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Perturbation theory for a class of close selfadjoint extensions

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Let operators A_1 and A_2 in a Hilbert space $\mathcal{H}$ be selfadjoint extensions of densely defined in $\mathcal{H}$ symmetric operators A' and A''. We call extensions A_1 and A_2 *close of the same class* if

- $(A_2 - z)^{-1} - (A_1 - z)^{-1}$ is a nuclear operator for some non-real z ;
- there is a joint selfadjoint extension A_0 in $\mathcal{H}$ of A' and A''.

Special features of perturbation theory for close extensions of the same class are discussed in this talk with illustrations for singularly perturbed Schrödinger operators in $\mathbf{L}^2(\mathbb{E}_3)$.

Feynman path integral and unharmonic oscillator

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New developments in the theory of infinite dimensional integrals are presented. The case of potentials of unharmonic type are given particular attention. Connection with evolution equations and semigroups are also discussed.

Orthogonal polynomials with discrete spectrum and converging recurrence coefficients

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Orthogonal polynomials on the real line satisfy a three-term recurrence relation

$$xp_n(x) = a_{n+1}p_{n+1}(x) + b_np_n(x) + a_np_{n-1}(x),$$

with $a_n > 0$ and initial conditions $p_0 = 1,\ p_{-1} = 0$. Rakhmanov [**4**] has shown that the recurrence coefficients for orthogonal polynomials on $[-1, 1]$ converge, with $a_n \to 1/2$ and $b_n \to 0$, whenever the orthogonality measure μ satisfies the Erdős-Turán condition, i.e., $\mu' > 0$ almost everywhere on $[-1, 1]$. The converse of Rakhmanov's result is not true: there exist measures on $[-1, 1]$ which are not absolutely continuous, but for which the orthogonal polynomials have recurrence coefficients for which $a_n \to 1/2$ and $b_n \to 0$. Delyon, Simon and Souillard [**1**] have given random examples of this, Lubinsky [**2**] gave a class of discrete measure on $[-1, 1]$ with converging recurrence coefficients, and later [**3**] showed that there are singularly continuous measures with converging recurrence coefficients. Finally Totik [**5**] has given a general construction for measures violating the Erdős-Turán condition but for which the recurrence coefficients converge. This talk is based on [**6**] where an explicit construction is made of a class of discrete measures, with jumps at zeros of Chebyshev polynomials, and the convergence rate of the recurrence coefficients is obtained by using sieved orthogonal polynomials.

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Asymptotic Formulas for Eigenvalues of the Multidimensional Schrödinger Operator

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We consider the d-dimensional Schrödinger operator $L(q(x))$, defined by the differential expression

$$Lu = -\Delta u + q(x)u$$

in d-dimensional paralellepiped F, with the Dirichlet boundary conditions

$$u|_{\partial F} = 0,$$

where ∂F denotes the boundary of the domain F, $x = (x_1, x_2, ..., x_d) \in R^d$, $d \geq 2$, $\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + ... + \frac{\partial^2}{\partial x_d^2}$ is the Laplace operator in R^d, and $q(x)$ is a real-valued function in $L_2(F)$.

Asymptotic formulas for the resonans and non-resonans eigenvalues of the operator $L(q(x))$ in an arbitrary dimension are obtained. We show that the eigenvalues of the operator $L(q(x))$ in the non-resonance domain are close to the eigenvalues of the unperturbed operator $L(0)$, and the eigenvalues in the resonance domain are close to the eigenvalues of the corresponding one-dimensional Sturm-Liouville operator.

Asymptotic formulas for the eigenvalues of the Polyharmonic operator $H(q(x)) = (-\Delta)^l + q(x)$, $l > \frac{1}{2}$ are discussed.

Spectral methods of nonunitary operators in correlation theory of discrete fields

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In this talk, we consider discret fields of the form: $X(n_1, n_2) = T_1^{n_1} T_2^{n_2} X_0$ (1) where T_1 and T_2 are quasiunitary contractions in Hilbert space H, $X_0 \in H$. Under spectral conditions on operators T_1 and T_2 , we give the general form of the correllation function $K((n_1, n_2), (m_1, m_2)) = \langle X(n_1, n_2); (m_1, m_2)\rangle_H$ and study his assymptotical behaviour. We also give in terms of correllation function the necessary and sufficient conditions for representation (1).

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Spectral theory of commutative Jacobi fields and its some applications

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The Jacobi field is a linear (with respect to parameter h belonging to real Hilbert space H) family A of selfadjoint commuting operators A(h) acting on a symmetric Fock space F(H) and having the Jacobi (i.e. three-diagonal) structure.The theorem on expansion in the generalized joint eigenvectors of family A is proved and corresponding spectral theory is constructed.Note, that the examples of Jacobi field are, in particular, the classical free field and Poisson field.The applications of this spectral theory to the random processes, to the theory of generalized functions of infinitely many variables and connected moment problems and to integration of some nonlinear equations are obtained.

Fredholm Operators and Bifurcations in Plateau Problem

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The problem to find out a two-dimensional minimal area surface with a given boundary was formulated at first by J. Lagrange in 1760. The study of minimal surfaces, critical points of area functional, was begun in the works of such authors as Laplace, Lagrange, Monge and Euler. Such a direction of variations calculus was called a "Plateau Problem" by the name of J. Plateau, who as one of the first observed minimal films in physical experiments in the 19th century. Detailed descriptions of the classical and modern results may be found in fundamental monographs by T. Rado, R. Courant, A. Fomenko and A. Tuzhilin, J. Jost and some others.

The effect of bifurcation of minimal surfaces families under the continuous deformation of its common boundary first was found in physical experiments with minimal films. The main well known examples of such bifurcations were found by J. Plateau in 1849. They are bifurcations of catenoids and helicoids families in the periodic boundary-valued problem for area functional. In author's works the general method to the study of bifurcations in Plateau problem was developed. It is based on the Crandall-Rabinowitz bifurcation theorem, finite-dimensional reduction of Lyapunov-Schmidt type for Fredholm maps of index 0, key function and some others constructions. The experiments by Plateau were described mathematically. It was proved, that in the case of catenoid there occurs the catastrophe of A_2("*fold*") type and in the case of helicoid – A_3("*cusp*") . All the data were found by the author's method fit the physical data of experiments. Many new bifurcations of minimal surfaces were found and were studied. More, the method was effective in some others variational problems connected with area functional.

Mourre's inequality and embedded bound states

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A self-adjoint operator H with an eigenvalue λ embedded in the continuum spectrum is considered. Let the operator P be the eigenprojection associated with λ. Then boundedness of all operators of the form $A^n P$ is proven. A is any self-adjoint operator satisfying Mourre's inequality in a neighborhood of λ and such that the higher commutators of H with A up to order $n+2$ are relatively bounded with respect to H.

The boundedness of $A^n P$ can in particular be used to generalize the notion of complex resonance to a class of quantum systems characterized only by Mourre's inequality and smoothness of the resolvent.

Selfadjointness of non-Jacobi infinite matrices

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Given a symmetric operator we may compare it with a known selfadjoint one applying commutator conditions. Then the boundedness of special commutators together with an assumption on domains of involved operators yield the essential selfadjointness of the symmetric operator. This, when applied to integral operators or, more specifically, infinite matrices allows us to find criteria for (essential) selfadjointness of these operators. In the case of matrices, the essential selfadjointness is ensured whenever the growth of entries is appropriately tempered. Thus non three-diagonal matrices are allowed.

Spectral Analysis of Dirac operators

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Let H_0 be the operator defined in the space $L_2(R^n; C^m)$ by

$$H_0 = \sum_{k=1}^{n} \alpha_k D_k + \alpha_{n+1},$$

where $D_k = i\frac{\partial}{\partial x_k}$ $(k = 1, ..., n)$, α_k $(k = 1, ..., n+1)$ are $m \times m$ Hermitian matrices which satisfy the anticommutation relations (or, so-called relations of Clifford)

$$\alpha_j \alpha_k + \alpha_k \alpha_j = 2\delta_{jk} \ (j, k = 1, ..., n+1).$$

The matrices α_k $(k = 1, ..., n+1)$ can be considered as elements of $GL(m, C)$ with $m = 2^{n/2}$ for $n = 2k$ and $m = 2^{(n+1)/2}$ for $n = 2k+1$ $(k \in N)$.

Let $Q(x)$ be a $m \times m$ Hermitian matrix-valued function and consider in the space $L_2(R^n; C^m)$ the operator Q of multiplication by $Q(x)$.

Our purpose is to study the spectral properties of the perturbed Dirac operator $H = H_0 + Q$ (see, for instance [1]). The main attention is paid on the study of the structure of the spectra of the Dirac operator H. In particular, the point, absolute continuous and singular continuous parts of the spectrum are investigated. Estimate formulae for the discrete spectrum are also given. The main results are obtained by the methods developed by us in [2], [3], [4].

