gamma \times [0,T], \quad f \in L_2(\Gamma, [0,T]),$$

we prove that the system is controllable in the space $H^1(\Omega) \times L_2(\Omega)$.

Some Topics from the Quantum Graphs Theory

Stanislav A. Molchanov
University of North Carolina at Charlotte, USA

The talk will contain the discussions of the three closely related topics. The transition from the stationary wave (Helmholtz) equation inside the network of the thin waveguides with Dirichlet boundary conditions to the quasi 1D equations on the limiting graph (with the appropriate gluing condition (GC) on the branching points). The approximation of the scattering on the junction of network by the scattering on the effective potential for the quantum graph with the Kirchhoff's GC. Non-linear waves on the quantum graphs.

Spectral Analysis of Jacobi Matrices with Some Essential Oscillations

Marcin Moszyński
Faculty of Mathematics, Warsaw University, Poland

We study spectral properties of some Jacobi matrices with coefficients possessing "essential" oscillations. In some regions of the real line we prove the absolute continuity or the pure pointness (possibly the discreteness) of the spectrum. We use subordination theory methods of Gilbert, Khan and Pearson and some new discrete versions of Levinson type theorems on asymptotic behaviour of solutions of linear difference equations systems. We try also to answer some questions concerning the spectral character in the other regions, where these theorems do not work.

The results presented here are partially included in the paper *New discrete Levinson type asymptotics of solutions of linear systems* by Jan Janas and M.M. (*Journal of Difference Equations and Applications, Vol.12 No. 2, February 2006, pp. 133-163*)

New Classes of Jacobi Matrices with Absolutely Continuous Spectrum

Wojciech Motyka
Institute of Mathematics of the Polish Academy of Sciences, Poland

The talk will concern the question of absolutely continuous Jacobi matrices. Two different approaches to this question will be presented. The first approach uses a new result on asymptotic behavior of generalized eigenvectors. The second one employs ideas of recent paper by R.Szwarc.

This talk is based on a joint work with J.Janas.

Lax-Phillips Scattering Revisited

Hagen Neidhardt
WIAS-Berlin, Germany

The well-known relation between the scattering matrix of the Lax-Phillips scattering theory and the characteristic function of Foias and Sz.-Nagy found by Adamyan and Arov is extended to a scattering theory of singular perturbations which includes the usual ones as a special case. The extended Lax-Phillips scattering theory applies to open quantum systems as dissipative and quantum transmitting Schrödinger-Poisson systems. Crucial is the notion of the Strauss extensions of symmetric operators which corresponds to the notion of transparent boundary conditions in the theory of open quantum systems. The spectral properties of the extensions of a symmetric operator will be described with the help of boundary triplets and associated Weyl functions and it will be shown how the scattering matrix can be expressed in terms of the Weyl function.

Infinite-Dimensional Elliptic Coordinates and Some Inverse Spectral Problems

Andrey Osipov
Scientific-Research Institute for System Studies,
Russian Academy of Sciences, Russia

A number of the equations in classical mechanics is solvable by separation of variables. In particular, some Hamiltonian systems in the finite-dimensional case are integrable in elliptic Jacobi coordinates. In recent years a generalization of elliptic coordinates to the infinite case has been offered. We consider this generalization and indicate the connection of infinite-dimensional elliptic coordinates with some inverse spectral problems for infinite Jacobi matrices and Sturm-Liouville operators (two spectra inverse problems).

A Riemann-Hilbert Approach to Some Theorems on Toeplitz Operators and Orthogonal Polynomials

Jorgen S. Ostensson
Katholieke Universiteit Leuven, Belgium

I will show how Riemann-Hilbert techniques can be used to prove various results, some old and some new, in the theory of Toeplitz operators and orthogonal polynomials on the unit circle (OPUC's). I will mainly focus on a result concerning the approximation of the inverse of a Toeplitz operator by the inverses of its finite truncations.

The talk is based on joint work with Percy Deift.