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Around the semiclassical behavior of "Quantum Fidelity"

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A lot of recent papers appeared in the physics literature about the notion of so-called "quantum fidelity" (or Loschmidt Echo), in order to measure the sensitivity of a classically chaotic quantum system with respect to small perturbations. Some of the results presented in these papers are even contradictory. We present a new approach of this problem in a semiclassical framework.We make use of a beautiful formula suggested by Mehlig and Wilkinson, about the metaplectic representation, that we establish in full generality.

One dimensional models of excitons in carbon nanotubes

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Excitons in carbon nanotubes may be modeled by two oppositely charged particles living on the surface of a cylinder. We derive three one dimensional effective Hamiltonians which become exact as the radius of the cylinder vanishes. Two of them are solvable.

On new developments in the spectral analysis of the Schrödinger and Dirac operators

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In this talk, we will discuss the recent results in the spectral analysis of the Dirac and Schrödinger operators. The focus of the talk will be on the multidimensional case, the one-dimensional problems might be covered briefly as well.

Quaternionic potentials in quantum mechanics

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We discuss the Schrödinger equation in presence of quaternionic potentials. The study is performed analytically as long as it proves possible, when not, we resort to numerical calculations. As well known, the central force problem plays an important role in many applications of quantum mechanics. After a brief introduction to tunneling effects by quaternionic potential barriers, we investigate the existence of bound states in presence of quaternionic attractive potential. Comparison with standard (complex) quantum mechanics could be useful to quantify and qualify new phenomena which witness an hidden quaternionic quantum dynamics in particle physics.

Scattering problem for "step-like" Jacobi operator

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We study the scattering problem for the "step-like" Jacobi operator with asymptotically periodic coefficients that are close to different periodic backgrounds of period 2 on the half-axes.

Let $L_\pm : \ell^2(\mathbb{Z}) \to \ell^2(\mathbb{Z})$ be two periodic Jacobi operators with the coefficients $a^\pm_{n+2} = a^\pm_n > 0$, $b^\pm_{n+2} = b^\pm_n \in \mathbb{R}$ respectively. The following condition is imposed on the mutual location of their spectra. Suppose that the operators $L_\pm$ have nonempty interior gaps it the spectra, and the auxiliary spectra of these operators do not intersect with the set of the bands edges. Suppose also, that the spectra of $L_\pm$ have one common band and the others are mutually disjoint.

Consider the Jacobi operator $L : \ell^2(\mathbb{Z}) \to \ell^2(\mathbb{Z})$

$$(Ly)_n = a_{n-1}y_{n-1} + b_n y_n + a_n y_{n+1}, \quad a_n > 0,\, b_n \in \mathbb{R},$$

such that

$$\sum_{n=0}^{\pm\infty} |n|^2 \{|a_n - a^\pm_n| + |b_n - b^\pm_n|\} < \infty. \tag{1}$$

For this operator we propose the characterization of its scattering matrix. By means of the Marchenko approach ([1]) we solve the direct/inverse scattering problem for the operator L in the non-resonance case.

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Vertex coupling approximations in quantum graphs

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It is well known that the probability current conservation does not fix the the vertex coupling in quantum graph models uniquely. We describe an optimal parametrization of such couplings and ask about the meaning of different elements of this class. The squeezing limit of thin graph-like manifolds is analyzed and shown to yield essentially the "free case" only. Other approximations consider the graph itself and employ a family of interactions supported in the vicinity of the vertex in analogy with one-dimensional point interactions; two results of this type are illustrated on the cases of δ and δ' vertex couplings.

A Model Hamiltonian for Condensed Matter Physics

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We consider a class of singular, zero-range perturbations of the quantum hamiltonian of a system consisting of a test particle and N harmonic oscillators (Rayleigh gas). Using the theory of quadratic forms we construct the self-adjoint and bounded from below perturbed hamiltonian and we give representation formulas for the resolvent and the unitary group. The one dimensional, two dimensional and three dimensional cases are discussed.

On the Laplacian on the halfplane with a periodic boundary condition

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We study spectral and scattering properties of the Laplacian $H^{(\sigma)} = -\Delta$ in $L_2(\mathbb{R}^2_+)$ corresponding to the boundary condition $\frac{\partial u}{\partial \nu} + \sigma u = 0$ for a wide class of periodic functions σ. For non-negative σ we prove that $H^{(\sigma)}$ is unitarily equivalent to the Neumann Laplacian $H^{(0)}$, the unitary equivalence being provided by the wave operators. In general, there appear additional channels of scattering corresponding

to surface states which we analyze in detail.

The results are published in
R. L. Frank, *On the scattering theory of the Laplacian with a periodic boundary condition. I. Existence of wave operators*, Documenta Math. 8 (2003), 547–565.
R. L. Frank, R. G. Shterenberg, *On the scattering theory of the Laplacian with a periodic boundary condition. II. Additional channels of scattering*, Documenta Math. 9 (2004), 57–77.

Transport Mobility Edge for Random Landau Hamiltonians

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We consider a two-dimensional Landau Hamiltonian in the continuum with a random potential. We show that near the Landau levels, in between regions of complete localization, there exists at least one energy E which is responsible for some non trivial transport, characterized by a transport exponent $\beta(E)$ greater than 1/4. To our best knowledge, this is the first time that non trivial transport is proved for quantum Hall systems. The proof combines several important results obtained by various people in the field (random Schrodinger operators) during the last 15 years. This is a joint work with A. Klein and J. Schenker.

Selberg zeta function and geometric scattering on compact manifolds

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A new approach to the Selberg trace formula on a compact Riemann surface X of a constant negative curvature K was proposed in [1] and [2]; this approach is based on some relations between the resolvent kernels of the Laplace-Beltrami operator on the surface and the Selberg zeta function Z_X of this surface. On the other hand, if Γ is a Fuchsian group of the first kind with non-compact fundamental domain, then the Selberg trace formula contains an information about the scattering matrix for the surface with cusps $X = \Gamma\backslash H$, where H is the Lobachevsky plane (the upper Poincaré half-plane) [3]. In the case of a compact surface X we can attach the semi-axis $\mathbf{R}_+$ to X at a point $q \in X$ with the help of boundary conditions at q; the corresponding scattering amplitude $S(k;q)$ in terms of the Green function of the Laplace–Beltrami operator on X is calculated in [4]. The following relation is among the main results of the talk:

$$\frac{\mathsf{Z}'(s)}{\mathsf{Z}(s)} = (2s-1)\Big[2(g-1)\psi(s) + \beta -$$

$$\frac{|\alpha|^2\sqrt{s(s-1)}}{4\pi(g-1)} \int\limits_X \left(\mathrm{Ca}(S(\sqrt{s(s-1}; q)) + \gamma\sqrt{s(s-1}\ \right)^{-1} dq\Big] .$$

Here α, β, γ ($\alpha \in \mathbf{C}$, $\beta, \gamma \in \mathbf{R}$) are parameters of the boundary condition, ψ is the logarithmic derivative of the Euler Γ-function, g is the genus of X, $\mathrm{Ca}(S)$ stands for the Cayley transform of S, and we assume $K = -1$.

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Discrete spectrum of complex Jacobi matrices and Pavlov's theorems

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Let

$$J = \begin{pmatrix} b_0 & c_0 & \dots & \dots & \dots \\ a_0 & b_1 & c_1 & \dots & \dots \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \ddots \end{pmatrix}$$

be a complex Jacobi matrix with

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = 1, \qquad \lim_{n\to\infty} b_n = 0$$

that is, the operator J is a compact perturbation of the discrete laplacian J_0. We say that J belongs to the class $\mathcal{P}(\beta)$ if

$$|a_n - 1| + |c_n - 1| + |b_n| \le C_1(\exp(-C_2 n^\beta)), \quad n \to \infty, \quad 0 < \beta < 1.$$

Theorem 1. *Let $J \in \mathcal{P}(\beta)$, $0 < \beta < \frac{1}{2}$ and let E be a limit set of the discrete spectrum $\sigma_d(J)$. Then E is the closed set of the Lebesque measure zero and its Hausdorff dimension obeys* $\dim E \le (1-2\beta)(1-\beta)^{-1}$. *Moreover, if $J \in \mathcal{P}(\frac{1}{2})$, then $E = \varnothing$ ($\sigma_d(J)$ is a finite set).*

Theorem 2. *For arbitrary $\varepsilon >> 0$ and $-1 < \lambda < 1$ there exists an operator $J \in \mathcal{P}(\frac{1}{2} - \varepsilon)$, such that $E = \{\lambda\}$.*

The above results can be viewed as the discrete analogs of Pavlov's theorems on non-selfadjoint Schrödinger operators on the halfline.

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Inverses of generators

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Let A be a linear operator on a Banach space B that generates a uniformly bounded C_0-semigroup of linear operators on B. We show that is possible that operator A is one-to-one with dense range but operator A^{-1} does not generate C_0-semigroup on B. This answers an open question from [1].

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Absence of quantum states corresponding to unstable classical channels

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We consider classical hamiltonians $h(x, \xi)$which are homogeneous of degree zero in the x variable and develop a general theory to show that there are no quantum states corresponding to unstable classical channels.

Inverse spectral problems for Sturm–Liouville operators in impedance form

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We consider Sturm–Liouville operators in $L_2(0, 1)$ generated by the differential expression

$$\ell(f) = -a^{-1}(af')'$$

subject to suitable boundary conditions. Here the impedance function a is positive and belongs to $W_p1(0, 1)$, $p \in [1, \infty)$.

For such operators, we give a complete description of the Dirichlet and Neumann–Dirichlet spectra and solve the inverse spectral problem of recovering the impedance function by these two spectra. The reconstruction procedure is based on the Krein equation and yields a global solution, as opposed to e.g. local results by Andersson [*Inverse Problems* **4** (1988), 353–397].

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A mathematical theory of the phase space Feynman path integral of the functional

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Let $z_j(x, \Pi)$ $(j = 1, 2, \ldots, N)$ be functions in phase space, where Π denotes the genuine kinetic momentum. For fixed $t_1 < \ldots < t_N$ in $[0, T]$ we define the functional $F(q, \Pi)$ by $\prod_{j=1}^{N} z_j(q(t_j), \Pi(t_j))$ on the space of all paths $(q, \Pi) = (q(\theta), \Pi(\theta))$ $(0 \leq \theta \leq T)$ in phase space. In this talk we study rigorously the phase space Feynman path integral of the functional $F(q, \Pi)$, which is defined by the limit of the time-slicing approximations. We give a necessary condition and a sufficient condition of F for the path integral of the functional F to be well-defined. In addition, we give the expression of the path integral of the functional F in terms of the operator notation, which was studied by Feynman himself heuristically. We will give the proof to Feynman's formulas stated in his celebrated paper [2].

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C_ρ^*-Algebras and Functional Calculus Homomorphism

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Let $0 < \rho \leq 1$ and A be a C_ρ^*-algebra (a ρ-Banach algebra whose norm has the C^*-property). We note that if $\rho = 1$, then it is a C^*-algebra. We study some of their basic properties and we generalize some of the known results in C^*-algebras to C_ρ^*-algebras. Specially we generalize continuous functional calculus homomorphism to C_ρ^*-algebras. In fact we prove that:

Theorem. *Let A be a unital C_ρ^*-algebra, a be a normal element of A and let $z : \sigma(a) \to \mathbb{C}$ be the inclusion map. Then there exists a unique unital $*$-homomorphism $\varphi : C(\sigma(a)) \to A$ such that*

(i) φ is a $*$-monomorphism, (ii) $\|\varphi(f)\| = \|f\|_\infty^\rho$,

(iii) $\varphi(p) = p(a, a^*)$, for every polynomial $p(z, \bar{z})$ in z and $\bar{z}$.

We also generalize Gelfand representation theorem for commutative C^*-algebra to C^*_ρ-algebra:

Theorem (Gelfand Representation).
Let A be a commutative C^*_ρ-algebra with identity 1. Then the Gelfand transform $\mathcal{G} : A \to C(\Delta A)$ is a $*$-isomorphism such that

$$\|\hat{a}\|_\infty = r(a) = \|a\|^{1/\rho} \qquad (a \in A),$$

and $\sigma(a) = \hat{a}(\Delta A)$ for every $a \in A$.

We show that every element of a C^*_ρ-algebra is a linear combination of positive elements and the set of positive elements is a cone and we get some other results, e.g.:

Theorem. *Every positive linear functional f on a C^*_ρ-algebra is continuous and if 1 is the identity of A, then for every $a \in A$,*

$$|f(a)| \leq f(1)\|a\|^{1/\rho}.$$

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Weak Annihilators for Non-Self-Adjoint Operators with Almost Hermitian Spectrum

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New results on the existence of an *outer weak annihilator*, i. e., a scalar outer analytic in the upper half-plane function $\gamma(\lambda)$ such that

$$w - \lim_{\varepsilon \downarrow 0} \gamma(L + i\varepsilon) = 0,$$

for a non-self-adjoint trace class perturbation L of a bounded self-adjoint operator A acting in Hilbert space H are described.

In particular, it is proved that the existence of an outer weak annihilator for the operator L is equivalent to the fact that the spectrum of the operator L is *almost Hermitian*, that is, $N_i^0 = H$ (or, equivalently, $N_i^0(L^*) = H$ for the adjoint operator L^*). Here N_i^0 is a singular spectral subspace of the operator L (introduced in [**1**], see also [**2**]), corresponding to a portion of real singular spectrum of L. This spectral subspace can be shown to play a crucial role in spectral analysis of a non-dissipative operator L (essentially all new spectral properties of a non-dissipative operator, compared to an orthogonal sum of a dissipative and an adjoint to dissipative ones, are due to the presence and properties of N_i^0) and in analysis of the similarity problem (to either a dissipative or a self-adjoint operator).

It is shown that in the case when the operator L possesses almost Hermitian spectrum the determinant of one of the factors in the Ginzburg-Potapov factorization of its characteristic function can be chosen as a weak annihilator for L.

In effect, these results provide a generalization of the Caley identity to the case of non-self-adjoint operators of the class considered.

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Adiabatic quasi-periodic Schrödinger operators Interactions between spectral bands

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In this talk, we present some recent spectral results for one dimensional periodic Schrödinger operators perturbed by a slowly varying incommensurate periodic potential (see [**1, 2**]). More precisely, we consider the following self-adjoint quasi-periodic Schrödinger operator acting on the real line

$$H_{z,\varepsilon} = -\frac{d^2}{dx^2} + (V(x-z) + \alpha\cos(\varepsilon x)).$$

where $V:\ \mathbb{R} \to \mathbb{R}$ is a non constant, locally square integrable, 1-periodic function, ε is a small positive number chosen such that $2\pi/\varepsilon$ be irrational, z is a real parameter and α is a strictly positive parameter that we will keep fixed in most of the paper.

We show that in certain energy regions the perturbation leads to resonance effects related to the ones observed in the problem of two resonating quantum wells. These effects affect both the geometry and the nature of the spectrum. In particular, they can lead to the intertwining of sequences of intervals containing absolutely continuous spectrum and intervals containing singular spectrum. Moreover, in regions where all of the spectrum is expected to be singular, these effects typically give rise to exponentially small "islands" of absolutely continuous spectrum. Another typical phenomenon is level repulsion (which in the standard double well problems gives rise to splitting). It is seen that this phenomenon is related with the nature of the spectrum.

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Schrödinger operators with singular interactions: a resonance model

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We discuss a generalized Schrödinger operator in $L^2(R^2)$ with an attractive singular interaction supported by a straight line and a finite family of points. It can be regarded as a model of a leaky quantum wire and a family of quantum dots. We analyze the discrete spectrum and show that the resonance problem can be explicitly solved in this setting.

Inverse problem for the discrete 1D Schrödinger operator with periodic potentials

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Consider the discrete 1D Schrödinger operator with a periodic potential. We study the Marchenko-Ostrovski mapping:" a periodic potential " to heights of vertical slits on the quasi-momentum domain (similar to the continuous case on the real line). We obtain the following results : i) this mapping is a local isomorphism outside some ball , ii) this mapping is a global isomorphism outside large ball. iii) we study the same problems for small potentials, iv) for above cases the asymptotics of the spectrum is obtained: firtsly, for large potentials and secondly for small potentials

Singular perturbations given by Jacobi matrices

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Let $A > m \geq 0$ be a strictly positive unbounded self-adjoint operator with domain $\mathcal{D}(A)$ in a separable Hilbert space $\mathcal{H}$ and $\dot{A}$ denote its symmetric restriction with infinite deficiency indices. Let $\mathrm{Ker}\dot{A} = \mathcal{N}$, $\mathcal{N} \cap \mathcal{D}(A) = 0$, denote the defect subspace of $\dot{A}$. And let J be a Jacobi matrix. We consider J as an operator parameter for construction of a new self-adjoint extension $\tilde{A} \neq A$ of $\dot{A}$, i.e., $\tilde{A}$ is a singular perturbation of A since the set $\mathcal{D} := \mathcal{D}(A) \cap \mathcal{D}(\tilde{A})$ is dense in $\mathcal{H}$ [**2, 3**]. With this aim we associate to a Jacobi matrix J the singular operator $T_J : \mathcal{H}_1 \to \mathcal{H}_{-1}$ acting in the A-scale of Hilbert spaces $\mathcal{H}_{-1} \supset \mathcal{H}_0 \equiv \mathcal{H} \supset \mathcal{H}_1$. Formally the singularly perturbed operator is given as the generalized sum [**1**] $\widetilde{A} = A\tilde{+}T_J$. In really we construct $\widetilde{A}$ using the Krein resolvent formula and taking elements of a Jacobi matrix J consequently as coefficients of rank one singular perturbations in some basic $\{\psi_j\}_{j=1}^{\infty}$ in subspace $\mathcal{N}$. We find conditions which insure the arising an additional point spectrum E_j in the operator $\widetilde{A}$.