Approximation by Point Potentials in the Norm Resolvent Sense

Kateřina Ožanová
Chalmers University of Technology, Sweden

We discuss Schrödinger operators perturbed by measures and the limit relations between them. For a large family of real-valued Radon measures m, including the Kato class, the operators $-\Delta + \varepsilon^2\Delta^2 + m$ tend to Schrödinger operator $-\Delta + m$ in the norm resolvent sense as ε tends to zero. On the other hand, weak convergence of finite measures m implies the norm resolvent convergence of the corresponding perturbed fourth-order operators, provided the dimension is smaller than four.

Thus the combination of both convergence results yields an approximation of Schrödinger operators and the presence of the fourth-order perturbation makes it possible to choose point potentials as the approximating measures. Since in that case the spectral problem is solvable, we obtain an alternative method for the numerical computation of the eigenvalues. We illustrate the approximation by numerical calculations of eigenvalues for one simple example of measure m.

This is joint work with Johannes Brasche. [math-ph/0511029]

Spectra of Schrödinger Operators on Equilateral Quantum Graphs

Konstantin Pankrashkin
Humboldt-University of Berlin, Germany

We provide a description of the spectra of Schrödinger operators on a class of quantum graphs in terms of the corresponding discrete Hamiltonians. This gives a rigorous justification of the de Gennes-Alexander correspondence between the tight-binding and quantum network spectra widely used in the physics literature and extend the use of Hill determinants for periodic Sturm-Liouville problems to more general structures. Using this correspondence we provide some global estimates for the spectral gaps, which hold independently of the graph structure.

Fitting of the Solvable Model for Helmholtz Resonator

Boris S. Pavlov
The University of Auckland, New Zealand

The Kirchhoff model gives a convenient Ansatz

$$\Psi(x,\nu,\lambda) = \Psi^N_{out}(x,\nu,\lambda + A_{out}\, G^N_{out}(x, x_{\Gamma_H}, \lambda)$$

for calculation of the scattered wave in the outer domain of the Helmholtz resonator, in terms of the scattered wave and the Green function of the Neumann Laplacian in the outer domain, with a pole x_Γ at the point-wise opening connecting the outer domain with the cavity. We suggest an explicit formula for the Kirchhoff coefficient A_{out}, and the corresponding formula for A_{int} in the cavity, based on construction of a solvable model for the Helmholtz resonator with narrow and short connecting channel.

A solvable model for Helmholtz resonator , with a relatively short $kH << 1$ and narrow $\delta/H < 1$ channel connecting the inner and outer domains , is suggested and fitted such that the Scattering matrix of the model gives a good for the resonator in a certain range of wave numbers k.

Current View at Classical Extremal Problems

Irina Peterburgsky
Suffolk University, USA

We introduce various operators over the classes of functions with codomains in normed linear spaces and study extremal problems for these operators. Several classical results for scalar-valued functions including well-known Landau coefficient problem have been generalized for function spaces and operators under consideration.

Various Types of Smoothness of Linearizations

Victoria Rayskin
International University Bremen, Germany

We discuss various types of smoothness of local linearizations near hyperbolic fixed point. First, we show that a hyperbolic diffeomorphism can be linearized in α-Holder class, with α being arbitrarily close to, but less than 1. Then, using a different technique, we prove that the linearizing homeomorphism in the Hartman–Grobman Theorem is differentiable at the fixed point (although the derivative is not necessarily continuous). Using the ideas of the last proof, we show that a linearizing homeomorphism has the C^1-dependence on perturbations of nonlinear terms of an underlying diffeomorphism. We also present an expression for the derivative of the linearization with respect to the perturbations of the nonlinear terms.

This is joint work with Misha Guysinsky and Boris Hasselblatt.

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Detection of Spectral Singularities via Asymptotics of Contractive Semigroups

Roman V. Romanov
Institute for Physics, St. Petersburg State University, Russia

The influence of spectral singularities on the asymptotical behaviour of contractive semigroups, Z_t, as large t, is studied. We focus on the problem of localisation of the singularities in terms of the asymptotics. In the case of finitely many singularities each of finite power order, we present a solution to this problem for the spectral singularities whose orders are closed to the maximal one. In this way, the role of eigenfunctions of the absolutely continuous spectrum corresponding to the singularities is revealed. In the case when the set of spectral singularities has rich structure, some upper estimates for local norms are obtained.