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On Isomorphism of Elliptic Operators

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The talk deals with a properly elliptic invertible differential operator P of even order $2m$ over R^n with constant coefficients which defines a natural mapping of the space $C_0^\infty(\Omega)$ of all infinitely smooth finite functions over a bounded domain $\Omega \subset R^n$ to itself. It turns out that there exists an extension operator to P which is an *isomorphism* between the standard Sobolev spaces $H^m(\Omega)$ and $H_0^{-m}(\Omega)$ and such that the inverse operator can be represented in an extremely simple form using the Fourier transform. In comparison with usual boundary value problems, here the extension to P is more complicated than the operator generated by the boundary value problem. On the contrary, the inverse operator is much more simple than the operators solving the usual boundary value problems. The isomorphism holds under an extra condition that the Dirichlet boundary-value problem for P in the domain *exterior* to Ω is elliptic and uniquely solvable (trivial kernel and cokernel).

Non-Szegö asymptotics for orthogonal polynomials on the unit circle

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Asymptotical properties of orthogonal polynomials from the so-called Szegő class are very well-studied.First, we discuss pointwise asymptotics for orthogonal polynomials from a class that is considerably larger than the Szegö class. Then we prove that these asymptotics hold in the $L2$-sense (with a weight) on the unit circle.

Discrete spectrum asymptotics for Schrödinger operators on lattice

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The Hamiltonian of a system of three quantum mechanical particles moving on the three-dimensional lattice $\mathbb{Z}^3$ with arbitrary "dispersion functions" having not necessarily of compact support and interacting via zero-range attractive potentials is considered.

For the two-particle energy operator $h(k)$, with $k \in \mathbb{T}^3 = (-\pi, \pi]^3$ the two-particle quasi-momentum, the existence of a unique positive eigenvalue below the bottom of the continuous spectrum of $h(k)$ is proven, provided that the operator $h(0)$ has a zero energy resonance.

The existence of infinitely many eigenvalues of the three particle discrete Schrödinger operator $H(0)$ is proven and the asymptotics for the number of eigenvalues $N(0, z)$ lying below $z < 0$,

$$\lim_{z \to 0-} \frac{N(0, z)}{|\log|z||} = \mathcal{U}_0 \qquad (1)$$

is found.

Moreover for all nonzero and small values of the three-particle quasi-momentum $K \in \mathbb{T}^3 = (-\pi, \pi]^3$ the finiteness of the number $N(K, \tau_{ess}(K))$ of eigenvalues below the bottom $\tau_{ess}(K))$ of the essential spectrum of the three particle discrete Schrödinger operator $H(K)$ is proven. The asymptotics for the number $N(K, 0)$ (resp. $N(K, \tau_{ess}(K)))$ of eigenvalues of $H(K)$ lying below zero (resp. below the bottom $\tau_{ess}(K))$) of the essential spectrum $\sigma_{ess}(H(K))$

$$\lim_{K \to 0} \frac{N(K, 0)}{|\log |K||} = 2\mathcal{U}_0 \quad (\text{resp.} \lim_{K \to 0} \frac{N(K, \tau_{ess}(K)}{|\log |K||} = U, \quad U \geq 4\mathcal{U}_0) \qquad (2).$$

is established. We remark that whereas the result (1) is similar to that of continuous case and, the results (2) and (3) are surprising and characteristic for the lattice systems, in fact they do not have any analogues in the continuous case.

The generalized Schur algorithm and inverse spectral problems

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The generalized Schur algorithm is used to obtain representations of J-unitary polynomial 2×2–matrix functions as a product of elementary factors. In the line case this is related to the inverse spectral problem of finding a canonical system which has the given generalized Nevanlinna function as its Titchmarsh-Weyl coefficient. The continuous analogue of this method is explained for a special class of Titchmarsh-Weyl functions.

Reflectionless point interactions

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For reflectionless potentials we construct a generalized point interaction having the same scattering data. It turns out that in this situation the extension space can not be chosen as a Hilbert space, but an indefinite inner product occurs.

Moreover we describe the time dependence of this model for a few-soliton solution of the Korteweg-de Vries equation.

Asymptotic behaviour of eigenvalues of some Jacobi matrices

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We investigate the asymptotic behaviour of the point spectrum for some special unbounded Jacobi operators J acting in the Hilbert space $l^2 = l^2(N)$. For given sequences of real numbers λ_n and q_n the Jacobi operator is given by $J = SW + WS^* + Q$, where $Q = diag(q_n)$ and $W = diag(\lambda_n)$ are diagonal operators, S is the shift operator in l^2 and the operator J acts on the maximal domain. By various methods we calculate the asymptotic behaviour of eigenvalues for different classes of Jacobi operators. We consider some cases of J when $q_n = \delta n^\alpha + o(n^\alpha)$ and $\lambda_n = n^\beta + o(n^\beta)$ for $\alpha \geq \beta$. For example we consider the case $q_n = \delta n$ and $\lambda_n = n + c$ for $\delta > 2$.

Eigenproblems on exterior domains and approximation of DN maps

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We consider eigenvalue problems for the Schrödinger operator

$$-\Delta + q(\cdot)$$

in an exterior domain $\Omega = \mathbb{R}^n \setminus \Omega^c$, where Ω^c is a simply-connected bounded domain with a smooth boundary, and $q(\cdot)$ is a complex-valued potential. These and similar problems arise in a variety of applications, including scattering of water waves by an obstacle. A typical way to solve such problems numerically is to truncate the infinite domain Ω by introducing an artificial outer boundary with some associated boundary conditions, and then solve the problem on the truncated domain using a suitable numerical approach.

In this talk we consider the effect of the truncation on the spectrum by considering its effect on the Dirichlet to Neumann (DN) and related maps on the (fixed) inner boundary. Our results fall into 3 classes: (a) guaranteed convergence results for some special cases; (b) characterization of 'spurious' eigenvalues for the general case; (c) nesting results for sets of DN maps for the general case. The results in (c) are similar in spirit to recent work of Amrein and Pearson for the self-adjoint case.

Eigenfunction Expansions and Spectral Projections for Isotropic Elasticity outside an Obstacle

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We consider the selfadjoint realization of $-\mathbf{\Delta} - \alpha\, \mathbf{grad}\, \mathrm{div}$ acting on a domain Ω, exterior to a compact obstacle with smooth boundary, subject to Dirichlet boundary conditions. The spectral projections of this operator are written in terms of an expansion of generalized eigenfunctions. The methods used follow those of N. Shenk [**1**] for the analogous problem for the Laplace operator.

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The parabolic Anderson model: The asymptotics of the statistical moments, Lifshitz tails and related topics

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We consider the parabolic Anderson model $\partial_t u = \kappa \triangle u + V u$ on $\mathbb{R}_+ \times \mathbb{Z}^d$ with initial condition $u(t, 0) \equiv 1$. Here $\triangle$ is the discrete Laplacian, κ is a diffusion constant and $V = \{V(x) : x \in \mathbb{Z}^d\}$ is an i.i.d. random field taking values in $\mathbb{R}$. The large-time behaviour of $u(t, 0)$ is strongly linked with the occurrence of high peaks in the fields V and $u(t, .)$. These intermittency peaks give the main contribution to the formation of the statistical moments and determine the correlation structure of $u(t, 0)$. The latter quantities are extensively studied in the stochastics literature using path integral methods and the theory of large deviations.
The parabolic Anderson model is also closely connected to the theory of Anderson localization. The large-time behaviour of the statistical moments and the correlation structure correspond to the asymptotics of the integrated density of states and to properties of localized eigenstates. Our aim is to study the relation between the stochastic and the spectral theoretic point of view with respect to these quantities.

Spectra of Singular Differential Operators on the Circle

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Let $m \in \mathbb{N}$, V be a (2-periodic) complex-valued distribution in the negative Sobolev space $H^{-m}(\mathbb{T})$. Spectra of the singular Schrödinger-type operators

$$S = (-1)^m \frac{d^{2m}}{dx^{2m}} + V(x)$$

in the Hilbert space $L^2(\mathbb{T})$ are studied. These operators are well defined owing to the KLMN theorem and the convolution lemma. The resolvent of any S is a compact operator. The operator S is self-adjoint iff the distribution V is real-valued (i.e. $\hat{V}(k) = \hat{V}(-k)$, $k \in \mathbb{Z}$, where $\hat{V}(k)$ denote the Fourier coefficients of V). The following estimates for the eigenvalues $\lambda_j = \lambda_j(m, V)$ which are enumerated with their algebraic multiplicity and ordered so that

$$Re(\lambda_j) < Re(\lambda_{j+1}),\ or\ Re(\lambda_j) = Re(\lambda_{j+1}), Im(\lambda_j) \leq Im(\lambda_{j+1}), j = 0, 1, 2, \ldots$$

are found:

(1) *For any* $m \in \mathbb{N}$, $V \in H^{-m}(\mathbb{T})$ the one-term asymptotic formulae

$$\lambda_{2n-1}, \lambda_{2n} = n^{2m}\pi^{2m} + o(n^m), n \longrightarrow \infty$$

is valid.

(2) *Let* $m \in \mathbb{N}$, $V \in H^{-m\alpha}(\mathbb{T})$, $\alpha \in [1/2, 1)$. *Then for any* $\varepsilon > 0$ *uniformly for bounded sets of distributions* V *in* $H^{-m\alpha}(\mathbb{T})$ *the asymptotic formulae*

$$\lambda_{2n-1}, \lambda_{2n} = n^{2m}\pi^{2m} + \hat{V}(0) \pm \sqrt{\hat{V}(-2n)\overline{\hat{V}(2n)}} + r_n^{\pm}(m, V), \quad (*)$$
$$r_n^{\pm} \in h^{m(1-2\alpha-\varepsilon)}$$

are hold. Here h^β, $\beta \in \mathbb{R}$, denote the weight Hilbert sequence spaces:

$$s_n \in h^\beta \Longleftrightarrow \sum |s_n|^2 (1+n)^{2\beta} =: \|s\|_\beta^2 \Longrightarrow s_n = o(n^{-\beta}), n \longrightarrow \infty.$$

(3) *Let* $m \in \mathbb{N}$, $V \in H^{-m\alpha}(\mathbb{T})$, $\alpha \in [0, 1/2)$. *Then uniformly for bounded sets of distributions* V *in* $H^{-m\alpha}(\mathbb{T})$ *formulae (*) are hold with the remainder terms in* $h^{m(1/2-\alpha)}$.

Similar estimates for the semiperiodic eigenvalues are found. Some applications to spectral analysis of the periodic Schrödinger operators with singular potentials are given.

The results are obtained joint with V.M. Molyboga.

Two-particle bounded states of transfer-matrix for Gibbsian fields (high temperature regime)

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It is possible to construct one- and two-particle invariant spaces for transfer-matrix of large class of Gibbsian fields on lattice $\mathbb{Z}^d$ $(d = 1, 2, 3)$. The restriction of transfer-matrix on two-particle subspace is similar to the family of operator like generalized Friedrich's model.

The talk is devoted to the description of the eigenvalues of these operators for high temperature (the criteria of existence, the position).

Simple and double eigenvalues of the Hill operator with a two term potential

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We analyze the instability zones and multiplicities of eigenvalues of Hill operator with a two term potential

Ly = - y" + a cos2x + b cos4x

with parametrization of its coefficients a = - 4qt, b = - 2q2. A proper gauge transform (W. Magnus and S. Winkler) kills a term with a higher frequency and gives us a chance to realize the similar to L operator with a tridiagonal matrix K. [But it is not a Jacobi matrix.] We sharpen Magnus/Winkler results on multiplicity of the L's eigenvalues and give complete structure of the spectrum Sp(L) with periodic or antiperiodic boundary conditions. Quantum type effect is observed when t is an integer. This links our analysis with the theory of quasi-exactly solvable differential equations (A. Turbiner, P. Olver and others). Asymptotics of spectral gaps for L when q approaches zero gives a series of surprising identities with squares of integers. [All of the above are joint results of the speaker and Plamen Djakov.] Our related papers are

1. Plamen Djakov and Boris Mityagin, The asymptotics of spectral gaps of 1D Dirac operator with cosine potential, Lett. Math. Phys. 65 (2003), 95 - 108.

2. Plamen Djakov and Boris Mityagin, Multiplicities of the eigenvalues of periodic Dirac operators, Ohio State Math. Research Inst. Preprints, 04-1, January 7, 2004, 32 p.,

but the talk's results are not there.

On some unbounded tridiagonal matrices generating C_0 semigroups in l^p spaces

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We study some (unbounded) Jacobi matrices acting in l^p spaces. We find some conditions on the sequences on the main, the lower, and the upper diagonal of the matrix, guaranteeing that the operator is the generator of a C_0 semigroup. [joint work with J. Banasiak and M.Lachowicz]

An inverse scattering problem for Sturm–Liouville operators on semiaxis

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Consider in the Hilbert space $L_2(\mathbb{R}_+)$ a Sturm-Liouville operator T given by the differential expression

$$Tf := l(f) = -a^{-1}(af')' + q$$

on the domain

$$\mathcal{D}(T) = \{f \in L_2(\mathbb{R}_+)\ :\ f, af' \in W^1_{1,\mathrm{loc}}(\mathbb{R}_+),\ l(f) \in L_2(\mathbb{R}_+),\ f(0) = 0\}.$$

We suppose that q is a real-valued function that belongs to the space $L_1\big(\mathbb{R}_+, (1+x)dx\big)$ and that the function $a \in W^1_{1,\mathrm{loc}}(\mathbb{R}_+)$ is positive and $(\log a)' \in L_1(0,\infty)$. For such operators we give a complete description of its scattering data and solve the inverse scattering problem of recovering the operator T.

Inverse spectrum problem for quantum graphs

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Quantum graph is a differential operator on a geometric graph with certain self adjoint boundary conditions at the vertices. The simplest example is the Laplace operator i.e. the second derivative operator with natural b.c. (the function is continuous and the sum of derivatives is zero). Following Gutkin and Smilansky we prove that the spectrum of free operator determine the graph if the lengths of edges are rationally independent. The proof is based on the trace formula connecting periodic orbits with the spectrum of a quantum graph.

We will also discuss physical boundary condition instead of natural. An example of two different isospectral graphs will be given (of course having rationally dependent edges).

Compact extremums of integral functionals

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A new approach to extreme problems for integral functionals is proposed. The difficulties arising in these questions are mainly closed to absence of the second strong derivative for general integral functionals and to realization of their extremums on some compact sets. These cases exclude application of the classical Frechet extreme conditions.

Let f be a real functional on some locally convex space E. Say that f has K-extremum at $x \in E$ if, for every absolutely convex compactum $C \subseteq E$, the restriction $f|_{x+E_C}$ has local extremum at x (here $E_C = span(C)$). Analogously, say that f is K-differentiable at x, if every restriction $f|_{x+E_C}$ is differentiable at x respective to $\|\cdot\|_C$, generated by C.

The sufficient K-extreme conditions in terms of K-derivative are proved both in the cases of one and many variables. The applications to Euler-Lagrange functional

$$\Phi(y) = \int_a^b f(x, y(x), y'(x))\, dx \quad (y(\cdot) \in W_2^1([a,b], E))$$

are considered.

The repeated K-differentiability of $\Phi(y)$ is proved. It allowed to obtain the sufficient K-minimum condition of $\Phi(y)$, namely, positive definiteness of $f''(x, y(x), y'(x))$ in second and third variables (here $y(\cdot)$ is extremal, $a \leqslant x \leqslant b$). In the case of nuclear space E this condition is expressed by means of the second derivatives of f.

A simple example of K-extremum $\Phi(y)$, which is not a local extremum, is constructed. The possible modifications of the approach above for diverse classes of integral functionals are discussed.

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Point perturbations as pseudopotentials

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We consider magnetic Schroedinger operators with zero-range potentials (point perturbations) within the framework of the operator extension theory. Under rather generic conditons on magnetic and electric potentials it is shown that such operators can be represented as the sum of the unperturbed operator and a certain singular linear operator called pseudopotential. Such an expression can be viewed as an operator in the space of distributions and gives a description of the point perturbation in a form closed to the description of usual Schrödinger operators with regular potentials; within this context, the point pertubation can be treated as a kind of additive perturbation, and the matrix of parameters becomes a matrix of coupling constants. An analytic form of the pseudopotential for several classes of unperturbed operators is given and the relationship with various types of regularization is discussed.

Transport properties and modelling of Quantum Networks

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We propose the description of one-particle transport in the quantum network based on observation that the resonance transmission of an electron across the wells, from one quantum wire to the other, happens due to the excitation of oscillatory modes in the well. This approach allows us to obtain also an approximate formula for the transmission coefficients, based on numerical data for eigenvalues and eigenfunctions of the discrete spectrum of some intermediate Schrödinger operator in certain range of energy. We interpret the corresponding approximate Scattering matrix as a Scattering matrix of an appropriate solvable model in form of a quantum graph with the resonance node. We also suggest a new version of the analytic perturbation technique based on factorization of the corresponding scattering matrix near the resonance.