Products of Non-Stationary Random Matrices and Multiperiodic Equations of Several Scaling Factors

Jörg Schmeling
Lund University, Sweden

Let $\beta > 1$ be a real number and $M : \mathbb{R} \to \mathrm{GL}(\mathbb{C}^{\mathrm{d}})$ be a uniformly almost periodic matrix-valued function. We study the asymptotic behavior of the product

$$P_n(x) = M(\beta^{n-1}x) \cdots M(\beta x)M(x).$$

Under some condition we prove a theorem of Furstenberg-Kesten type for such products of non-stationary random matrices. Theorems of Kingman and Oseledec type are also proved. The obtained results are applied to multiplicative functions defined by commensurable scaling factors. We get a positive answer to a Strichartz conjecture on the asymptotic behavior of such multiperiodic functions. The case where β is a Pisot–Vijayaraghavan number is well studied.

Hyperbolic Solution Operators: Symmetry and Entropy

Achim Schroll
Lund University, Sweden

Heinz Kreiss pointed out the central role of symmetrizers in connection with well–posedness of hyperbolic problems. In particular, well–posedness does not depend on a bounded zero–order (source) term. It is only the leading part of the PDE operator that matters.

This picture changes completely, when the zero–order term turns unbounded like in relaxation problems. We review the concept of uniform well–posedness for relaxation problems and relate it to the existence of convex entropies. A uniformly well–posed system arising as a model of two–phase flow in oil recovery is discussed for which (despite of well–posedness) no quadratic entropy exists.

The talk presents joint work with Jens Lorenz at University of New Mexico, Albuquerque, USA.

Separable Degenerate Differential Operators with Parameters

Veli B. Shakhmurov
Engineering Faculty, Istanbul University, Turkey

The nonlocal boundary value problems for degenerate anisotropic partial differential-operator equations with the parameters

$$(L+\lambda)\,u=\sum_{k=1}^{n} t_k D_k^{[l_k]} u\,(x)+A_\lambda\,(x)\,u\,(x)+\sum_{|\alpha:l|<1}\prod_{k=1}^{n} t_k^{\frac{\alpha_k}{l_k}} A_\alpha\,(x)\,D^{[\alpha]} u\,(x)=f\,(x) \tag{4}$$

$$L_{kj}u = \alpha_{kj}u^{[m_{kj}]}\,(G_{k0})+\sum_{i=1}^{N_{kj}}\delta_{kji}u^{[m_{kj}]}\,(G_{ki})=0,\ j=1,2,\ldots,\ d_k, \tag{5}$$

$$L_{kj}u = \beta_{kj}u^{[m_{kj}]}\,(G_{kb})+\sum_{i=1}^{N_{kj}}\delta_{kji}u^{[m_{kj}]}\,(G_{ki})=0, j=d_k+1,\ldots,l_k,\ k=1,2,\ldots,n,$$

are studied, where

$$G = \{x=(x_1,x_2,\ldots,x_n)\,,\ 0<x_k<b_k,\}\,,\ \alpha=(\alpha_1,\alpha_2,\ldots,\alpha_n)\,,$$

$$D^{[\alpha]} = D_1^{[\alpha_1]}D_2^{[\alpha_2]}\ldots D_n^{[\alpha_n]},\ D_k^{[i]}=\left(\gamma_k\,(x_k)\,\frac{\partial}{\partial x_k}\right)^i,\ \int_0^{x_k}\gamma_k^{-1}\,(y)\,dy<\infty,$$

$$0 \le m_{kj}\le l_k-1, G_{k0}=(x_1,x_2,\ldots,x_{k-1},0,x_{k+1},\ldots,x_n)\,,$$

$$G_{kb} = (x_1, x_2, ..., x_{k-1}, b_k, x_{k+1}, ..., x_n), \; G_{ki} = (x_1, x_2, ..., x_{k-1}, x_{ki}, x_{k+1}, ..., x_n),$$
$$x_{ki} \in (0, b_k), k = 1, 2, ..., n, l = (l_1, l_2, ..., l_n), |\alpha : l| = \sum_{k=1}^{n} \frac{\alpha_k}{l_k};$$