Spectral asymptotics of quantum harmonic oscillator perturbed by almost periodic potential

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Consider the Hamiltonian of a perturbed quantum harmonic oscillator

$$T = -\frac{d^2}{dx^2} + x^2 + q(x) \quad \text{in} \quad L^2(\mathbb{R}),$$

supposing that the perturbation q satisfies $\|q\| < \infty$, where

$$\|q\| = \|q\|_\infty + \|q'\|_\infty + \|Q\|_\infty, \quad Q(x) = \int_0^x q(s)\,ds$$

and $\|\cdot\|_\infty$ is the norm in $L_\infty(\mathbb{R})$. In particular, q may be periodic or almost periodic. The spectrum of T is purely discrete: $\sigma(T) = \{\mu_n\}_{n=0}^\infty$, where $|\mu_n| \le |\mu_{n+1}|$. Generalizing the result of [1], we determine the term of the spectral asymptotics, linear in the perturbation q. For $\|q\|n^{-1/4} \to 0$ we obtain

$$\mu_n - (2n+1) = \int_{-\infty}^{\infty} q(x)\psi_n^0(x)^2\,dx + O(\|q\|^2 n^{-1/2}) = O(\|q\| n^{-1/4}), \tag{1}$$

where $\{\psi_n^0\}_{n=0}^\infty$ is the set of orthonormed real unperturbed eigenfunctions.

Consider the operator

$$H = -\partial_x^2 + (-i\partial_y - x)^2 + q(x) \quad \text{in} \quad L_2(\mathbb{R}^2).$$

This operator is the quantum Hamiltonian of an electron on a plane in homogeneous magnetic field perpendicular to the plane, in the presence of the electric field $q(x)$.

For real q satisfying $\|q\| \le \infty$ the operator H is self-adjoint and its spectrum $\sigma(H)$ has neither discrete nor singular continuous components. Using the asymptotics (1), we prove that there exists an absolute constant $c > 0$ such that for any integer $N > c\|q\|^4$ we have

$$\sigma(H) \bigcap [2N, \infty) = \bigcup_{n=N}^{\infty} [\mu_n^-, \mu_n^+], \quad \mu_n^- \le \mu_n^+ < \mu_{n+1}^-,$$

where

$$\mu_n^\pm - (2n+1) = \pm \sup_{\tau\in\mathbb{R}} \left\{ \pm \frac{1}{2\pi} \int_{-\pi}^{\pi} q(\tau + \sin\theta\sqrt{2n+1})\, d\theta \right\} + O\left(\frac{\|q\|}{n^{1/3}} \right).$$

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Subfields of a Jacobi Field

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By definition, a Jacobi field $J = (\tilde{J}(\phi))_{\phi \in H_+}$ is a family of commuting self-adjoint three-diagonal operators in the Fock space $\mathcal{F}(H)$. The operators $J(\phi)$ are indexed by the vectors of a real Hilbert space H_+. The function $\phi \mapsto J(\phi)\Phi$ is assumed to be linear and continuous for any finite vector $\Phi \in \mathcal{F}(H)$.

Let H_- stand for the dual of H_+. Consider a real Hilbert space T_- and a bounded operator $K^+ : T_- \to H_-$. We suppose $\mathrm{Ker}(K^+) = \{0\}$. The spectral measure ρ of the field J is defined on H_-. The operator K^+ takes ρ to a probability measure ρ_K defined on T_- w.r.t. the formula

$$\rho_K(\Delta) = \rho((K^+)^{-1}(\Delta))$$

($(K^+)^{-1}(\Delta)$ denotes the full preimage of Δ). The main objectives of my talk are:

1. To obtain a family J_K of commuting selfadjoint operators with its spectral measure equal to ρ_K.
2. To obtain the chaotic decomposition for the space $L^2(T_-, d\rho_K(\omega))$.

Noteworthily, the family J_K appears to be isomorphic to a subfield of the initial field J. If K^+ is an invertible operator then J_K is isomorphic to J.

The talk is based on my joint work with Prof. Yurij M. Berezansky and Dr. Eugene W. Lytvynov.

Trace formula and high energy spectral asymptotics for the Landau Hamiltonian

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A two-dimensional Schrödinger operator with a constant magnetic field perturbed by a smooth compactly supported potential is considered. The spectrum of this operator consists of eigenvalues which accumulate to the Landau levels. We call the set of eigenvalues near the n'th Landau level an n'th eigenvalue cluster, and study the distribution of eigenvalues in the n'th cluster as $n \to \infty$. A complete asymptotic expansion for the eigenvalue moments in the n'th cluster is obtained and some coefficients of this expansion are computed. A trace formula involving the eigenvalue moments is obtained.

The phenomenon of instability of the absolutely continuous spectrum of nonself-adjoint ordinary differential operators.

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We show that the absolutely continuous spectrum of dissipative ordinary differential operators of the second order is unstable under slowly decaying perturbation of the imaginary part. The main result here is that if the additive perturbation has imaginary part which is not from L^1, then the absolutely continuous subspace is trivial. This result is in sharp contrast with the situation of the self-adjoint theory, where the absolutely continuous spectrum is preserved as soon as the perturbation is in L^2.

On the absolutely continuous spectrum of multi-dimensional Schrödinger operators

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We consider a multi-dimensional Schrödinger operator $-\Delta + V$ in $L^2(\mathbb{R}^d)$ and find conditions on the potential V which guarantee that the absolutely continuous spectrum of this operator is essentially supported by the positive real line. We prove some results which go beyond the case $L^1 + L^p$ with $p < d$.

Existence and Completeness of wave operators

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The aim of the present paper is to show the existence and the completeness of the wave operators $W_\pm$ and other related results. One of the main objects of scattering theory is that the absolutely continuous spectrum of the self-adjoint operator T_0 do not change when one add perturbation V to T_0. The definition of $W_\pm$ is central.

Let T_0 and T be a self-adjoint operators on a Hilbert spaces H_0 and H respectively. By J we denote a bounded operator from H_0 into H. J is also called the identification operator. A fundamental question one asks in scattering theory is when do the strong limit

$W_\pm f = s - \lim_{t\to\pm\infty} e^{itT} \; Je^{-itT} f$ exist, for every f in the absolutely continuous subspace H_0^{ac} of T_0. This limits (if exist) define the wave operators $W_\pm$.

If $W_\pm(T, T_0, J)$ exist and are isometric on T_0^{ac} then $W_\pm(T, T_0, J)$ is called complete if Ran $W_\pm(T, T_0, J) = H^{ac}$.($H^{ac}$ is the absolutely continuous subspace of T.)

This implies that, the scattering operator S, defined by $S = W_+^* W_-$, is unitary on H_0^{ac} if $W_\pm$ are complete. Also S intertwines T_0, i.e; $e^{-itT_0} S = S \; e^{-itT_0}$, $t \in R$.

We construct on trace class method in dealing with the existence and completeness of the wave operators for the self-adjoint operators T_0, T. The original result of trace class scattering theory is the "Kato-Rosenblum-Pearson theorem". Kato-Rosenblum theorem was extended to the case where the difference of the resolvents of the unperturbed operator T_0 and perturbed operator T is a trace operator.

We shall study some examples to show the existence and completeness of the wave operators $W_\pm(T, T_0), T = T_0 + V$, under suitable conditions on T_0 and V. The main example study an operator considered a generalization of the fractional Laplacian T in the differential expression $T = \left(\sum_{\alpha\le m} a_\alpha(x) D^\alpha\right)^{\frac{s}{m}}, 0 \preceq s \preceq m$ on R^n with L_2-domain is equal to $W_{2,s} = \Lambda^{-s}(L_2)$. Hence $\Lambda^{-s} = (I - \Delta)^{\frac{s}{2}}$. We apply trace class theory to prove the existence and completeness of $W_\pm$ under mild conditions.

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Spectral Theory of Time Dispersive and Dissipative Systems

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I will discuss linear time dispersive and dissipative systems. Very often such systems are not conservative and the standard spectral theory can not be applied. I will describe a mathematically consistent framework, based on the classical theorem of Bochner, allowing (i) to constructively determine if a given time dispersive system can be extended to a conservative one; (ii) to construct that very conservative system (which is essentially unique). Time permitting, I will illustrate the method by applying it to the scattering of light off a dispersive, absorbing scatterer.

Weak disorder expansion for localization lengths of quasi-1D systems

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A perturbative formula for the lowest Lyapunov exponent of an Anderson model on a strip is presented. It is expressed in terms of an energy dependent doubly stochastic matrix, the size of which is proportional to the strip width. This matrix and the resulting perturbative expression for the Lyapunov exponent are evaluated numerically. Dependence on energy, strip width and disorder strength are thoroughly compared with the results obtained by the standard transfer matrix method. Good agreement is found for all energies in the band of the free operator and this even for quite large values of the disorder strength.

Spectral portraits and pseudospectrum for the Orr–Sommerfeld equation and associated model problem

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We will remark that the well-known Orr–Sommerfeld equation with large Reynolds numbers is closely related to a model problem

$$-i\varepsilon z'' + q(x)z = \lambda z,$$
$$z(-1) = z(1) = 0$$

with small $\varepsilon > 0$. We investigate the spectrum behaviour and estimate the resolvent growth of this problem (localization of the pseudospectrum) as $\varepsilon \to 0$. We prove that for some classes of analytic profiles $q(x)$ the spectrum of the problem is located near some critical curves in the complex plane $\mathbb{C}$ as $\varepsilon \to 0$. We find the quantization formulas for the eigenvalue distribution near critical curves. We consider also important particular profiles $q(x) = x^k$, $k \in \mathbb{N}$.

Periodic magnetic Schrodinger operator with degenerate lower edge of the spectrum

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We investigate the structure of the lower edge of the spectrum of the periodic magnetic Schrodinger operator. It is known that in the nonmagnetic case the energy is a quadratic form of the quasimomentum in the neighbourhood of the lower edge of the spectrum of the operator. We construct an example of the magnetic Schrodinger operator for which the energy is partially degenerated with respect to one component of the quasimomentum.

Absence of Accumulation Points in the Pure Point Spectrum of Jacobi Matrices

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Consider the sequences $\{b_n\}_{n=1}^\infty \subset \mathbb{R}_+$ and $\{q_n\}_{n=1}^\infty \subset \mathbb{R}$. A Jacobi matrix $\bar{J}$, regarded as an operator in $l^2(\mathbb{N})$, is the closure of a symmetric operator J whose domain is the set of all sequences $\{u_n\}_{n=1}^\infty$ having a finite number of non-zero elements and defined by

$$(Ju)_n = b_{n-1}u_{n-1} + q_n u_n + b_n u_{n+1} \quad n \in \mathbb{N} \setminus \{1\}$$
$$(Ju)_1 = q_1 u_1 + b_1 u_2 \,.$$

Ever since the subordinacy theory of Gilbert and Pearson [**1**] was carried over into the discrete domain [**3**], various asymptotic methods have been used to study the spectral properties of Jacobi matrices. However, the spectral analysis given by the theory of subordinacy leaves unanswered the question of whether the pure point parts of the spectrum have points of accumulation in finite intervals. For answering this question we developed a method whose ingredients are, firstly, a uniform generalization of the Levinson theorem on the asymptotics of solutions of discrete linear systems, secondly, the smooth properties of solutions of Levinson type systems, and finally, some basic properties of symmetric operators. The method has a wide range of applicability. As an illustration of this we prove that, for operator $\bar{J}$, the pure point part of the spectrum is actually discrete, when either of the following is satisfied:

(1) $b_n = n^a + c_n$ and $q_n = 0$, $\forall n \in \mathbb{N}$, where $a \in (0;1]$ and $\{c_n\}_{n=1}^\infty$ is a T-periodic positive sequence ($T \geq 2$).
(2) $b_n = n^a(1 + \frac{c_n}{n})$ and $q_n = 0$, $\forall n \in \mathbb{N}$, where $a > 1$, $\{c_n\}_{n=1}^\infty$ is a $2N$-periodic positive sequence ($N \in \mathbb{N}$), and J is self-adjoint.

In [**2**] it was established that operators $\bar{J}$-1 and $\bar{J}$-2 have pure point spectrum in some intervals, but the question of discreteness remained open.

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Generic singular continuous spectrum for geometric disorder

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Geometric disorder is defined with the help of Delone sets. We show that under some mild restrictions the associated Hamiltonians exhibit purely singular continuous spectrum generically.

The main analytical input is a result of Simon known as the "Wonderland theorem". Here, we provide an alternative approach and actually a slight strengthening by showing that various sets of measures defined by regularity properties are generic in the set of all measures on a locally compact metric space.

Scattering Theory for Jacobi Operators with Quasi-Periodic Background

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Scattering theory for Jacobi operators is by now well-understood in the case of a constant background operator. The case of periodic respectively quasi periodic background operators has only recently been started. We will investigate inverse scattering theory which is particulary important for application of the inverse scattering transform to the Toda equation.

Analysis of decoherence in a two-particle system

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We consider a two-particle system in the limit of small mass ratio, with an initial state given in a product form and the light particle in a scattering state. We derive the asymptotic dynamics of the system with an explicit control of the error and we discuss an application to the analysis of the decoherence effect induced on the heavy particle.

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Functional model for operators with spectrum on a curve and its applications

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We extend results of the paper A.S. Tikhonov "Functional model and duality of spectral components for operators with continuous spectrum on a curve" (St.Petersburg Math.J, Vol 14 (2003), No.4, 655–682) to the case of multiply connected domains. A main result is the existence of functional model of Sz.-Nagy-Foias-Naboko type for an arbitrary trace class perturbation of normal operator with spectrum on a curve. The functional model framework is applied to establish the duality of spectral components of such operators. In particular, we consider the duality of the absolutely continuous and singular spectral components which is a generalization of the corresponding well-known decomposition for self-adjoint operators and the canonic $C_0. - C_1.$ triangulation for contractions.

Remarks on the spectrum of the Neumann problem with magnetic field in the half space

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We consider a Schrödinger operator with a constant magnetic field in a half 3-dimensional space, with Neumann type boundary conditions. It is known from the works by Lu-Pan and Helffer-Morame that the lower bound of its spectrum is less than the intensity b of the magnetic field, provided that the magnetic field is not normal to the boundary.

We prove that the spectrum under b is a finite set of eigenvalues (each of infinite multiplicity).

In the case when the angle between the magnetic field and the boundary is small, we give a sharp asymptotic expansion of the number of these eigenvalues.

Generalized quadratic forms and robust states in complex linear spaces

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Let $\mathfrak{X}$ be a complex linear space, $\overline{\overline{\mathbb{R}}}\colon = \mathbb{R} \cup \{-\infty, +\infty, \nu\}$ — the set of real numbers extended with symbols of infinities and symbol ν that means indeterminate quantity. We naturally extend operations of addition and multiplication to $\overline{\overline{\mathbb{R}}}$ (at that they remain commutative and associative). Further the notion of limit of network of real numbers generalized to $\overline{\overline{\mathbb{R}}}$.

Let $\iota\colon \mathfrak{X} \to \overline{\overline{\mathbb{R}}}$ be an arbitrary fixed function. The couple $\langle \mathfrak{X}, \iota \rangle$ we call *state space. Domain of finiteness* is the set $Fin(\mathfrak{X}, \iota)\colon = \{x \in \mathfrak{X} \mid \iota(x) \in \mathbb{R}\}$. We will be say that function $\iota\colon \mathfrak{X} \to \overline{\overline{\mathbb{R}}}$ *preserve finiteness with regard to scalar multiplication* if for all $x \in Fin(\mathfrak{X}, \iota)$ and $\alpha \in \mathcal{C}$ $\alpha x \in Fin(\mathfrak{X}, \iota)$. A state x from domain of finiteness $Fin(\mathfrak{X}, \iota)$ we call *robust* (with regard to ι) if for any $y \in Fin(\mathfrak{X}, \iota)$ $x + y \in Fin(\mathfrak{X}, \iota)$. The set of all such states we denote $RFin(\mathfrak{X}, \iota)$. $RFin(\mathfrak{X}, \iota)$ is linear space.

Function $\iota\colon \mathfrak{X} \to \overline{\overline{\mathbb{R}}}$ that satisfies conditions: 1) $\iota(x + y) + \iota(x - y) = 2\iota(x) + 2\iota(y)$ $(x, y, x + y \in Fin(\mathfrak{X}, \iota))$ (*the rule of parallelogram*); 2)$\iota(\alpha x) = |\alpha|^2 \iota(x)$ $(\alpha \in \mathbb{C}, x \in \mathfrak{X})$ (*quadratic homogeneity*) we call *generalized quadratic form*. Using well-known *polarization identity* for ι we turn $RFin(\mathfrak{X}, \iota)$ to inner product space.

In different physical theories space $\langle \mathfrak{X}, \iota \rangle$ with generalized quadratic form ι means the space of states $x \in \mathfrak{X}$ where ι play a role of either certain energetic form or certain probabilistic characteristic of physical quantity for some physical system being in the state x. Presence of set $\{-\infty, +\infty, \nu\}$ in codomain of ι is related with such natural fact that characteristics indicated above may be infinite or may be indeterminate at all for some states. Note, that, for example, in the case of interpretation of ι as energetic form robustness of state x means that no perturbation can lead to a state with infinite energy or indeterminate state.