where α_{kji}, β_{kji}, δ_{kji}, λ are complex numbers and t_k are positive parameters; $A_\lambda = A + \lambda$, A, $A_\alpha(x)$ for $x \in G$ are possible unbounded operators in Banach space E and γ_k are positive measurable functions on G. For $l_1 = l_2 = \cdots = l_n = 2m$ from (1) we obtain the following degenerate elliptic DOE with parameters

$$\sum_{k=1}^{n} t_k D_k^{[2m]} u(x) + Au(x) + \sum_{|\alpha|<2m} \prod_{k=1}^{n} t_k^{\frac{\alpha_k}{2m}} A_\alpha(x) D^\alpha u(x) = f(x).$$

Let $L_p(G; E)$ denote E-valued L_p space and $W_{p,\gamma}^{[l]}(G; E(A), E)$ denotes E-valued weighted Sobolev-Lions type spaces with norm

$$\|u\|_{W_{p,\gamma}^{[l]}(G;E(A),E)} = \sum_{k=1}^{n} \left\| D_k^{[l_k]} u \right\|_{L_p(G;E)} + \|Au\|_{L_p(G;E)} < \infty.$$

We say that the problem $(1)-(2)$ is L_p-separable, if for all $f \in L_p(G; E)$ there exists an unique solution $u \in W_{p,\gamma}^{[l]}(G; E(A), E)$ of the problem $(1)-(2)$ satisfying this problem almost everywhere and there exists a positive constant C independent on f, such that has the coercive estimate

$$\sum_{k=1}^{n} \left\| D_k^{[l_k]} u \right\|_{L_p(G;E)} + \|Au\|_{L_p(G;E)} \leq C \|f\|_{L_p(G;E)}.$$

The above estimate implies that if $f \in L_p(G; E)$ and u is the solution of the BVP's $(1)-(2)$ then all terms of the equation (1) belong to $L_p(G; E)$ (i.e. all terms are separable in $L_p(G; E)$). The principal parts of the appropriate generated differential operators are non self-adjoint. Several conditions for the maximal regularity and the fredholmness in Banach-valued L_p- spaces of these problems are given. These results permit us to establish that the inverse of corresponding differential operators belong to Schatten q- class. Some spectral properties of the operators are investigated. In applications, the existence, unique and maximal smoothness of solution of a degenerate reaction-diffusion system modelling photochemical generation and atmospheric dispersion of pollutants is established. Note that, there are many positive operators in different Banach spaces. Therefore, putting concrete Banach spaces instead of E and concrete positive differential, pseudo-differential operators or finite or infinite matrices for instance, instead of operator A on the differential-operator equations (1) we can obtained different class of separable and Fredholm BVP's for partial differential and pseudo-differential equations and its systems.

This is joint work with Aida M. Shahmurova.

Spectral Properties of Some Classes of Jacobi Matrices with Rapidly Growing Weights

Luis O. Silva P.
Departament of Mathematical and Numerical Methods IIMAS-UNAM, Mexico

We establish sufficient conditions for self-adjointness on a class of unbounded Jacobi operators defined by matrices with main diagonal sequence of very slow growth and rapidly growing off-diagonal entries. With some additional assumptions, we also prove that these operators have only discrete spectrum.

Dynamical Systems and Operator Algebras. A Fruitful Interplay

Sergei Silvestrov
Lund University, Sweden

Crossed product and generalised crossed product operator algebras incorporate both the properties of spaces and dynamics on them, and are central objects for quantum mechanics and quantum field theory. They are intimately connected to properties of composition operators and other closely related classes of operators, such as for example those given by Jacobi matrices.

This talk will be devoted to the fruitful interplay between dynamical systems and operator algebras and some connections with composition operators, moment problems, operator monotonicity, Jacobi matrices and topological dynamics.