We consider different examples of spaces with generalized quadratic form.

Lieb-Thirring inequalities in Quantum wires

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We discuss Lieb-Thirring type bounds on trapped modes in strip- und tube-like domains, perturbed by potentials, geometric deformations or local changes of boundary conditions. This is a joint work with P. Exner and in part also with H. Linde.

Algebras dense in L^2 spaces

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Let μ be a finite positive Borel measure defined on a σ-algebra of subsets of a set $\mathbf{X}$. Using operator techniques we provide some criteria for finitely generated algebras to be dense in the space $L^2(\mu)$. The main tools are symmetric and selfadjoint operators of multiplication and Nussbaum's criterion for essential selfadjointness.

On generalized sum rules for Jacobi matrices

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The work is in a stream initiated by a paper of Killip and Simon [*Ann. of Math.*, 2003]. Using methods of Functional Analysis and the classical Szegö Theorem we prove sum rule identities in a very general form. Then, we apply the result to obtain new asymptotics for orthonormal polynomials. We reformulate Simon's conjecture, formulated initially for orthogonal polynomials on the unit circle, for polynomials orthogonal on an interval of the real axis and show that the conjecture holds true on an infinite series of examples that were proposed by Laptev, Naboko, Safronov.

New results in control of discrete, continuous and band spectra of Shrödinger equation in inverse problem and supersymmetry approach

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In recent years we have achieved a breakthrough in understanding quantum mechanics due to new approach: using inverse problem theory and supersymmetry with computer visualization (spectral scattering and decay control, particularly for periodic structures). We have just published our new book "Submissive Quantum Mechanics: New Status of the Theory in inverse problem approach" in Russian and put it and its first part in English into my homepage http://thsun1.jinr.ru/ zakharev/ for free access. The new theory reveals the elementary and universal constituents ("bricks" and building blocks) for construction (at least theoretically) of quantum systems with the given properties (spectral control) "as with a children toy constructor set". The fundamental depth of the discovered algorithms is combined with the extremely clear presentation. Our recent achievement is the theory of spectral control of periodic structures (shifting spectral zones, changing the degree of forbiddenness at arbitrary energy point in forbidden zone – intensity of tunneling through the zone interval. We have even understood how to solve Schroedinger equation 'mentally'). A resonance mechanism (unknown before) of forbidden zone formation will be higlighted. That enables also to understand better (in a qualitative way) the wave behavior in the allowed bands (beatings of waves as a result of violation of a conformity between wave internodal length and the potential period). This deepens the Bloch's theory of waves in periodical structures. As an paradoxical examples will be also demonstrated: 1.Dense spectrum of localized states and discrete spectrum of scattering states for Shrödinger equation. 2. Multichannel systems having simultaneously for the same interaction matrix and at the same energy value (but different boundary conditions): a) bound state solutions, b) solutions with absolute transparency of the target, c) solutions with absolute reflectivity of the target . Such simple instructive example enrich our quantum intuition.

Spectral properties of pseudorelativistic hamiltonians of atoms and positive ions with nuclei of finite masses

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We study the discrete spectrum structure of pseudorelativistic hamiltonians H of atoms and positive ions with nuclei of finite masses and with n electrons for any $n \geq 1$. Since the separation of the center of mass of such systems is impossible we investigate the spectrum of the restriction H_p of the operator H to the subspace of the states with fixed value P of the total momentum of the system. Such restriction forms the hamiltonian H_p of the relative motion (with fixed P). For the operators H_p we discovered:

A) Two-sides estimates of counting function of the discrete spectrum $\sigma_d(H_p)$ of the operator H_p in the terms counting functions of some effective two-particle operators;

B) The principal term of spectral asymptotics of $\sigma_d(H_p)$ near the lower bound $\inf \sigma_{ess}(H_p)$ of essential spectrum H_p.

It is interesting that the effective operators in A) and spectral asymptotics in B) depend on some integral properties of ground states of the operators, which determine $\inf \sigma_{ess}(H_p)$.

These results were established with application some approaches from [2, 3] and use some assertions from [1, 4, 5]. Early the discrete spectrum structure of the considered systems was discovered only for $n = 1$ [2]. For any $n \geq 1$ the discrete spectrum was studied solely for the case of infinitely heavy masses [6, 3], where fixing the total momentum was not done, since it was not necessary and where the operators were considerably more single for the investigation a compared with H_p.

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On the asymptotic behaviour of the number of eigenvalues for a class of weakly regular elliptic operators

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Let us assume that for $h > 0$ the differential operators $A_h = a(h, x, hD)$ are self-adjoint in the Hilbert space $L2(\mathbf{R}^d)$ and $a(h, x, \xi) = \sum h^n a_n(x, \xi)$ holds with sufficiently regular a_n $(n = 0, 1, ...)$. If the real number E satisfies the inequality

$$E < \liminf_{|x|+|\xi|\to\infty} a_0(x, \xi),$$

then there exists $h_0 > 0$ such that the spectrum of A_h is discrete in $]-\infty;\ E]$ for $h \in]0;\ h_0]$ and we can define the counting function $\mathcal{N}(A_h, E)$ as the number of eigenvalues smaller than E (counted with their multiplicities). We are interested in the semiclassical approximation of $\mathcal{N}(A_h, E)$ given by the Weyl formula

$$\mathcal{N}(A_h, E) = c_E h^{-d} + W_E(h), \qquad \text{(WF)}$$

where c_E is a constant (depending on E and a_0) and the remainder $W_E(h)$ is $o(h^{-d})$ when $h \to 0$.
It is well known that a development of the microlocal analysis based on the theory of Fourier integral operators of L. Hörmander, gives (WF) with $W_E(h) = O(h^{1-d})$ under the hypothesis

$$a_0(x, \xi) = E \implies \nabla a_0(x, \xi) \neq 0. \qquad \text{(NC)}$$

The aim of the talk is to discuss the following problems:
a) to estimate $W_E(h)$ when the condition (NC) is violated,
b) to describe estimates of $W_E(h)$ depending on the regularity of coefficients for operators with non-smooth coefficients,
c) to deduce classical Weyl formula for globally elliptic operators in $\mathbf{R}^d$.

Sum Rules for Jacobi Matrices and Divergent Lieb-Thirring Sums

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Let E_j be the eigenvalues outside $[-2, 2]$, and μ' the density of the a.c. part of the spectral measure for the vector δ_1, of a Jacobi matrix with $a_n - 1 \in \ell^2$ and $b_n \to 0$. We show that if $b_n \notin \ell^4$, $b_{n+1} - b_n \in \ell^2$, then

$$\sum_j (|E_j| - 2)^{5/2} = \infty,$$

and if $b_n \in \ell^4$, $b_{n+1} - b_n \notin \ell^2$, then

$$\int_{-2}^{2} \ln(\mu'(x))(4 - x^2)^{3/2}\, dx = -\infty.$$

We also show that if $a_n - 1, b_n \in \ell^3$, then the above integral is finite if and only if $a_{n+1} - a_n, b_{n+1} - b_n \in \ell^2$. We prove these and other results by deriving sum rules in which the a.c. part of the spectral measure and the eigenvalues appear on opposite sides.

Abstracts not included into the original book of abstracts

Kaczmarz algorithm in Hilbert space

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Let $\{e_n\}_{n=0}^{\infty}$ be a sequence of unit vectors in Hilbert space $\mathcal{H}$. We are interested in reconstructing vectors x from the sequence of the coefficients $\{(x, e_n)\}_{n=0}^{\infty}$. The Kaczmarz algorithm provides approximate solutions x_n according to the formulas $x_{-1} = 0$ and

$$x_n = x_{n-1} + (x - x_{n-1}, e_n)e_n.$$

Kaczmarz [1] proved that if $\mathcal{H}$ has finite dimension N, the sequence $\{e_n\}_{n=0}^{\infty}$ is N-periodic and linearly dense then $x_n \to x$, for any $x \in \mathcal{H}$.

We will discuss the Kaczmarz algorithm for infinite dimensional Hilbert spaces and give necessary and sufficient conditions for the sequence $\{e_n\}_{n=0}^{\infty}$ for which the algorithm reconstructs vectors. Such sequences will be called *effective*. We will also provide sufficient conditions for sequences $\{e_n\}_{n=0}^{\infty}$ such that every truncated sequence $\{e_n\}_{n=k}^{\infty}$ is effective as well.

The work was inspired by a paper by S. Kwapień and J. Mycielski [2], who studied stationary sequences of unit vectors, when the quantities (e_i, e_j) depend only on the difference of the indices. We will recover their result from ours, which will give entirely different approach to that case.

References

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