Some Remarks on the Cesàro Operator Acting on Spaces of Analytic Functions

Anna-Maria Simbotin
Lund University, Sweden

This talk concerns the Cesàro operator acting on various spaces of analytic functions on the unit disc. The Cesàro operator has received a lot of attention recently. The remarkable fact that this operator is subnormal when acting on the Hardy space H^2 has lead to extensive studies of its spectral picture on other spaces of this type. I intend to present some of the methods that have been used to obtain information about the spectrum of the Cesàro operator acting on Hardy and Bergman spaces and give a unified approach to these problems which yields new results in this direction.

Spectral Theory for Orthogonal Polynomials with Perturbed Periodic Recursion Coefficients

Barry Simon
Caltech, USA

I will describe joint work with Damanik and Killip that proves analogs of the Denissov-Rakmanov theorem, Szego's theorem and the Killip-Simon theorem for perturbations about OPRL and OPUC with periodic recursion coefficients. In these results, approach to a single "free" limit is replaced by approach to an isospectral torus. A key ingredient is to relate this approach to the approach of an associated matrix valued polynomial family to the free case.

Spectral Phase Transition Example for Some Class of Jacobi Operators

Sergey A. Simonov
Physical Faculty of St. Petersburg State University, Russia

Self-adjoint unbounded Jacobi matrices with entries $q_n = n$ on the diagonal and weights $\lambda_n = c_n n$ are considered, where c_n is a 2-periodical sequence of real numbers. Spectral properties of such matrices are investigated. The parameter space $(c_1; c_2)$ is decomposed into several separate regions, where the spectrum is either purely absolutely continuous or discrete. This constitutes an example of the spectral phase transition of the first order. The lines where the spectral phase transition occurs are studied. The following main result holds: either the interval $(-\infty; \frac{1}{2})$ or the interval $(\frac{1}{2}; +\infty)$ is covered by the purely absolutely continuous spectrum. The remainder of the spectrum is of pure point type and supposedly discrete. The proof is based on finding asymptotics of generalized eigenvectors via the Birkhoff-Adams Theorem and application of the subordinacy theory. We consider the spectral recurrence relation and reduce it to a such form that the Birkhoff-Adams Theorem is applicable. This yields the asymptotics of even and odd components of generalized eigenvectors. Combining them properly we obtain the desired asymptotics of generalized eigenvectors. In the case when the absolutely continuous spectrum covers the interval $[\frac{1}{2}; +\infty)$, we establish the discreteness of the remainder of the spectrum via the Glazman lemma, which is applicable due to the semiboundedness of the operator in that situation. We prove this fact by the estimation of the quadratic form of the operator. Also the degenerate case is considered, which is the case when one of the parameters c_1, c_2 turns to zero. This constitutes another example of the spectral phase transition.

Orthogonality of Multi-Variable Polynomials

Jan Stochel
Jagiellonian University, Poland

Criteria for orthogonality of multi-variable polynomials with respect to a Hermitian (or positive definite) linear functional are formulated in terms of the three term recurrence relations modulo a proper ideal in the ring of complex multivariable polynomials. Sufficient conditions for orthogonality of multi-variable polynomials with respect to a finite positive Borel measure on an algebraic set are supplied. A class of ideals V, called ideals of type C, for which the three term recurrence relations modulo V automatically imply the existence of an orthogonalizing measure is distinguished. The class is shown to be wide enough to contain the ideals composed of all polynomials vanishing on algebraic sets of types A or B which are strongly related to multidimensional moment problems. An example of a non-zero proper ideal which is not of type C is exhibited.

Large Time Asymptotics for 2-Component KdV System

Vladimir V. Sukhanov
St. Petersburg Univ., Russia

2 -component KdV system is nonlinear integrable system which is connected to energy dependent Schroedinger operator. Corresponding inverse problem for such operator can be constructed with the help of Riemann-Hilbert problem. The asymptotic analysis of such Riemann-Hilbert problem gives large time asymptotics for solutions of 2-component KdV system.

Analysis on Fractals as Infinitesimal Limits of Quantum Graphs

Alexander Teplyaev
Department of Mathematics, Gakushuin Univ., Japan

We show that many fractals, such as the Sierpinski gasket, can be viewed as infinitesimal limits of sequences of quantum graphs in harmonic coordinates. In particular, the energy (Dirichlet) form, and the energy measure Laplacian are the limits of corresponding objects on the quantum graphs. We generalize several previously known results by defining sets with finitely ramified cell structure. In particular, we consider post-critically infinite fractals. We prove that if Kigami's resistance form satisfies certain assumptions, then there exists a weak Riemannian metric such that the energy can be expressed as the integral of the norm squared of a weak gradient with respect to an energy measure. Furthermore, we prove that if such a set can be homeomorphically represented in harmonic coordinates, then for smooth functions the weak gradient can be replaced by the usual gradient. We show that the gradient and Laplacian allow to define and study certain notions of Differential geometry on fractals, such as the de Rham cohomology, harmonic forms and the Hodge theorem. We also discuss the spectral theory on some self-similar fractals.

Trace Formulas for Jacobi Operators in Connection with Scattering Theory for Quasi-Periodic Background

Gerald Teschl
University of Vienna, Austria

We investigate trace formulas for Jacobi operators which are perturbations of quasi-periodic finite-gap operators using Krein's spectral shift theory.

Conservative Curved Systems (Inverse Problems)

Alexey S. Tikhonov
Taurida National University, Ukraine

We introduce conservative curved systems over multiply connected domains and study relationships of such systems with related notions of functional model, characteristic function, and transfer function. In contrast to standard theory for the unit disk, characteristic functions and transfer functions are essentially different objects. We study possibility to recover the characteristic function for a given transfer function. As the result we obtain the procedure to construct the functional model for a given conservative curved system.

Born-Oppenheimer-Type Approximations for Degenerate Potentials

Françoise Truc
Institut Fourier, France

We consider a semi-classical Schrödinger operator with a degenerate potential V(x,y) =f(x) g(y) , where g is assumed to be a homogeneous positive function of m variables , smooth outside 0, and f is a smooth and strictly positive function of n variables, with a minimum in 0.

The potential is degenerate in the sense that the Weyl formula may be not valid . However, in the case where f tends to infinity at infinity, the operator has a compact resolvent and we gave in a previous work the asymptotic behaviour, for small values of the small parameter h, of the number of eigenvalues less than a fixed energy .

In this talk , without assumptions on the limit of f, we give a sharp localization of the low eigenvalues, using a Born Oppenheimer approximation.

With a refined approach we localize also higher energies . In the case when the degree of homogeneity is not less than 2, we can even assume that the order of these energies is like the inverse power of the square of h.

Finally we apply the previous results to a class of potentials in R^d, $d \geq 2$, which vanish on a regular hypersurface.

This is a joint work with Abderemane Morame.

Lifschitz Tails for Schrödinger Operators with Random Breather-Type Potential

Ivan Veselic'
Fakultät für Mathematik, Technische Universität Chemnitz, Germany

We derive bounds on the integrated density of states of Schrödinger operators with a random, ergodic potential. The potential depends on a sequence of random variables, not necessarily in a linear way. An example of such a random Schrödinger operator is the breather model, as introduced by Combes, Hislop and Mourre. For these models we show that the the integrated density of states near the bottom of the spectrum behaves according to the so called Lifshitz asymptotics. This enables us to prove localisation in certain energy/disorder regimes.

We report on results obtained jointly with Werner Kirsch.

Steady Rotational Water Waves with Surface Tension

Erik Wahlén
Lund University, Sweden

I consider two-dimensional steady water waves governed by gravity and surface tension. In the irrotational case, that is, when the underlying current is uniform, this is a well-studied problem, and it is known that there exist both periodic, solitary and "generalized" solitary waves. I will discuss some new similar existence results for rotational waves, that is, waves with a non-uniform underlying current. The results are based on bifurcation theory.

Parseval Formula for Wave Equations with Dissipative Term of Rank One

Kazuo Watanabe
Department of Mathematics, Gakushuin Univ., Japan

We show Parseval formula (the spectral resolution) for the generator of wave equations with some dissipative terms of rank one. We deal with the following equation :

$$\partial_t^2 u(x,t) + \langle \partial_t u, \varphi \rangle_0 \varphi(x) - \partial_x^2 u(x,t) = 0, \quad (x,t) \in \mathbb{R} \times \mathbb{R}_+ \tag{6}$$

where $\partial_t = \partial/\partial t$, $\partial_x = \partial/\partial x$, $\varphi(x) \in L_s^2(\mathbb{R})$ for some $s > 1/2$ and $\langle \cdot, \cdot \rangle_0$ is the inner product of $L^2(\mathbb{R})$.

Moreover we resolve the solutions for that equations into sevaral modes by using Parseval formula.

This is joint work with M. Kadowaki(Ehime univ.) and H. Nakazawa(Chiba inst. Univ.)

Inverse Scattering at a Fixed Energy

Ricardo Weder
IIMAS-UNAM, Mexico

We discuss the inverse scattering problem at a fixed energy for the Schrödinger equation in $\mathbf{R}^d, d \geq 3$. We prove that the scattering amplitude at a fixed energy, and the electric potential and the magnetic field in the complement of a ball determine uniquely the electric potential and the magnetic field everywhere in $\mathbf{R}^d$.

Then, we prove that the forward singularity of the scattering amplitude uniquely determines the asymptotics at infinity of electric potentials and magnetic fields that are asymptotic sums of homogeneous terms.

The combination of both results prove that the scattering amplitude at a fixed energy uniquely determines electric potentials and magnetic fields that have a regular behaviour at infinity.

Part of this work was done in collaboration with Dimitri Yafaev.

On Inverse Problems for Trees

Rudi Weikard
UAB, USA

We prove that the Dirichlet to Neumann map of a finite simply connected tree determines uniquely the potential on the tree. This generalizes the fact that the Titchmarsh-Weyl m-function determines uniquely the potential on an interval. We also prove a generalization of the Borg-Levinson theorem, namely that the Dirichlet eigenvalues and the Neumann data of the Dirichlet eigenfunctions determine the potential. Here potentials may be complex-valued.

This is joint work with Malcolm Brown, Cardiff.

The Ornstein-Uhlenbeck Semigroup in Exterior Lipschitz Domains

Ian G. Wood
School of Mathematics, Cardiff University, UK

We consider operators of the form

$$\mathcal{L}u(x) := \Delta u(x) + Mx \cdot \nabla u(x), \quad x \in \Omega$$

in $L^p(\Omega)$, where $1 < p < \infty$, $M \in \mathbb{R}^{n\times n} \setminus \{0\}$. Using known results for the case $\Omega = \mathbb{R}^n$ and on maximal L^p-regularity for the Dirichlet-Laplacian in bounded Lipschitz domains, we show that $\mathcal{L}$ with a suitable domain is the generator of a quasi-contractive C_0-semigroup on bounded and exterior Lipschitz domains for a certain range of exponents p near 2. When the domain Ω satisfies a uniform exterior ball condition and $1 < p \leq 2$, the domain of the operator is given by

$$D(\mathcal{L}) := \{u \in W^{2,p}(\Omega) \cap W_0^1(\Omega) : Mx \cdot \nabla u \in L^p(\Omega)\},$$

and the semigroup satisfies L^p-L^q-estimates.

Limit Periodic Jacobi Matrices with a Singular Continuous Spectrum and the Renormalization of Periodic Matrices

Peter Yuditskii
Bar Ilan University, Israel

In a recent joint work with Franz Peherstorfer and Sasha Volberg we built a certain machinery that allows us to construct a wide class of limit periodic (special case of almost periodic) Jacobi matrices with a singular continuous spectrum. Roughly speaking, constructing -in a regular iterative way- a Cantor set (the spectrum of a limit periodic matrix) it is enough to follow this strategy: on each step the approximating set should have a form of an inverse polynomial image, or, in other words, it should be a spectrum of a periodic matrix. The above statement becomes a theorem if on each step we remove a sufficiently large part of the previous set.

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